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Engineering Mechanics





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Chapter One: Introduction to Engineering Mechanics

1.1 General introduction

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies that are subjected to the action of forces. In general, this subject can be subdivided into three branches:

- Rigid-body mechanics (our focus).
- Deformable-body mechanics.
- Fluid mechanics.

Rigid body mechanics is divided into two areas:

I. Statics: deals with the equilibrium of bodies that are either at rest or move with a constant velocity. Examples are the weight of a body, a car resting on a bridge, and pushing a wall. See **Figure 1**.



Figure 1. Static load example

II. Dynamic: deals with the motion of bodies (accelerated motion). Cars that move on a bridge are a perfect example of dynamic loads.

1.2 Fundamental concepts

- **Length:** length is used to locate the position of a point in space and thereby describe the size of a physical system. Once a standard unit of length is defined, one can then use it to define distances and geometric properties of a body as multiples of this unit.
- **Time:** time is conceived as a succession of events. Although the principles of statics are time-independent, this quantity plays an important role in the study of dynamics.
- **Mass:** mass is a measure of a quantity of matter that is used to compare the action of one body with that of another
- **Force:** force represents the action of one body on another. characterized by its magnitude, direction of its action, and its point of application

1.3 Idealization

- **Particle:** a particle has a mass, but a size that can be neglected. For example, the size of the earth is insignificant compared to the size of its orbit, and therefore the earth can be modeled as a particle when studying its orbital motion. When a body is idealized as a particle, the principles of mechanics reduce to a rather simplified form since the geometry of the body will not be involved in the analysis of the problem.
- **Rigid body:** a rigid body can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after applying a load. This model is important because the body's shape does not change when a load is applied, and so we do not have to consider the type of material from which the body is made. In most cases, the actual deformations occurring in structures, machines, mechanisms, and the like are relatively small, and the rigid-body assumption is suitable for analysis.
- **Concentered force:** A concentrated force represents the effect of a loading which is assumed to act at a point on a body. We can represent a load by a concentrated force,

provided the area over which the load is applied is very small compared to the overall size of the body. An example would be the contact force between a wheel and the ground (see **Figure 2**).

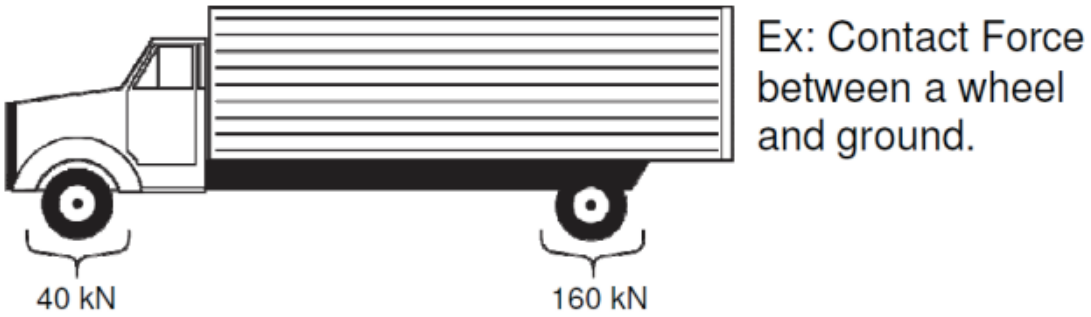


Figure 2. Concentrated load

1.4 Newton's laws (review)

- 1st law: an object at rest will remain at rest or an object in motion will remain in motion with the same speed and direction unless acted upon unbalanced force. In other words, objects continue to do what they are doing unless some external force acts on them.

$$\sum F = 0$$

- 2nd law: a force on a body is equal to its mass times its acceleration.

$$\sum F = m*a$$

- 3rd law: for every action, there is an equal but opposite direction (see **Figure 3**).

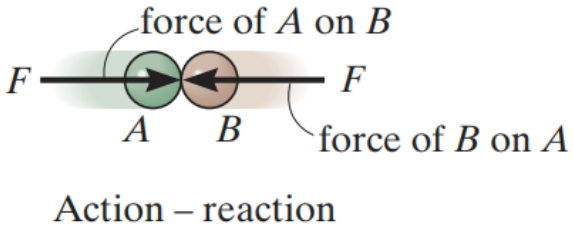


Figure 3. Illustration of Newton's 3rd law

1.5 Newton's law of gravitational

The weight of a body is required to be computed in Statics as well as Dynamics. If a particle is located at or near the surface of the earth, the only significant gravitational force is that between the earth and the particle.

$W = m * g$

Where,

$m = \text{mass of the body (kg)};$

$g = \text{acceleration due to gravity } (g = 9.81 \frac{m}{s^2});$

$W = \text{weight of the body}$

Therefore, a body of mass 1 kg has a weight of 9.81N

1.6 Scalar and vectors

- Scalar: a quantity described by magnitude alone. Mass (M), temperature (T), time (t), length (L) are examples of scalers.
- Vector: a quantity described by both magnitude and direction. Acceleration (a), velocity (v), force (F), and displacement (x, y, or z) – **will talk about vector representation in class**.

1.7 Units of measurements

See **Table 1**, **Table 2**, and **Table 3** for details

Table 1. System of units

Name	Length	Time	Mass	Force
International System of Units (SI)	Meter (m)	Second (s)	Kilogram (kg)	Newton (N)
US Customary (FPS)	Foot (ft)	Second (s)	Slug	Pound (lb)

Table 2. Conversion factors

Quantity	FBS	Equal	SI
Force	lb	=	4.448 N
Mass	Slug	=	14.59 kg
Length	ft	=	0.3048 m

1 foot = 12 inches.

1 minute = 60 seconds.

1 hour = 60 minutes.

1 day = 24 hours.

Table 3. Prefixes

Multiple	Exponential Form	Prefix	SI Symbol
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

1.8 Trigonometry review

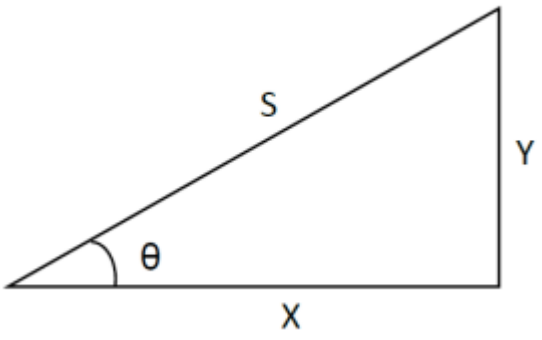
(you have studied this in high school 😉)

$$\sin\theta = \frac{Y}{S};$$

$$\cos\theta = \frac{X}{S};$$

$$\tan\theta = \frac{Y}{X}$$

$S = \sqrt{X^2 + Y^2}$ – Pythagorean Theorem



1.9 Example 1

Convert 2 km/h to m/s How many ft/s is this?

SOLUTION

Since 1 km = 1000 m and 1 h = 3600 s, the factors of conversion are arranged in the following order, so that a cancellation of the units can be applied:

$$2\ \text{km/h} = \frac{2\ \cancel{\text{km}}}{\cancel{\text{h}}}\left(\frac{1000\ \text{m}}{\cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right)$$

$$= \frac{2000\ \text{m}}{3600\ \text{s}} = 0.556\ \text{m/s} \qquad \textit{Ans.}$$

From Table 1–2, 1 ft = 0.3048 m. Thus,

$$0.556\ \text{m/s} = \left(\frac{0.556\ \cancel{\text{m}}}{\text{s}}\right)\left(\frac{1\ \text{ft}}{0.3048\ \cancel{\text{m}}}\right)$$

$$= 1.82\ \text{ft/s} \qquad \textit{Ans.}$$

NOTE: Remember to round off the final answer to three significant figures.

1.10 Example 2

Convert the quantities 300 lb · s and 52 slug/ft^3 to appropriate SI units.

SOLUTION

Using Table 1–2, 1 lb = 4.448 N.

$$300\ \text{lb}\cdot\text{s} = 300\ \cancel{\text{lb}}\cdot\text{s}\left(\frac{4.448\ \text{N}}{1\ \cancel{\text{lb}}}\right)$$

$$= 1334.5\ \text{N}\cdot\text{s} = 1.33\ \text{kN}\cdot\text{s} \qquad \textit{Ans.}$$

Since 1 slug = 14.59 kg and 1 ft = 0.3048 m, then

$$52\ \text{slug/ft}^3 = \frac{52\ \cancel{\text{slug}}}{\cancel{\text{ft}}^3}\left(\frac{14.59\ \text{kg}}{1\ \cancel{\text{slug}}}\right)\left(\frac{1\ \cancel{\text{ft}}}{0.3048\ \text{m}}\right)^3$$

$$= 26.8(10^3)\ \text{kg/m}^3$$

$$= 26.8\ \text{Mg/m}^3 \qquad \textit{Ans.}$$

1.11 Example 3

Evaluate each of the following and express with SI units having an appropriate prefix: (a) (50 mN)(6 GN), (b) (400 mm)(0.6 MN)^2, (c) 45 MN^3/900 Gg.

SOLUTION

First convert each number to base units, perform the indicated operations, then choose an appropriate prefix.

Part (a)

$$\begin{aligned}(50\text{ mN})(6\text{ GN}) &= \left[50(10^{-3})\text{ N}\right]\left[6(10^{9})\text{ N}\right]\\ &= 300(10^{6})\text{ N}^2\\ &= 300(10^{6})\,\text{N}^2\left(\frac{1\text{ kN}}{10^{3}\,\text{N}}\right)\left(\frac{1\text{ kN}}{10^{3}\,\text{N}}\right)\\ &= 300\text{ kN}^2 \qquad \textit{Ans.}\end{aligned}$$

NOTE: Keep in mind the convention $\text{kN}^2 = (\text{kN})^2 = 10^6\text{ N}^2$.

Part (b)

$$\begin{aligned}(400\text{ mm})(0.6\text{ MN})^2 &= \left[400(10^{-3})\text{ m}\right]\left[0.6(10^{6})\text{ N}\right]^2\\ &= \left[400(10^{-3})\text{ m}\right]\left[0.36(10^{12})\text{ N}^2\right]\\ &= 144(10^{9})\text{ m}\cdot\text{N}^2\\ &= 144\text{ Gm}\cdot\text{N}^2 \qquad \textit{Ans.}\end{aligned}$$

We can also write

$$\begin{aligned}144(10^{9})\text{ m}\cdot\text{N}^2 &= 144(10^{9})\text{ m}\cdot\text{N}^2\left(\frac{1\text{ MN}}{10^{6}\,\text{N}}\right)\left(\frac{1\text{ MN}}{10^{6}\,\text{N}}\right)\\ &= 0.144\text{ m}\cdot\text{MN}^2 \qquad \textit{Ans.}\end{aligned}$$

Part (c)

$$\begin{aligned}\frac{45\text{ MN}^3}{900\text{ Gg}} &= \frac{45(10^{6}\text{ N})^3}{900(10^{6})\text{ kg}}\\ &= 50(10^{9})\text{ N}^3/\text{kg}\\ &= 50(10^{9})\,\text{N}^3\left(\frac{1\text{ kN}}{10^{3}\,\text{N}}\right)^3\frac{1}{\text{kg}}\\ &= 50\text{ kN}^3/\text{kg} \qquad \textit{Ans.}\end{aligned}$$

1.12 Example 4

A concrete column has a diameter of 350 mm and a height of 2 m. If the density (mass/volume) of concrete 2400 kg/m^3. Determine the weight of the column in pounds.

Solution

$$V = \frac{\pi}{4}(d)^2 * h = \frac{\pi}{4}\left(\frac{350}{1000}\right)^2 * 2 = 0.192 m^3$$

$$\rho = \frac{m}{V} \rightarrow m = \rho * V = \frac{2400 kg}{m^3} * 0.192 m^3 = 460.8 kg$$

$$W = m * g = 460.8 kg * 9.81 \frac{m}{s^2} = 4520.448 N = 4.52 x 10^3 N$$

$$W(\text{in pounds}) = 4.52 x 10^3 N * \left(\frac{1 lb}{4.448}\right) = 1016.187 lb = 1.016 kip$$

1.13 Example 5

What is the weight in newtons of an object that has a mass of (a) 8 kg, (b) 0.04 kg, and (c) 760 Mg?

Solution

We will do it together in class.

Chapter Two: Co-planer System of Forces

2.1 What is force?

- Force is an action that has the ability to change motion.
- Force is a vector quantity.
- Force acting of an object may cause the object to change its shape, to start moving, to stop moving, to accelerate, or decrease.

For example, take a look at the bracket shown in **Figure 4**; the effect of the force applied to the bracket depends on P, the angle θ, and the location of the point of application. If any of these specifications will change, it alters the effect on the bracket. So magnitude, direction, sense, and point of application are characteristics of the force.

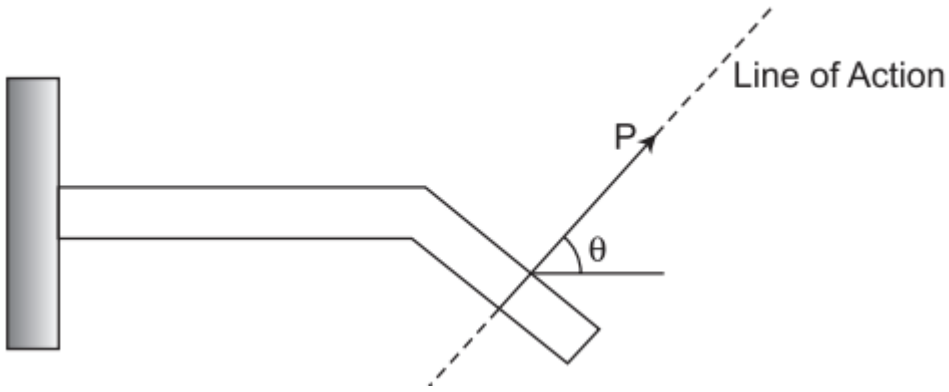


Figure 4. Force acting on a bracket

Therefore, the force is characterized by the following:

- Magnitude.
- Direction (angle).
- Sense (how the force moves along that line).
- Point of application.

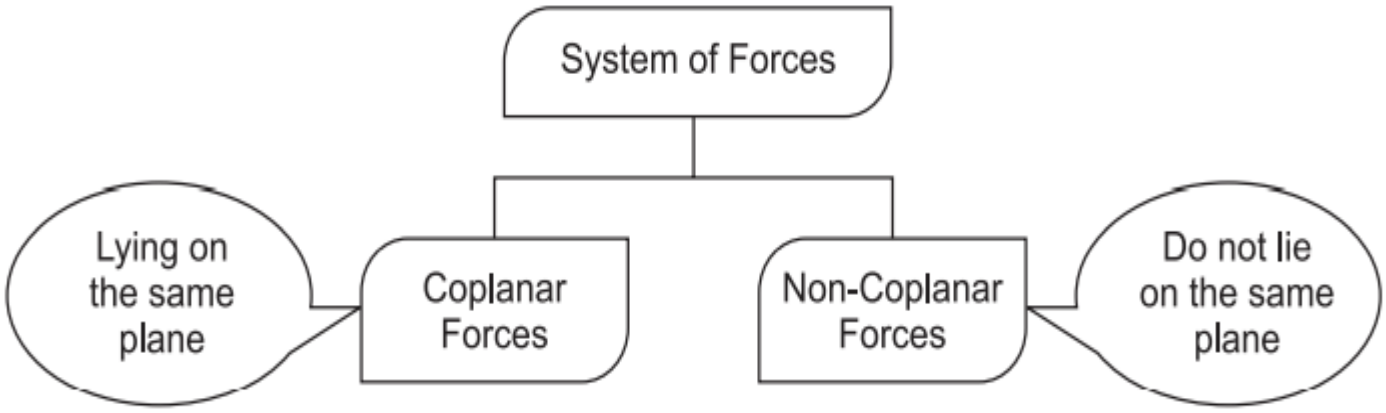
2.2 Classification of forces

- Contact force: direct physical contact between two objects; friction forces.
- Body force; wight.

Forces may be further classified as concentrated or distributed. Also, forces can be classified as internal and external forces.

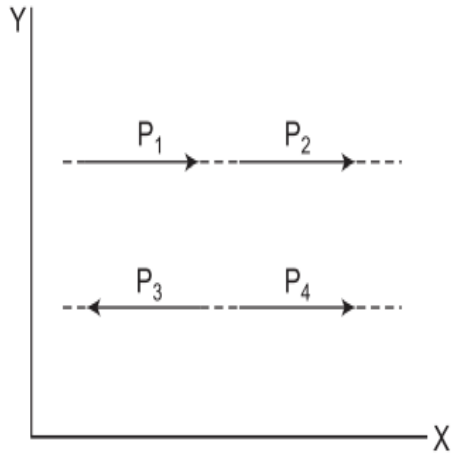
2.3 System of forces

When two or more forces act on a body, they are called to form a system of forces. Most practical problems of statics involve a system of force. System of forces is simply a term used to describe a group of forces. A system of forces may be classified as follows:

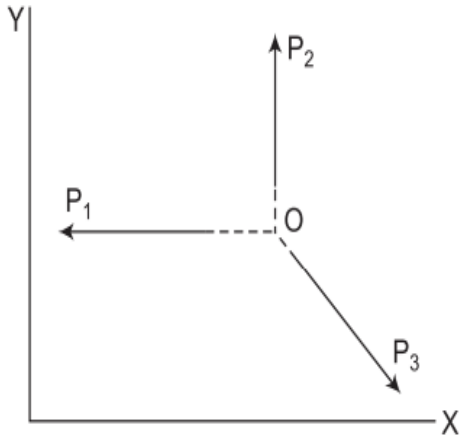


Coplanar forces are those forces whose lines of action lies on the same plane. Non-Coplanar forces may be concurrent, non-concurrent, and parallel. Non-coplanar forces are also known as space forces or spatial force systems, as forces pass-through space when they change their plane.

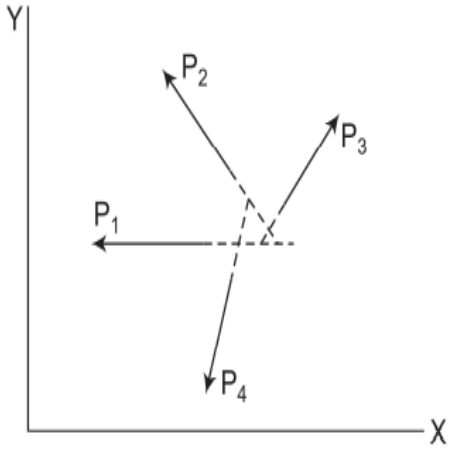
- **Collinear forces.**



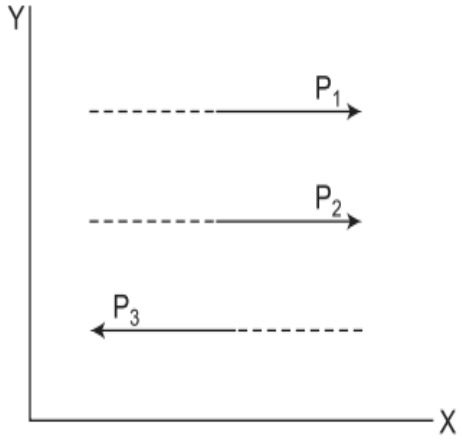
- **Coplanar Concurrent Forces**



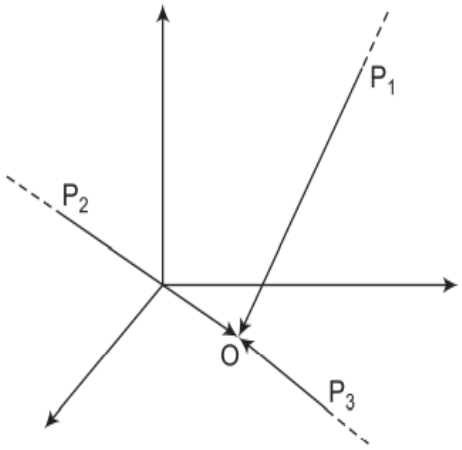
- **Coplanar non-concurrent Forces**



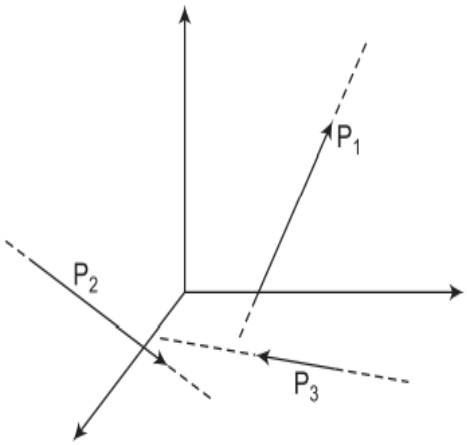
- **Coplanar parallel forces**



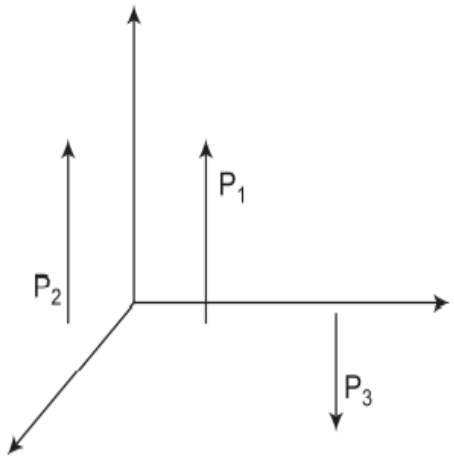
- **Non-Coplanar Concurrent Forces**



- **Non-Coplanar Non-Concurrent**



- **Non-Coplanar Parallel Forces**

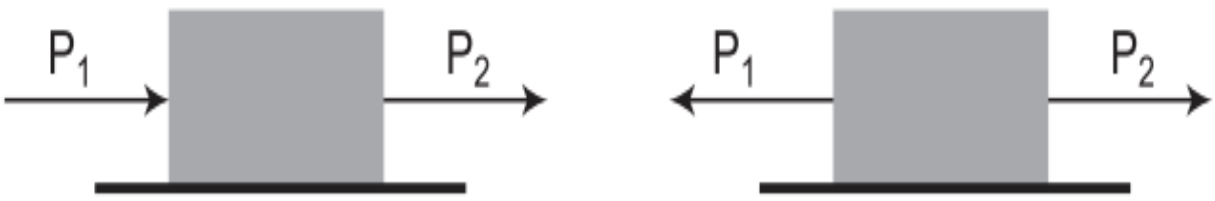


2.4 Resultant force

When a number of forces are acting on an object then it is possible to replace them with a single force that produces the same effect as produced by all the given forces, this single force is known as resultant force. The resultant force is denoted by **R**. The procedure to find out a single resultant force of a system of forces is known as the composition of forces.

I. Resultant of collinear forces

The resultant of two or more collinear forces is simply their algebraic sum and acts on the same line on which lines of action of all collinear forces lie.



$R = P1 + P2$

$R = P2 - P1$

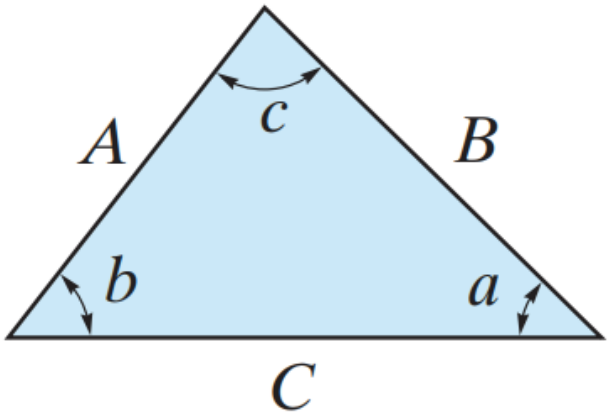
As a matter of convenience;

$\rightarrow (+)$

$\leftarrow (-)$

II. Resultant of concurrent forces

Law of Cosines: used to find the magnitude of the resultant.



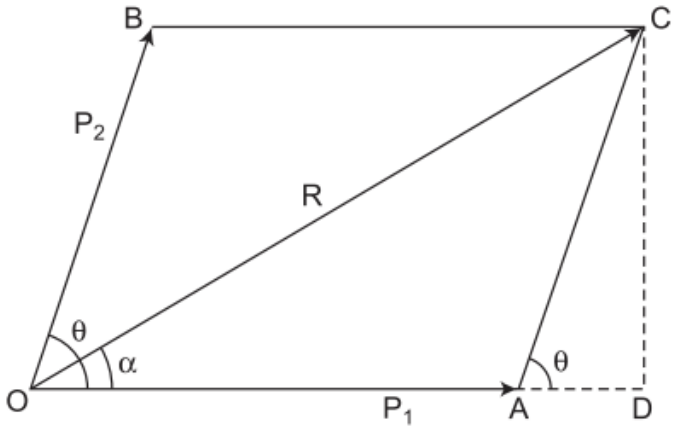
$C^2 = A^2 + B^2 - 2*A*B*\cos c$

$C = \sqrt{A^2 + B^2 - 2*A*B*\cos c}$

Law of Sines: used to find the direction of the resultant force.

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

If we want to prove the cosine law; we can 😊



in magnitude and direction by OA and OB respectively. By parallelogram law the resultant R is represented in magnitude and direction by OC.

We now find magnitude and direction of resultant as below:

Let, θ be the angle between two forces P_1 and P_2 and α is the angle between force P_1 and resultant R.

Now from C draw a perpendicular CD to OA produced.

Now $< DAC = < AOB = \theta$ …(Corresponding angle)

In ΔDAC, $CD = P_2 \sin\theta$ and $AD = P_2 \cos\theta$

In ΔDOC,

$$OC^2 = OD^2 + CD^2 = (OA + AD)^2 + CD^2$$

$$\therefore \quad R^2 = (P_1 + P_2 \cos\theta)^2 + (P_2 \sin\theta)^2$$

$$\therefore \quad R^2 = P_1^2 + 2P_1P_2 \cos\theta + P_2^2 \cos^2\theta + P_2^2 \sin^2\theta$$

$$\therefore \quad R^2 = P_1^2 + 2P_1P_2 \cos\theta + P_2^2 (\cos^2\theta + \sin^2\theta)$$

$$\therefore \; R = \sqrt{P_1^2 + P_2^2 + 2P_1P_2 \cos\theta} \; \text{... Magnitude of resultant} \qquad ...(1.1)$$

Now in ΔDOC,

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD}$$

$$\therefore \quad \tan\alpha = \frac{P_2 \sin\theta}{P_1 + P_2 \cos\theta} \; \text{... Direction of resultant} \qquad ...(1.2)$$

Particular Cases

1. If $\theta = 90°$, i.e. when the two forces act at right angle, then

   $R = \sqrt{P_1^2 + P_2^2}$ and $\tan\alpha = \dfrac{P_2}{P_1}$ (since $\cos 90° = 0$ and $\sin 90° = 1$)

2. If the two forces are equal, i.e. $P_1 = P_2$

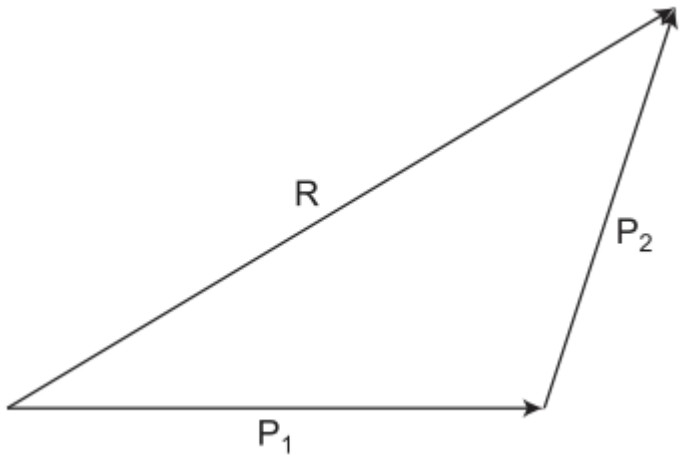
$$R = \sqrt{P_1^2 + P_1^2 + 2P_1^2\cos\theta} = \sqrt{2P_1^2(1+\cos\theta)}$$

$$= \sqrt{2P_1^2 \times 2\cos^2\frac{\theta}{2}} \qquad \left(\text{since } 1+\cos\theta = 2\cos^2\frac{\theta}{2}\right)$$

$$= \sqrt{4P_1^2\cos^2\frac{\theta}{2}}$$

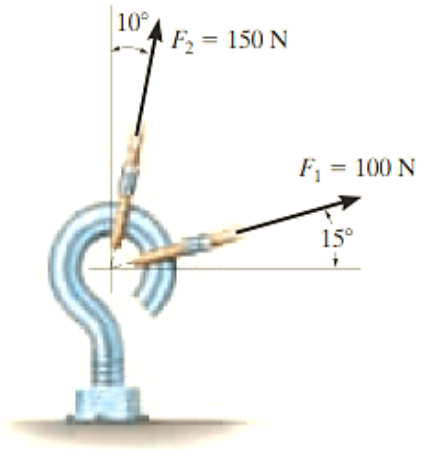
$$\therefore \qquad R = 2P_1\cos\frac{\theta}{2}$$

Triangle law of forces: If two forces acting simultaneously at a point be represented in magnitude and direction by two sides of a triangle taken in order, their resultant may be represented in magnitude and direction by the third side of the triangle taken in the opposite order.

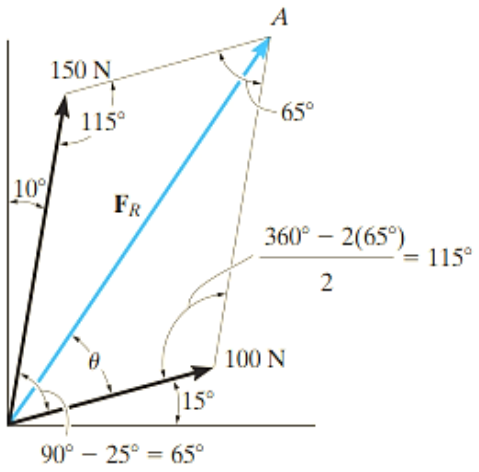


Example 1

The screw eye in Fig. 2–11*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



(a)



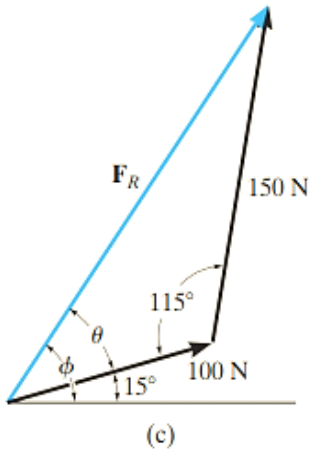
(b)

SOLUTION

Parallelogram Law. The parallelogram is formed by drawing a line from the head of $\mathbf{F}_1$ that is parallel to $\mathbf{F}_2$, and another line from the head of $\mathbf{F}_2$ that is parallel to $\mathbf{F}_1$. The resultant force $\mathbf{F}_R$ extends to where these lines intersect at point A, Fig. 2–11*b*. The two unknowns are the magnitude of $\mathbf{F}_R$ and the angle θ (theta).

Trigonometry. From the parallelogram, the vector triangle is constructed, Fig. 2–11*c*. Using the law of cosines

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115°}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N} \qquad \textit{Ans.}$$



(c)

Fig. 2–11

Applying the law of sines to determine θ,

$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115°} \qquad \sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(\sin 115°)$$
$$\theta = 39.8°$$

Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

$$\phi = 39.8° + 15.0° = 54.8° \qquad \textit{Ans.}$$

Example 2

Resolve the horizontal 600-lb force in Fig. 2–12*a* into components acting along the *u* and *v* axes and determine the magnitudes of these components.

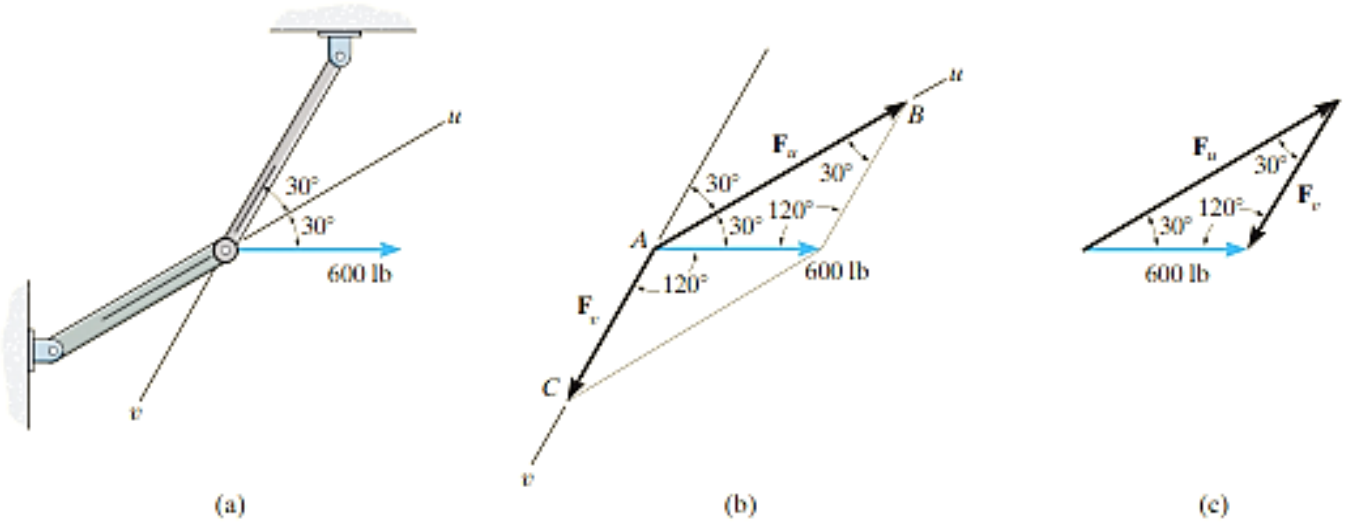


Fig. 2–12

SOLUTION

The parallelogram is constructed by extending a line from the *head* of the 600-lb force parallel to the *v* axis until it intersects the *u* axis at point *B*, Fig. 2–12*b*. The arrow from *A* to *B* represents $\mathbf{F}_u$. Similarly, the line extended from the head of the 600-lb force drawn parallel to the *u* axis intersects the *v* axis at point *C*, which gives $\mathbf{F}_v$.

The vector addition using the triangle rule is shown in Fig. 2–12*c*. The two unknowns are the magnitudes of $\mathbf{F}_u$ and $\mathbf{F}_v$. Applying the law of sines,

$$\frac{F_u}{\sin 120^\circ} = \frac{600\text{ lb}}{\sin 30^\circ}$$

$$F_u = 1039\text{ lb} \qquad \textit{Ans.}$$

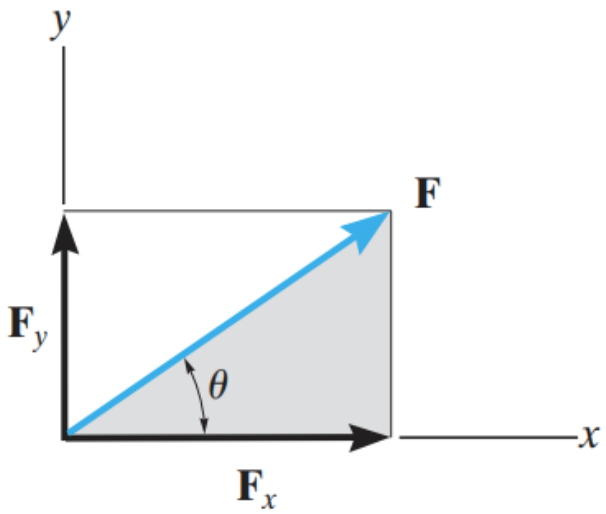
$$\frac{F_v}{\sin 30^\circ} = \frac{600\text{ lb}}{\sin 30^\circ}$$

$$F_v = 600\text{ lb} \qquad \textit{Ans.}$$

2.5 Rectangular components (Cartesian coordinates)

When a force is resolved into two components along the x and y axes, the components are then called

rectangular components.



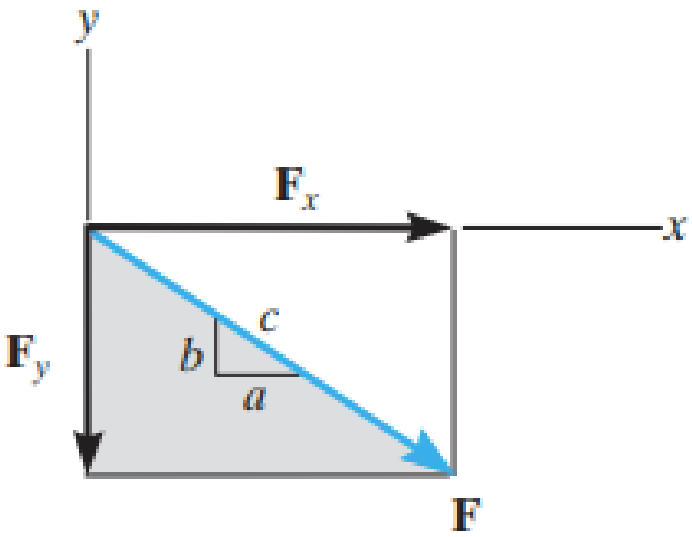
$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

$$F = \sqrt{F_x{}^2 + F_y{}^2}$$

$$\theta = \tan^{-1}\frac{F_y}{F_x}$$

Instead of using the angle u, however, the direction of "F" can also be defined using a small "slope" triangle, as in the example shown in the figure below.



$$\frac{F_x}{F} = \frac{a}{c}$$

$$F_x = F\left(\frac{a}{c}\right)$$

$$\frac{F_y}{F} = \frac{b}{c}$$

$$F_y = -F\left(\frac{b}{c}\right)$$

Here the "y" component is a negative scalar since F_y is directed along the negative y-axis.

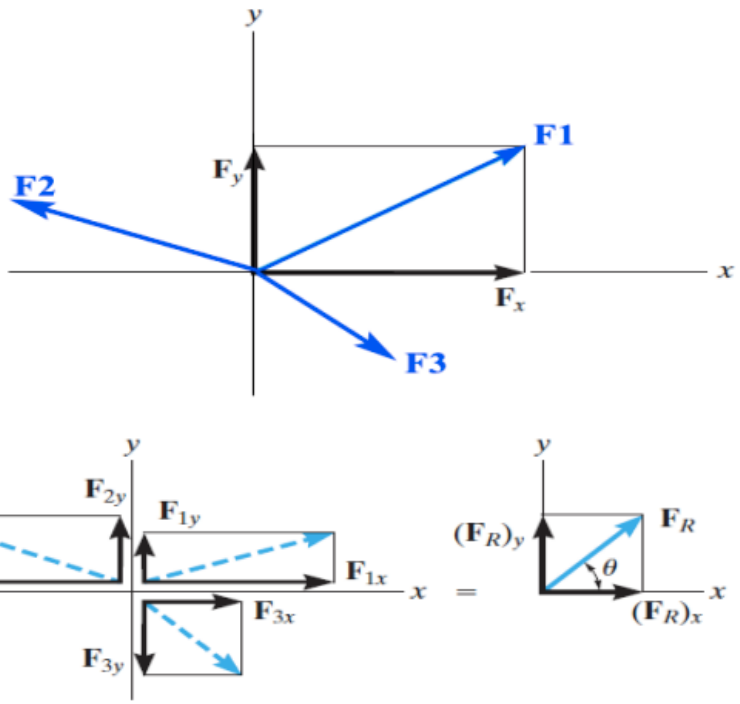
Note: we can represent the components of the resultant force of any number of coplanar forces symbolically by the algebraic sum of the x and y components of all the forces, i.e.,

$$\sum F_x = F_{1x} + F_{2x} + ... + F_{xn}$$

$$\sum F_y = F_{1y} + F_{2y} + ... + F_{yn}$$

$$F(Fr) = \sqrt{\sum F_x{}^2 + \sum F_y{}^2}$$

$$\theta = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right)$$



See Table 4 for a force resolution

Table 4. The resolution of a force into perpendicular components

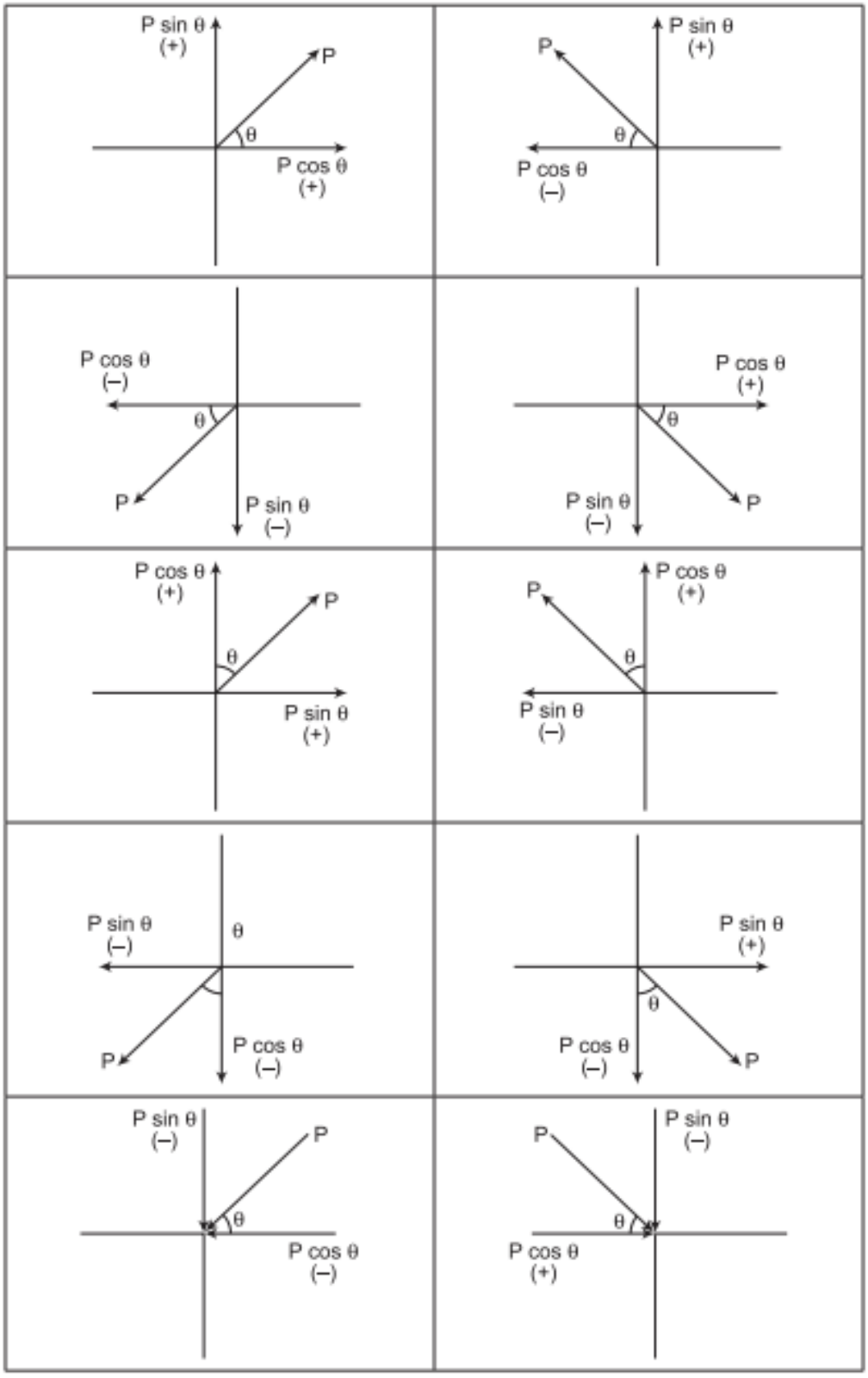


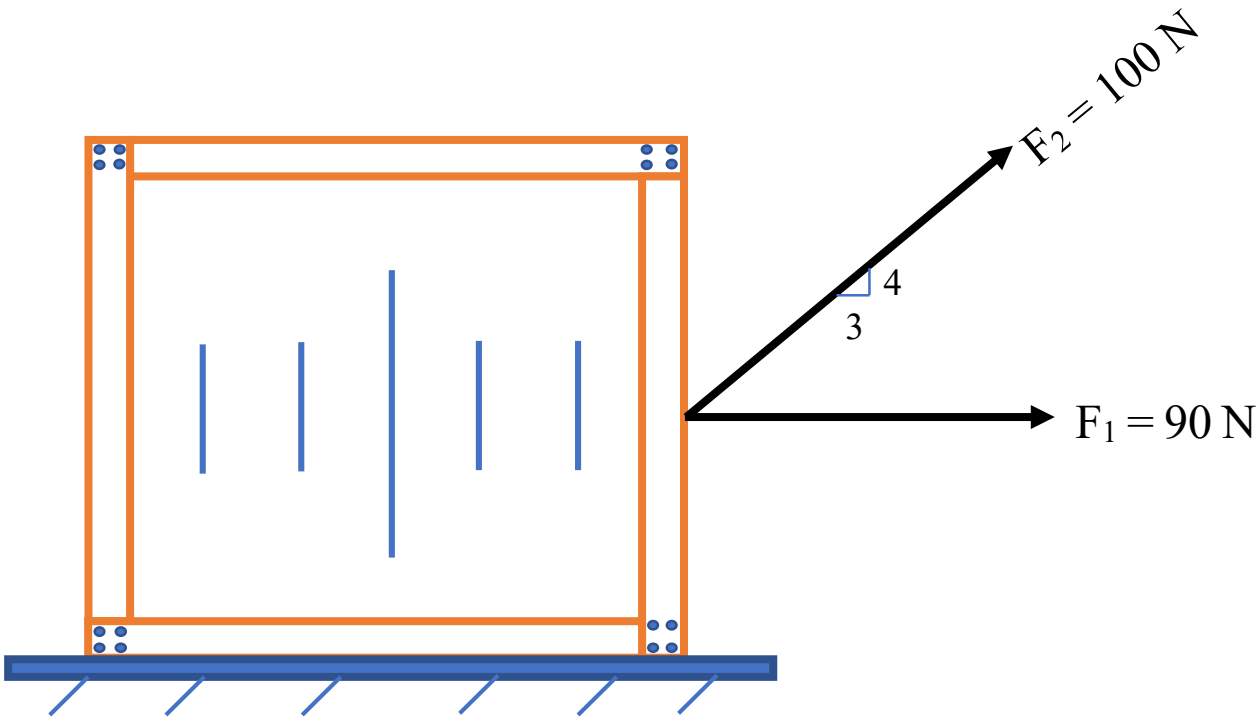
Table 4 (continued)



Example 3

Find the resultant force on the crate shown in the figure using:

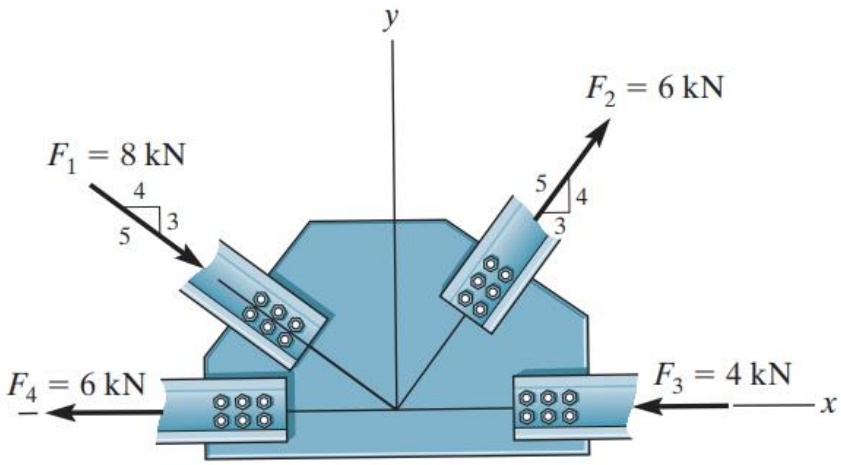
- The Triangle Rule.
- The Cartesian Coordinates.



Look at Examples 2.3, 2.4, and 2.6 on page 37 of the textbook. Also, you can try to solve the fundamental problems that start on page 28.

Example 4

Determine the x and y components of each force acting on the gusset plate of a bridge truss. Show that the resultant force is zero.



Solution

F1:

$$F1x = 8*\cos 36.87^{O} = 6.4kN \rightarrow orF1x = 8*\left(\frac{3}{5}\right) = 6.4kN(\rightarrow)$$

$$F1y = 8*\sin 36.87^{O} = 4.8kN(\downarrow)$$

F2:

$$F2x = 6*\cos 53.13^{O} = 3.6kN \rightarrow orF2x = 6*\left(\frac{3}{5}\right) = 3.6kN(\rightarrow)$$

$$F2y = 6*\sin 53.13^{O} = 4.8kN(\uparrow)$$

F3:

$$F3x = 4kN(\leftarrow)$$

$$F3y = 0kN$$

F4:

$$F4x = 6kN(\leftarrow)$$

$$F4y = 0kN$$

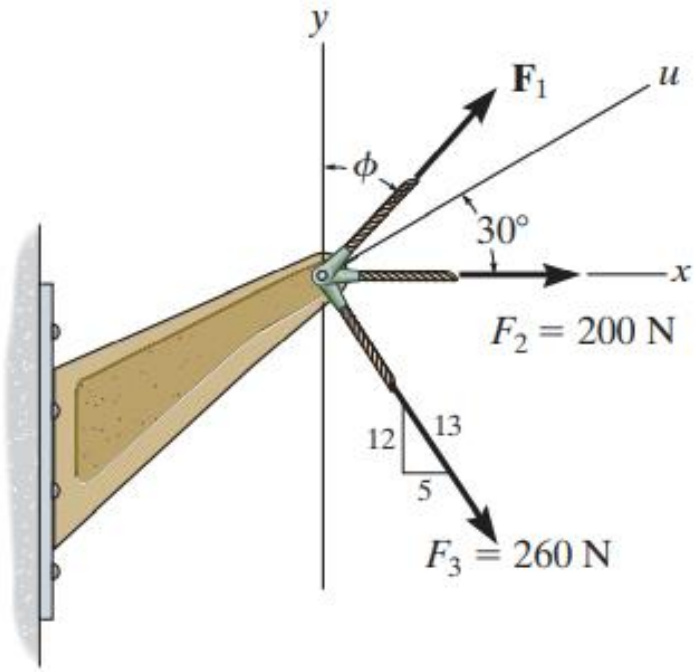
$$\sum Fx = 6.4 + 3.6 - 4 - 6 = 0kN$$

$$\sum Fy = -4.8 + 4.8 = 0kN \rightarrow Fr = \sqrt{Fx^2 + Fy^2} = \sqrt{0 + 0} = 0$$

Example 5

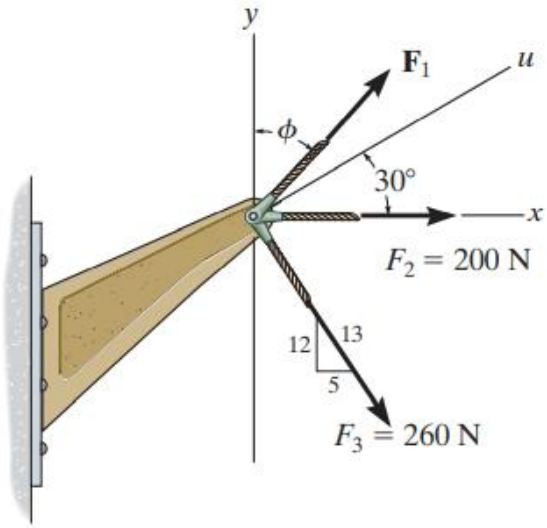
If the resultant force acting on the bracket is required to be a minimum, determine the magnitudes of F1and the resultant force. Set Φ = 30°. **We will solve it during the lecture. Deal?**





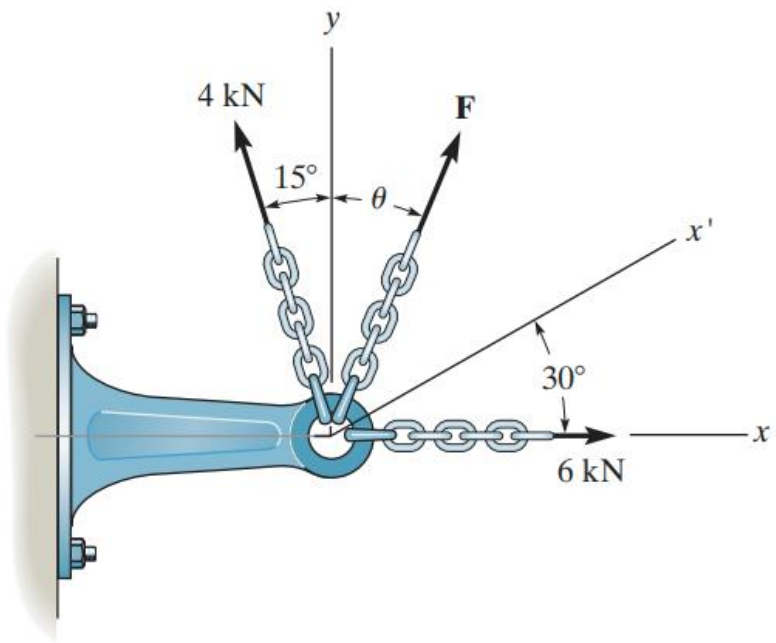
Assignment 1

If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive u axis, determine the magnitude of F1 and its direction Φ.



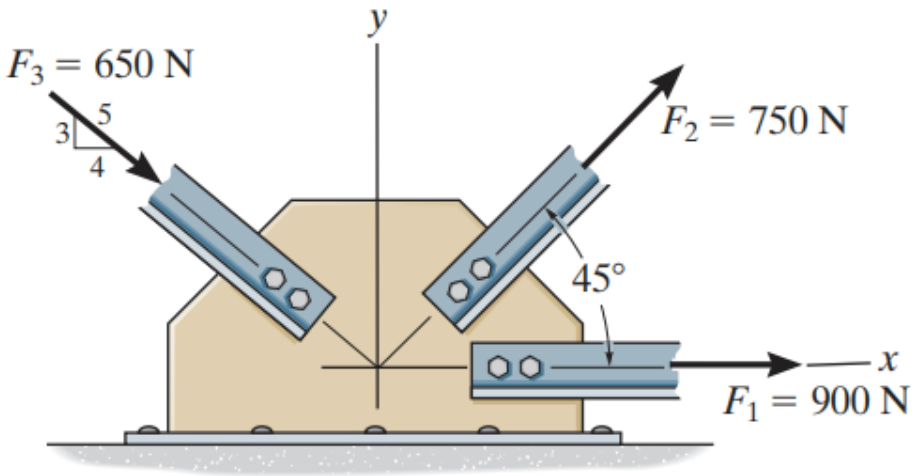
Assignment 2

If F = 5 kN and θ = 30°, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



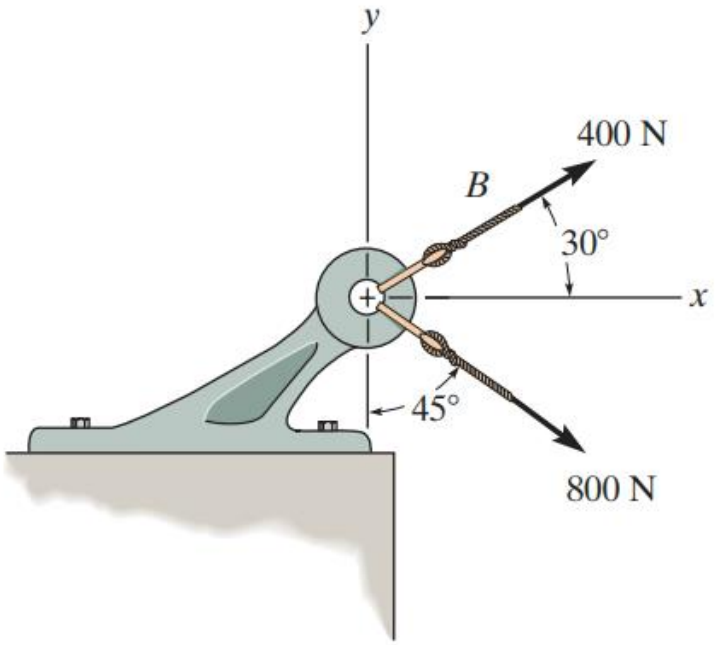
Assignment 3

Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x-axis.



Assignment 4

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x-axis.



2.6 Moment

When a force causes an object to turn, this turning effect is called moments. So the moment of a force is a measure of its tendency to cause a body to rotate around a specific point or axis. The moment is the product of the force multiplied by the perpendicular distance from the line of action of the force to the pivot or point where the object will turn. The point about which moment is taken is known as the moment center and the perpendicular distance is known as the moment arm, as shown in **Figure 5**. If moment arm d is zero, i.e. the line of action of the force P is passing through the moment center then the moment due to this force will be zero.

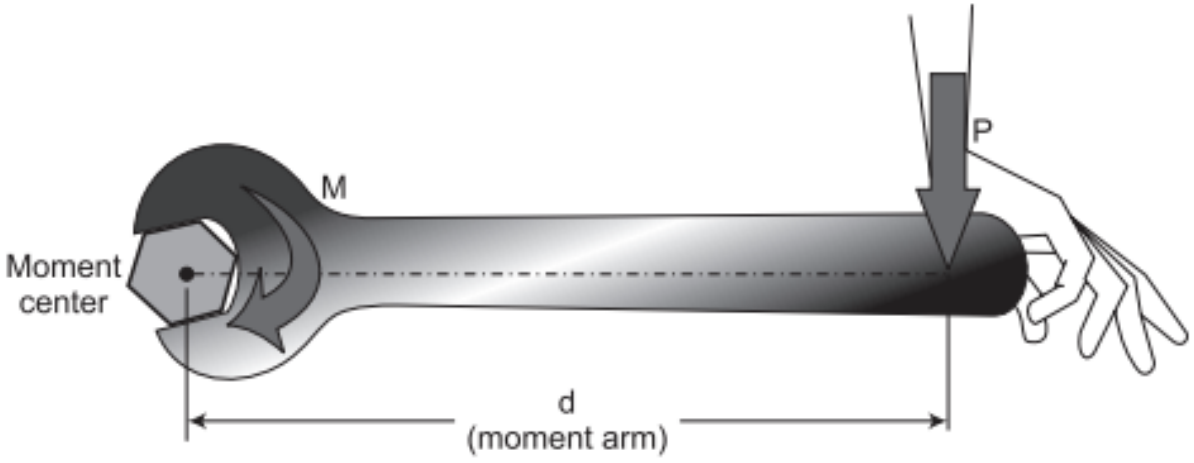


Figure 5. Moment example

Moment = Force * Distance

$$M = F*d$$

Since the moment is a product of force and perpendicular distance, its S.I. unit is N-m or KN-m. The direction of the rotation resulting from a moment is either clockwise or counterclockwise. When a force causes an object to turn in the clockwise direction, it is called the clockwise moment and when a force causes an object to turn in the counterclockwise direction, it is called the counterclockwise moment. Anticlockwise moments are regarded as positive, while clockwise moments are negative.

Counterclockwise moment (+)

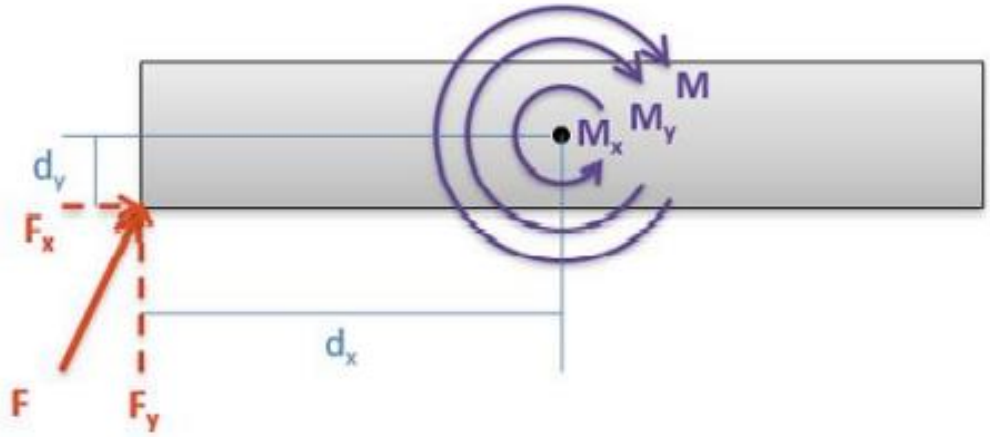


Clockwise moment (-)



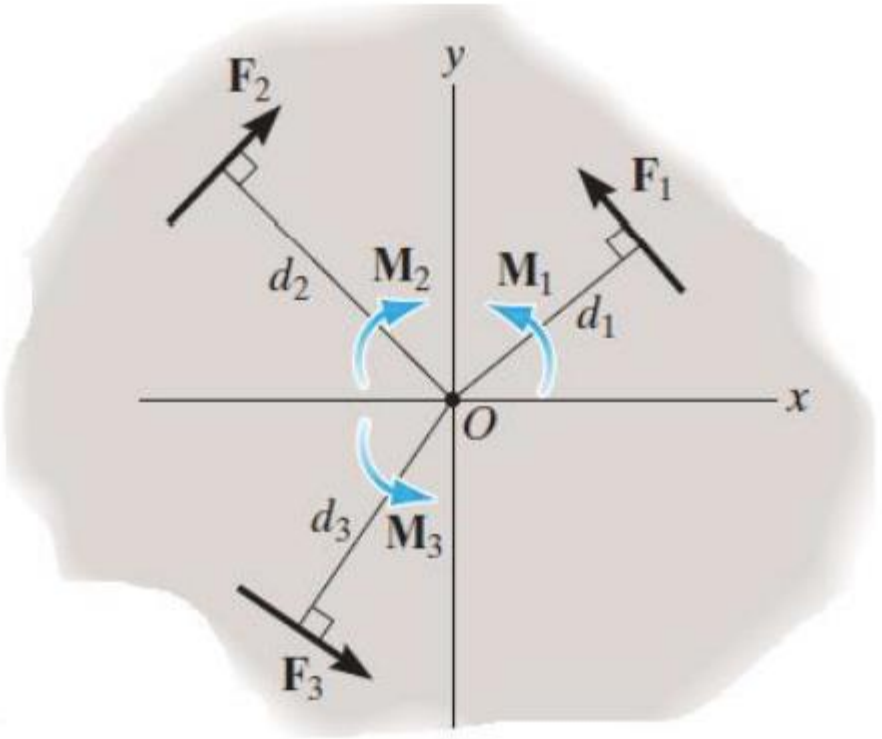
2.7 Varignon's Theorem

One of the most useful principles of mechanics is Varignon's theorem, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.



$$M = M_X + M_y = (F_X * d_y) + (F_y * d_X)$$

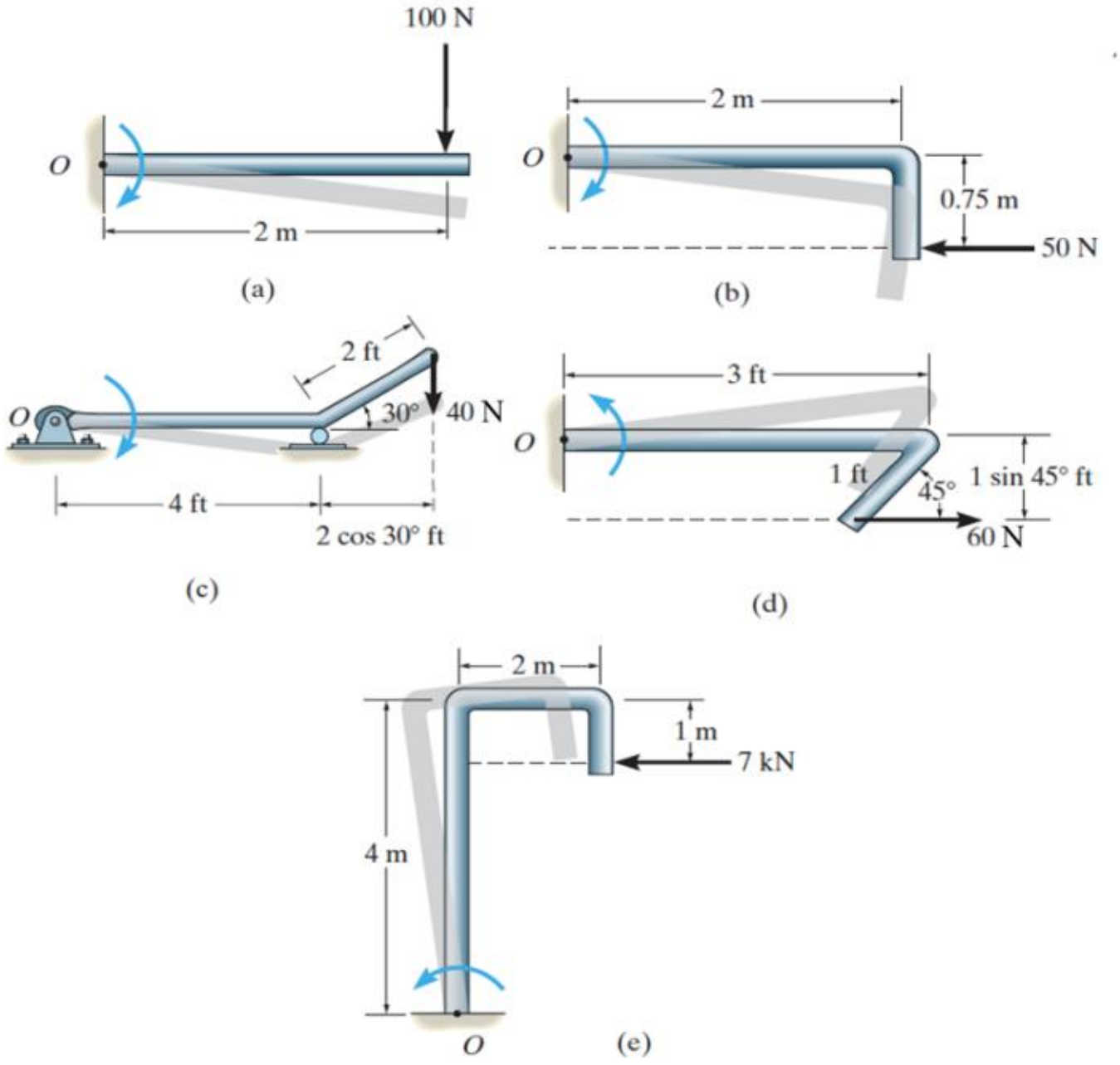
2.8 Resultant moment



$$(M_R)_O = F_1 * d_1 - F_2 * d_2 + F_3 * d_3$$

Example 1

For each case illustrated in the figures (a – e), determine the moment of the force about point O.



$$M_O = (100\ \text{N})(2\ \text{m}) = 200\ \text{N} \cdot \text{m}\ \circlearrowright$$

$$M_O = (50\ \text{N})(0.75\ \text{m}) = 37.5\ \text{N} \cdot \text{m}\ \circlearrowright$$

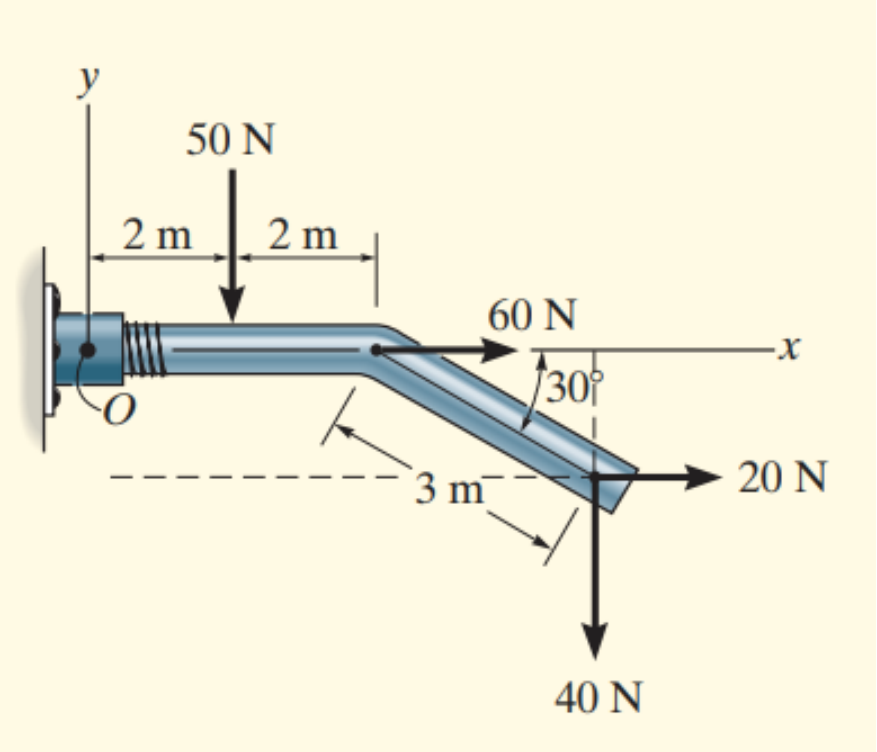
$$M_O = (40\ \text{lb})(4\ \text{ft} + 2\cos 30^\circ\ \text{ft}) = 229\ \text{lb} \cdot \text{ft}\ \circlearrowright$$

$$M_O = (60\ \text{lb})(1\sin 45^\circ\ \text{ft}) = 42.4\ \text{lb} \cdot \text{ft}\ \circlearrowleft$$

$$M_O = (7\ \text{kN})(4\ \text{m} - 1\ \text{m}) = 21.0\ \text{kN} \cdot \text{m}\ \circlearrowleft$$

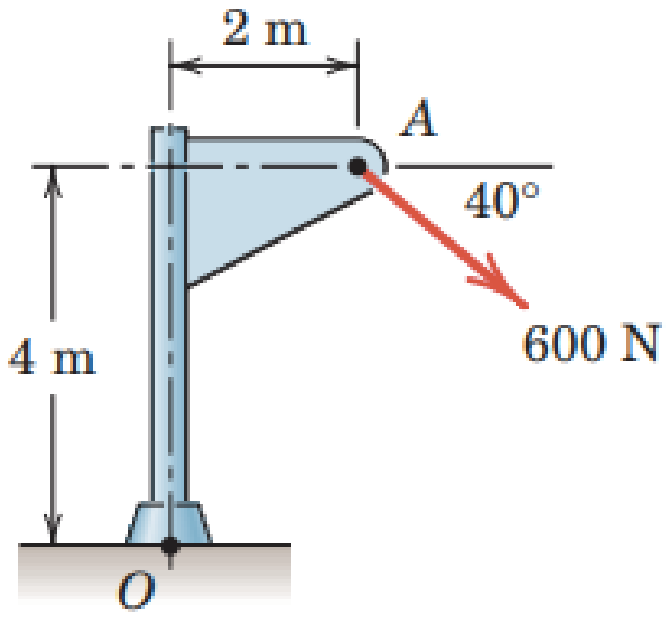
Example 2

Determine the resultant moment of the four forces acting on the rod shown in the figure below about point O. **Ans = 334 N-m (clockwise).**



Example 3

Calculate the magnitude of the moment about the base point O of the 600-N force. **Ans = 2610 N-m (clockwise).**



Example 4

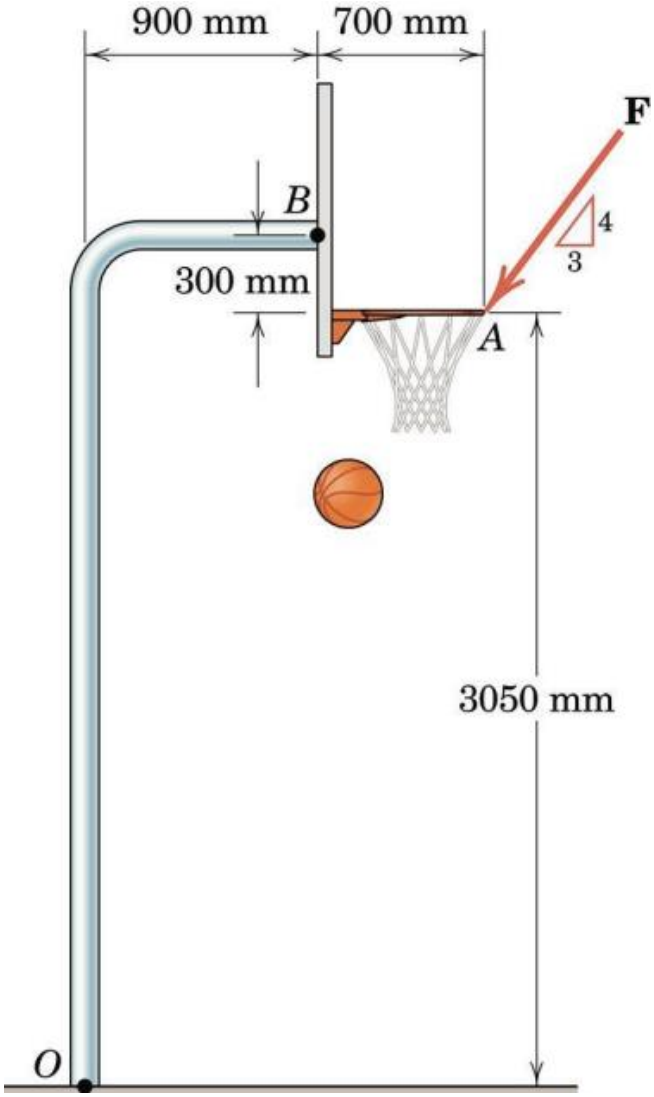
An experimental device imparts a force of magnitude F = 225 N to the front edge of the rim at A to simulate the effect of a slam dunk. Determine the moments of the force F about point O and point B. Finally, locate, from the base at O, a point C on the ground where the force imparts zero moment.

Ans

Mo = 123.75 N-m (counterclockwise)

MB = 166.5 N-m (clockwise)

d = 687.5 mm



2.9 Couple

A couple consists of a pair of two forces which has the following properties:

- Equal magnitude and opposite in direction.
- Act along parallel lines of action.
- Separated by a perpendicular distance "d" known as arm of couple. A couple is shown in **Figure 6**.

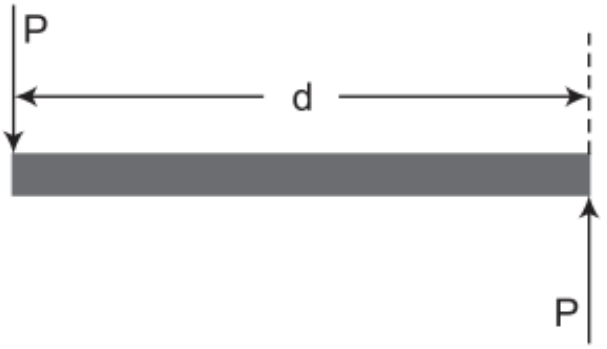


Figure 6. The representation of couple

2.10 Moment of a couple

Consider a couple as shown in **Figure 7**.

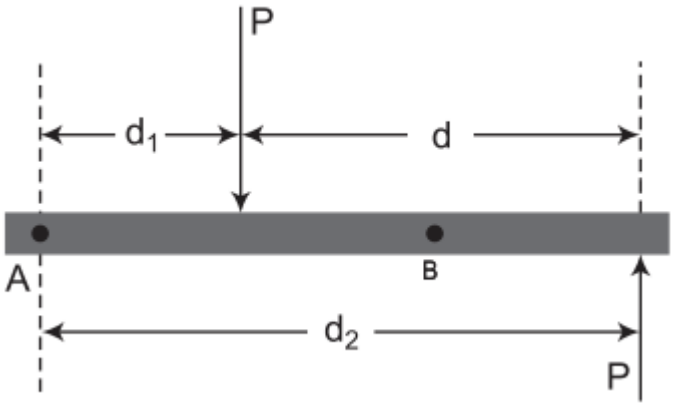


Figure 7. Mathematical representation of couple

The moment of couple about point A is:

$$M_A = P*d_2 - P*d_1$$

$$M_A = P\left(d_2 - d_1\right)$$

$$\because d_2 - d_1 = d$$

$$M_A = P*d$$

The moment of a couple is the product of magnitude of one of the forces and arm of couple.

Now consider point B at distance $\frac{d}{2}$ from force P, the moment of couple about point B is,

$$M_B = P*\frac{d}{2} + P*\frac{d}{2}$$

$$M_B = P\left(\frac{d}{2} + \frac{d}{2}\right)$$

$$M_B = P*d$$

From above we see that, the moment of couple about any pivot point is P * d. It is important to note here that a couple does not have moment center, like moment of force. A couple has the same moment about all points on a body.

2.11 Force-couple system

Refer to **Figure 8 (a)**, consider a force P is acting at point A and it is required to transfer from point A to point B without changing its magnitude and direction.

Now apply two forces of equal magnitude (their magnitude are same as that of force P acting at point A) and opposite in direction at point B, as shown in Figure **8 (b)**.

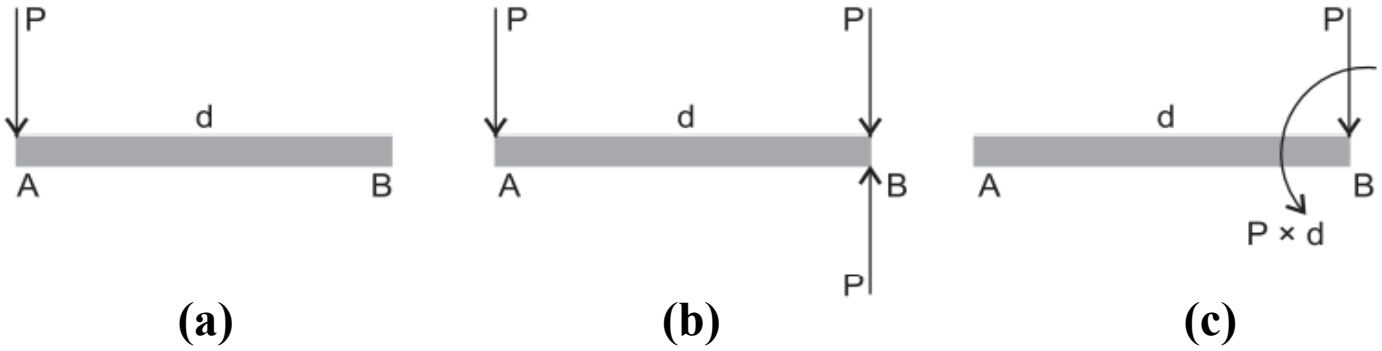


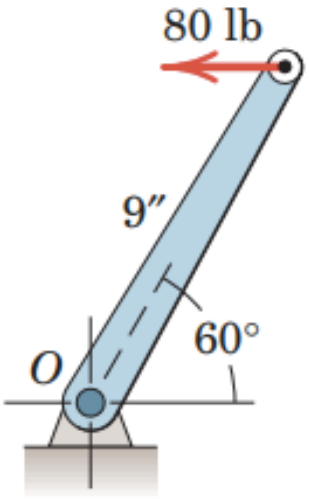
Figure 8. Force-couple system

Observing **Figure 8 (b)**, we see that two forces are acting in opposite direction at point A and B form a couple. Moment of this couple is P × d, counterclockwise. Thus, to shift a force to point B, a couple is required to be added to the system

This system which consisting of a force and a couple at point B, as shown in **Figure 8 (c)** is known as force-couple system. So we can transfer a force parallel to itself at any point with force-couple system.

Example 1

Replace the horizontal 80-lb force acting on the lever by an equivalent system consisting of a force at O and a couple. **Ans = 624 lb-in (counterclockwise).**



Example 2

Determine the resultant couple moment of the three couples acting on the plate in the figure below. **Ans = 950 lb-ft (clockwise).**

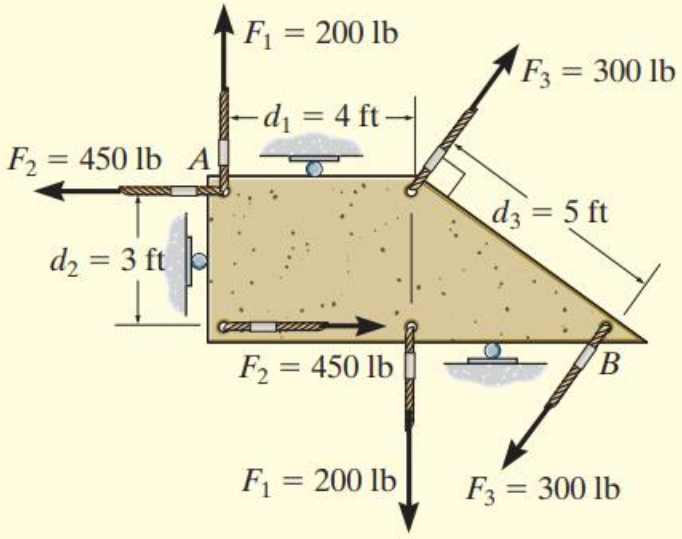


Fig. 4–30

SOLUTION

As shown the perpendicular distances between each pair of couple forces are $d_1 = 4$ ft, $d_2 = 3$ ft, and $d_3 = 5$ ft. Considering counterclockwise couple moments as positive, we have

$$\curvearrowleft + M_R = \Sigma M;\ M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$

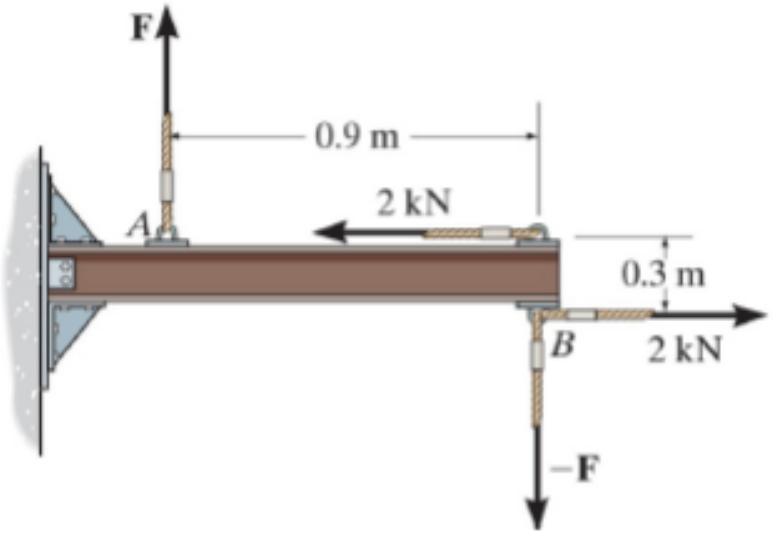
$$= -(200\text{ lb})(4\text{ ft}) + (450\text{ lb})(3\text{ ft}) - (300\text{ lb})(5\text{ ft})$$

$$= -950\text{ lb}\cdot\text{ft} = 950\text{ lb}\cdot\text{ft}\ \curvearrowright \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{M}_R$ has a clockwise rotational sense.

Example 3

Two couples act on the beam as shown in the figure. Find the magnitude of F so that the resultant couple moment is 1.5 kN-m (clockwise).



Solution

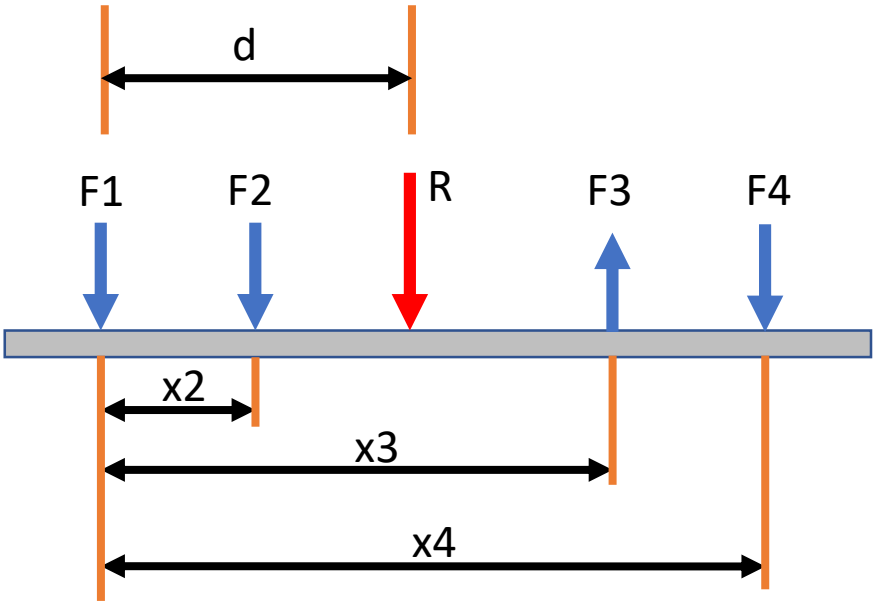
$M_R = -1.5 kN.m$

$M_R = -F*0.9 + 2*0.3$

$-1.5 = -F*0.9 + 0.6$

$\therefore F = 2.333 kN$

2.12 Resultant of Parallel Force System

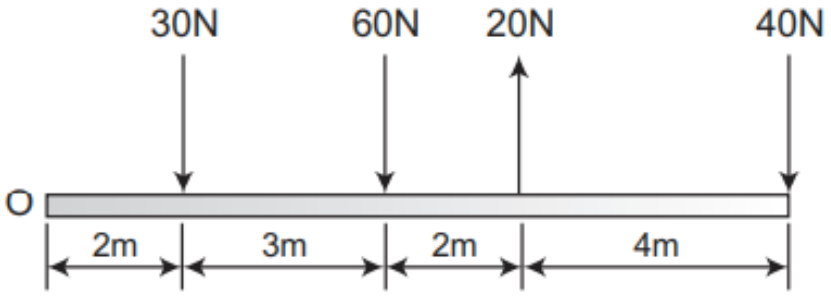


$$R = -F_1 - F_2 + F_3 - F_4$$

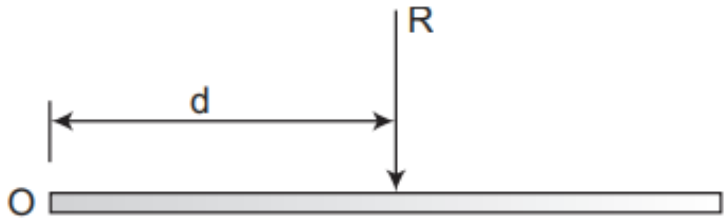
$$R * d = -F_2 * x_2 + F_3 * x_3 - F_4 * x_4$$

Example 1

Determine the magnitude of the resultant parallel force system shown in the figure below. Also, determine the position of the resultant force from point O.



Solution



$R = -30-60+20-40 = -110N$

To determine the location of the resultant, take moment about point "O"

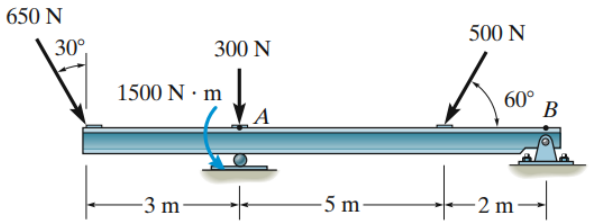
$-R*d = -30*2-60*5+20*7-40*11 \rightarrow$

$\therefore d = 6m$

2.13 Additional examples regarding the resultant force system

Example 1

Replace the loading system acting on the post by an equivalent resultant force and couple moment at point A.



SOLUTION

Equivalent Resultant Force And Couple Moment at Point *A*.

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 650 \sin 30° - 500 \cos 60° = 75 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = -650 \cos 30° - 300 - 500 \sin 60°$$

$$= -1295.93 \text{ N} = 1295.93 \text{ N} \downarrow$$

As indicated in Fig. *a*,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{75^2 + 1295.93^2} = 1298.10 \text{ N} = 1.30 \text{ kN} \quad \textbf{Ans.}$$

And

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{1295.93}{75}\right) = 86.69° = 86.7° \measuredangle \quad \textbf{Ans.}$$

Also,

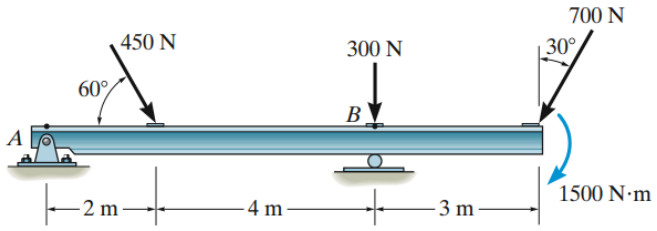
$$\circlearrowleft + (M_R)_A = \Sigma M_A; \quad (M_R)_A = 650 \cos 30° (3) + 1500 - 500 \sin 60° (5)$$

$$= 1023.69 \text{ N} \cdot \text{m}$$

$$= 1.02 \text{ kN} \cdot \text{m} \ (\textbf{counter clockwise}) \quad \textbf{Ans.}$$

Example 2

Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from end A.



SOLUTION

$$\stackrel{+}{\rightarrow} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 450\cos 60° - 700\sin 30° = -125\ \text{N} = 125\ \text{N} \leftarrow$$

$$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = -450\sin 60° - 700\cos 30° - 300 = -1296\ \text{N} = 1296\ \text{N} \downarrow$$

$$F = \sqrt{(-125)^2 + (-1296)^2} = 1302\ \text{N} \qquad \textbf{Ans.}$$

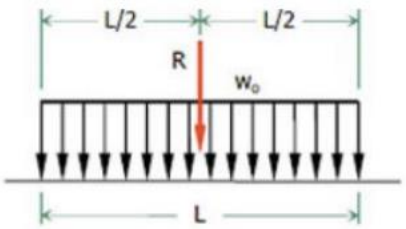
$$\theta = \tan^{-1}\left(\frac{1296}{125}\right) = 84.5° \qquad \textbf{Ans.}$$

$$\circlearrowleft + M_{RA} = \Sigma M_A; \qquad 1296(x) = 450\sin 60°(2) + 300(6) + 700\cos 30°(9) + 1500$$

$$x = 7.36\ \text{m} \qquad \textbf{Ans.}$$

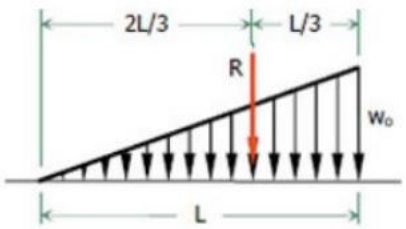
2.14 Resultant of distributed loads

The resultant of a distributed load is equal to the area of the load diagram. It is acting at the centroid of that area as indicated. The figure below shows the three common distributed loads namely; rectangular load, triangular load, and trapezoidal load.



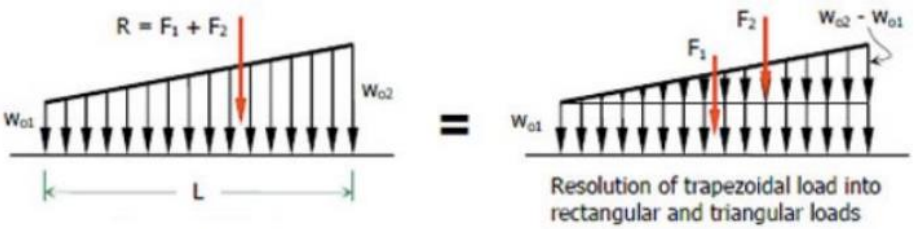
Rectangular Load

$R = w_o L$



Triangular Load

$R = \frac{1}{2} w_o L$

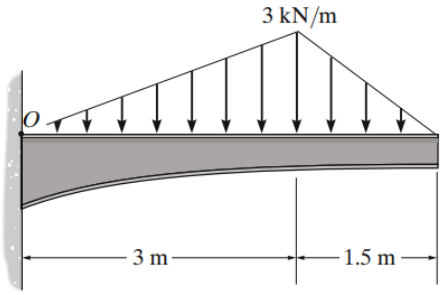


Trapezoidal Load

$$R = w_{o1}L + \tfrac{1}{2}(w_{o2} - w_{o1})L$$

Example 1

Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point O.



SOLUTION

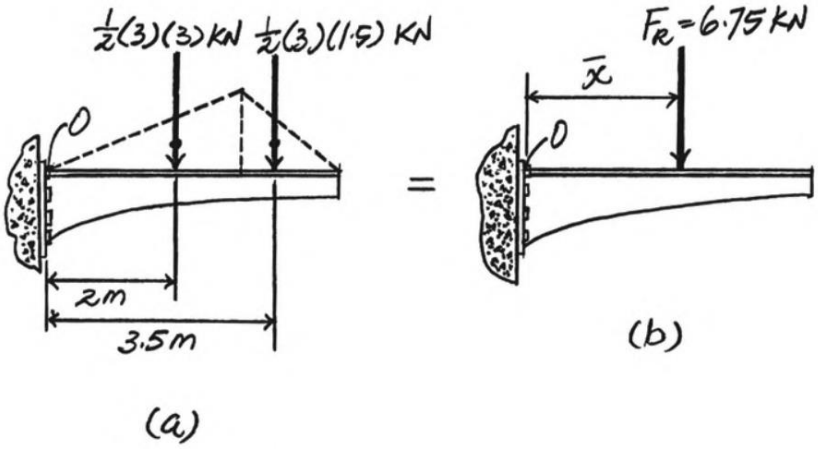
Loading: The distributed loading can be divided into two parts as shown in Fig. *a*.
Equations of Equilibrium: Equating the forces along the y axis of Figs. *a* and *b*, we have

$$+\downarrow F_R = \Sigma F; \qquad F_R = \frac{1}{2}(3)(3) + \frac{1}{2}(3)(1.5) = 6.75\ \text{kN} \downarrow \qquad \textbf{Ans.}$$

If we equate the moment of F_R, Fig. *b*, to the sum of the moment of the forces in Fig. *a* about point O, we have

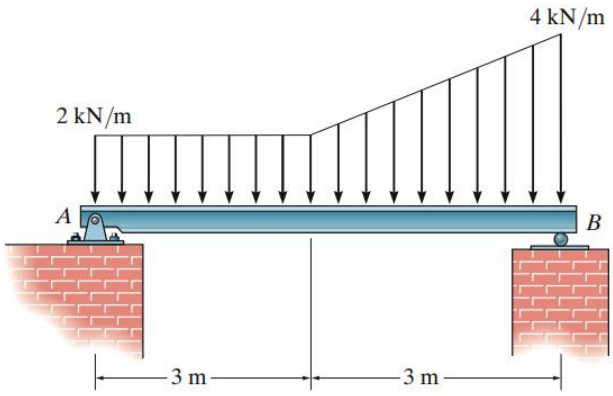
$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad -6.75(\bar{x}) = -\frac{1}{2}(3)(3)(2) - \frac{1}{2}(3)(1.5)(3.5)$$

$$\bar{x} = 2.5\ \text{m} \qquad \textbf{Ans.}$$



Example 2

Replace the distributed loading by an equivalent resultant force, and specify its location on the beam, measured from the pin at A.



SOLUTION

Equivalent Resultant Force. Summing the forces along the y axis by referring to Fig. a,

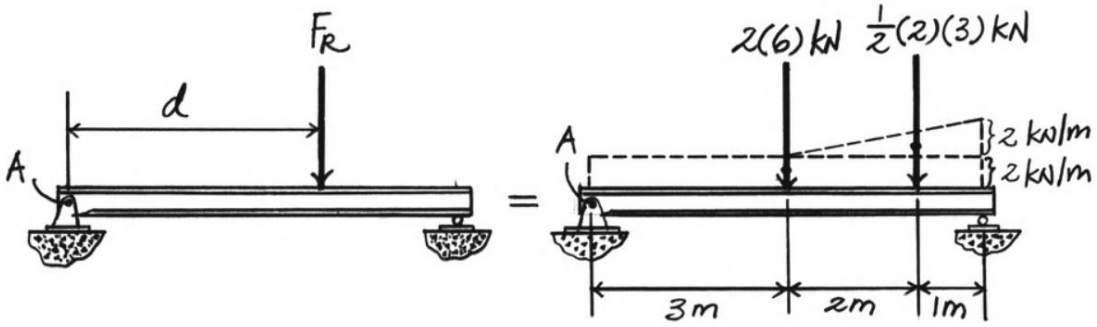
$$+\uparrow (F_R)_y = \Sigma F_y; \quad -F_R = -2(6) - \frac{1}{2}(2)(3)$$

$$F_R = 15.0 \text{ kN} \downarrow \qquad \textbf{Ans.}$$

Location of the Resultant Force. Summing the Moments about point A,

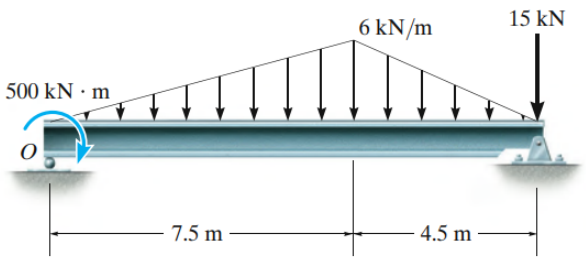
$$\circlearrowleft + (M_R)_A = \Sigma M_A; \quad -15.0(d) = -2(6)(3) - \frac{1}{2}(2)(3)(5)$$

$$d = 3.40 \text{ m} \qquad \textbf{Ans.}$$



Example 3

Replace the loading by an equivalent force and couple moment acting at point *O*.



SOLUTION

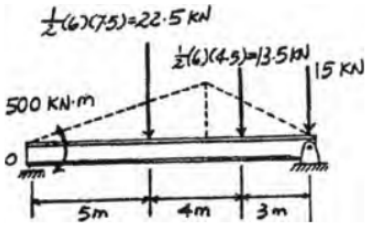
$+\uparrow F_R = \Sigma F_y;$ $\quad F_R = -22.5 - 13.5 - 15.0$

$= -51.0\ \text{kN} = 51.0\ \text{kN}\downarrow$ **Ans.**

$\zeta + M_{R_o} = \Sigma M_o;$ $\quad M_{R_o} = -500 - 22.5(5) - 13.5(9) - 15(12)$

$= -914\ \text{kN}\cdot\text{m}$

$= 914\ \text{kN}\cdot\text{m}$ ***(Clockwise)*** **Ans.**



Chapter Three: Equilibrium

3.1 General introduction

Equilibrium is the state of the rest of a body i.e. the body does not move and also does not rotate about any point under the action of forces. For a rigid body in static equilibrium, the external forces and moments are balanced and will impart no translational or rotational motion to the body; however, it could be translating or rotating at a constant velocity.

- **Rigid body:** a rigid body in a non-deformable body; the distance between any points in the body remains fixed.
- **Static equilibrium:** the state of the body in which the sum of the forces and moments acting on a body is equal to zero.

In other words, When an object is in equilibrium under the action of a system of forces, the following conditions are satisfied:

1. The algebraic sum of the components of the forces in any direction must be equal to zero.
2. The algebraic sum of the moments of the forces about any point must equal zero. **Table 5** states equilibrium conditions for different force systems.

Table 5. Equilibrium conditions

Force System	Equilibrium Condition
Collinear force system	$\Sigma P = 0$ $\therefore$ Resultant, R = 0
Concurrent force system	$\Sigma H = 0$ (sum of horizontal components) and $\Sigma V = 0$ (sum of vertical components) $\therefore$ Resultant, $R = 0$
Parallel force system	$\Sigma P = 0$ and $\Sigma M_O = 0$ (O is any point in the plane of the forces)
Non-concurrent force system	$\Sigma H = 0$ (sum of horizontal components) $\Sigma V = 0$ (sum of vertical components) and $\Sigma M_O = 0$ (O is any point in the plane of the forces)

3.2 Equilibrium equations

$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M_O = 0$

3.3 Free body diagram (FBD)

A free-body diagram of a body is a diagrammatic representation or a sketch of a body in which the body is shown completely separated from all surrounding bodies, including supports. The term free implies that all supports have been removed and replaced by the forces (reactions) that they exert on the body.

Forces that we show on a free body diagram can be divided into two categories:

(a) Reactive Forces: Reactive forces known as reactions are those forces that are exerted on a body by the supports to which it is attached. Reactions at supports and connections are of only two types:

- If support or connection prevents translation in some direction, then a force may be developed in that direction.
- If support or connection prevents rotation about some axis, then a moment may be developed around that axis.

(b) Applied forces: Forces acting on a body that is not provided by the supports are called applied forces.

Different supports and their reactions are shown in **Table 6**.

Table 6. Supports and their reactions

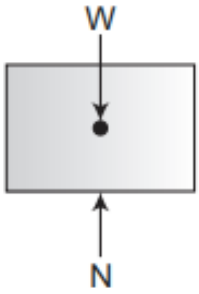
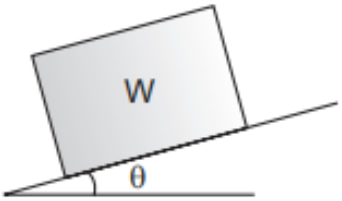
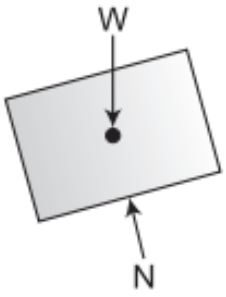
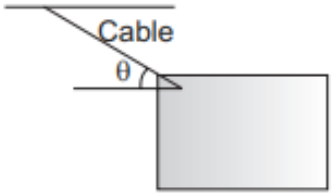
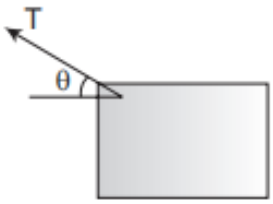
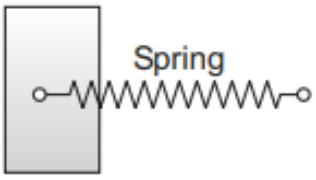
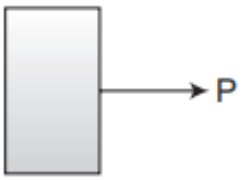
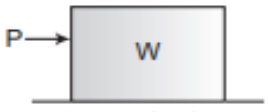
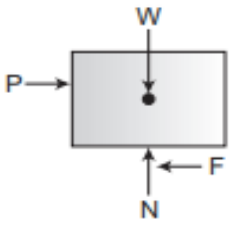
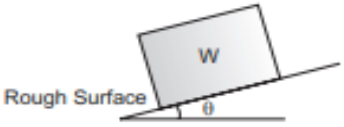
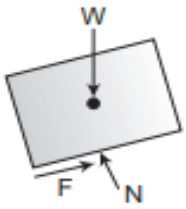
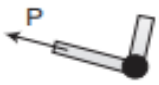
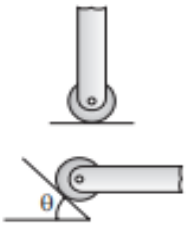
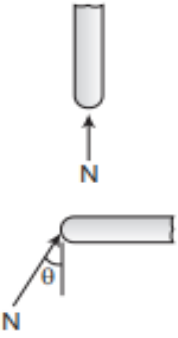
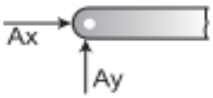
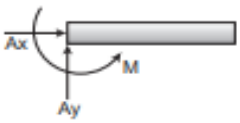
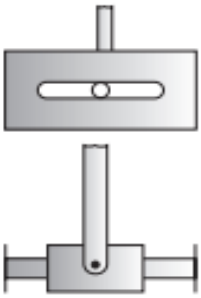
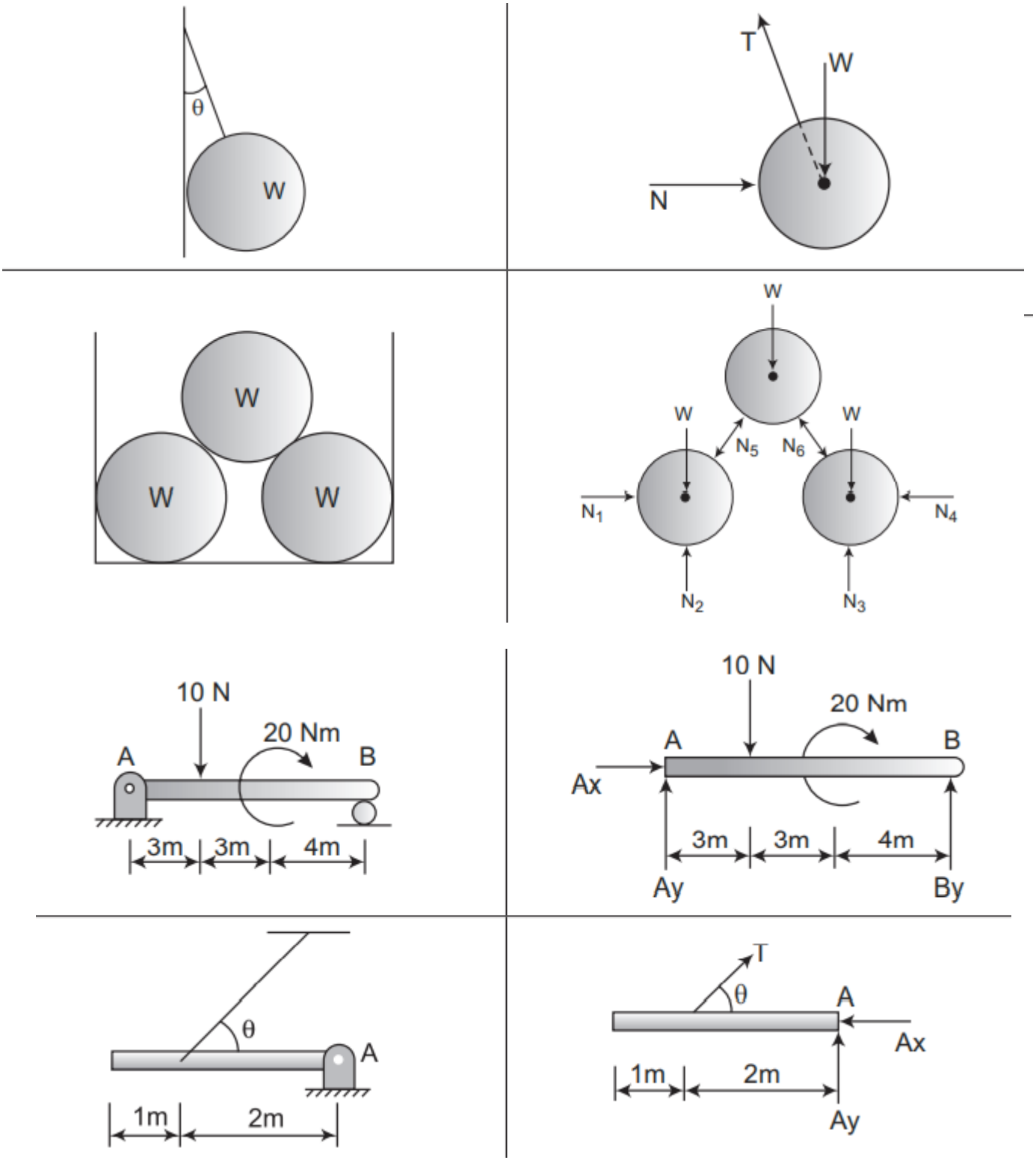
Support	Example	Free Body Diagram (F.B.D.)	Description
Normal contact on horizontal smooth surface			Normal force oriented perpendicular to surface. Direction of normal force is towards the body.
Normal contact on inclined smooth surface			Normal force oriented perpendicular to inclined surface. Direction of normal force is towards the body.
Rope, cable, wire			Tension oriented along cable. Direction of tension is away from the body.
Spring			Spring force oriented along long axis of spring. Force is pull if spring is in tension, and force is push if spring is in compression.

Table 6 (continued)

Normal contact on horizontal surface with friction			Normal force oriented perpendicular to surface. Friction force parallel to the surface.
Normal contact on inclined surface with friction			Normal force oriented perpendicular to inclined surface. Friction force parallel to the inclined surface.
Link			Force oriented along the link. Force can be push or pull
Roller			Normal force oriented perpendicular to the surface supporting the roller.
Pin/Hinge			Force in terms of components oriented along *x* and *y*-axes.
Fixed			Force in terms of components oriented along *x* and *y*-axes. Moment about *z*-axis.
Sliding in guide			Normal force oriented perpendicular to guide.

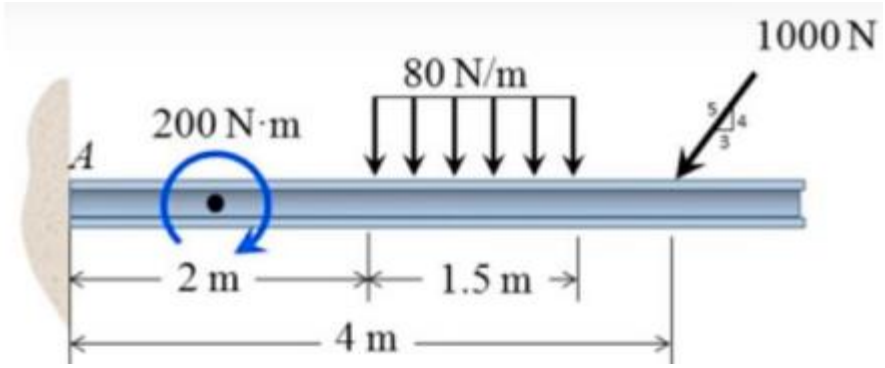
3.4 Examples of free body diagram (FBD)

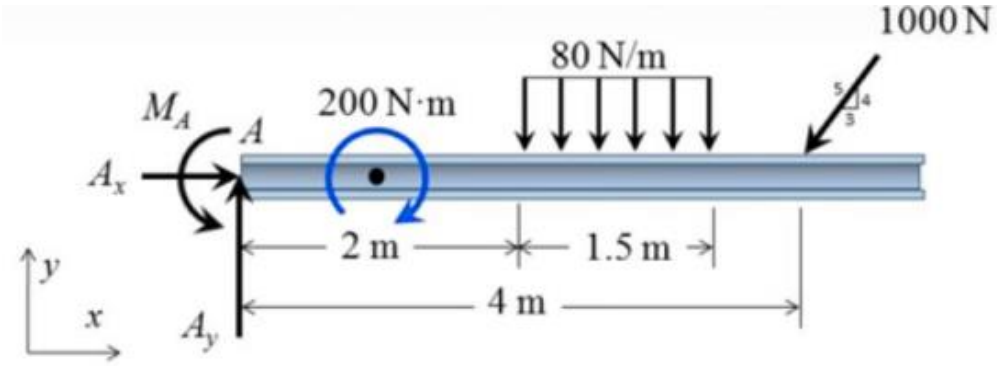


3.5 Constrains and statical determinacy

Statically determinate ------ unknowns = No. of equilibrium equations.

Statically indeterminate ------ unknowns ≠ No. of equilibrium equations.





Unknowns	No. of unknowns	Equilibrium equations	No. equilibrium equations
Ax Ay MA	3	ΣFx=0 ΣFy=0 ΣMA=0	3

3.6 Analysis – how to solve a problem

Procedure for Analysis

Coplanar force equilibrium problems for a rigid body can be solved using the following procedure.

Free-Body Diagram.

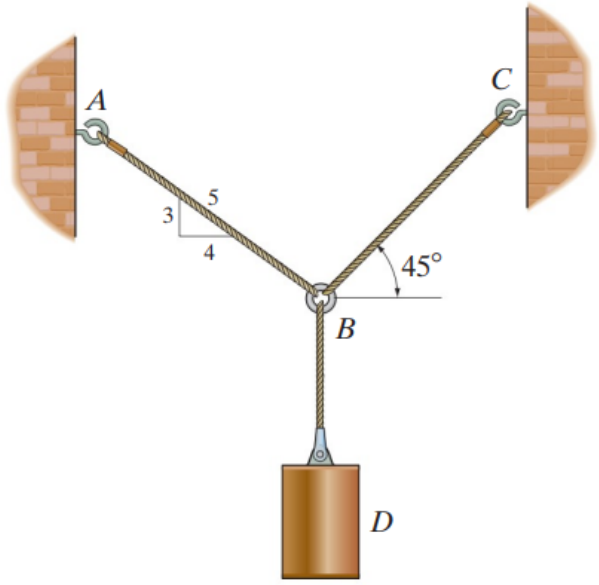
- Establish the x, y coordinate axes in any suitable orientation.
- Remove all supports and draw an outlined shape of the body.
- Show all the forces and couple moments acting on the body.
- Label all the loadings and specify their directions relative to the x or y axis. The sense of a force or couple moment having an *unknown* magnitude but known line of action can be *assumed*.
- Indicate the dimensions of the body necessary for computing the moments of forces.

Equations of Equilibrium.

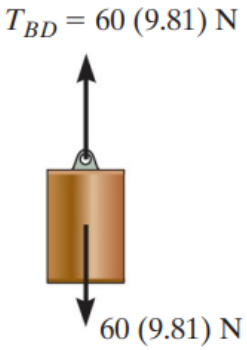
- Apply the moment equation of equilibrium, $\Sigma M_O = 0$, about a point (O) that lies at the intersection of the lines of action of two unknown forces. In this way, the moments of these unknowns are zero about O, and a *direct solution* for the third unknown can be determined.
- When applying the force equilibrium equations, $\Sigma F_x = 0$ and $\Sigma F_y = 0$, orient the x and y axes along lines that will provide the simplest resolution of the forces into their x and y components.
- If the solution of the equilibrium equations yields a negative scalar for a force or couple moment magnitude, this indicates that the sense is opposite to that which was assumed on the free-body diagram.

Example 1

Determine the tension in cables BA and BC necessary to support the 60-kg cylinder in Fig. 3–6a.



(a)



(b)

Equations of Equilibrium. Applying the equations of equilibrium along the x and y axes, we have

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad T_C \cos 45° - \left(\tfrac{4}{5}\right)T_A = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad T_C \sin 45° + \left(\tfrac{3}{5}\right)T_A - 60(9.81)\,\text{N} = 0 \qquad (2)$$

Equation (1) can be written as $T_A = 0.8839T_C$. Substituting this into Eq. (2) yields

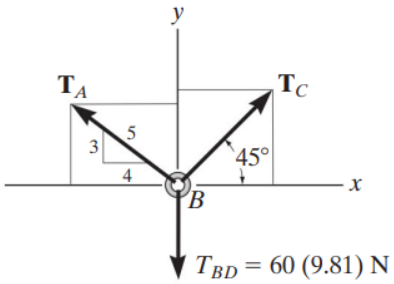
$$T_C \sin 45° + \left(\tfrac{3}{5}\right)(0.8839T_C) - 60(9.81)\,\text{N} = 0$$

so that

$$T_C = 475.66\,\text{N} = 476\,\text{N} \qquad \textit{Ans.}$$

Substituting this result into either Eq. (1) or Eq. (2), we get

$$T_A = 420\,\text{N} \qquad \textit{Ans.}$$

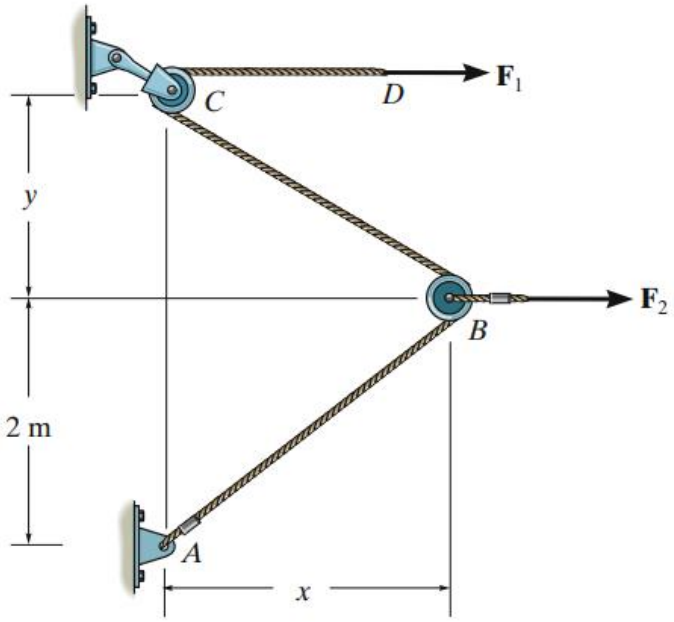


(c)

Fig. 3–6

Example 2

Determine the magnitude of F1 and the distance y if x = 1.5 m and F2 = 1000 N.



Solution

Equations of Equilibrium. The tension throughout rope ABCD is constant, that is F1 . Referring to the FBD shown in Fig.

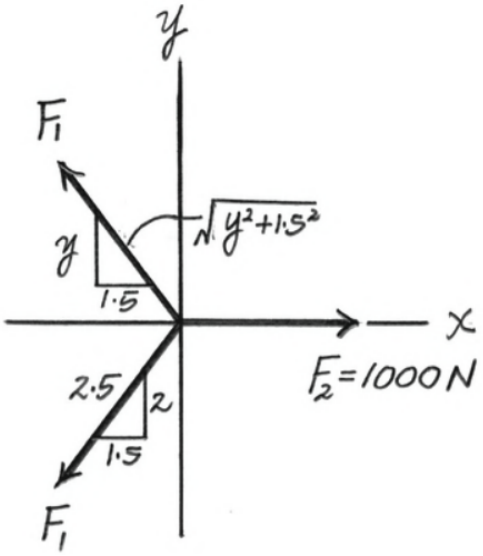
$$+\uparrow \Sigma F_y = 0; \qquad F_1\left(\frac{y}{\sqrt{y^2 + 1.5^2}}\right) - F_1\left(\frac{2}{2.5}\right) = 0$$

$$\frac{y}{\sqrt{y^2 + 1.5^2}} = \frac{2}{2.5}$$

$$y = 2 \text{ m} \qquad \textbf{Ans.}$$

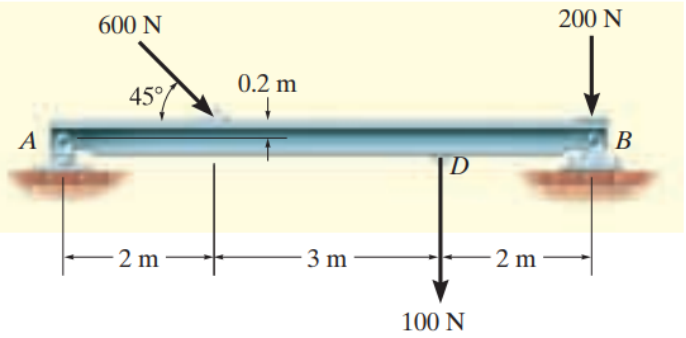
$$\pm \rightarrow \Sigma F_x = 0; \qquad 1000 - 2\left[F_1\left(\frac{1.5}{2.5}\right)\right] = 0$$

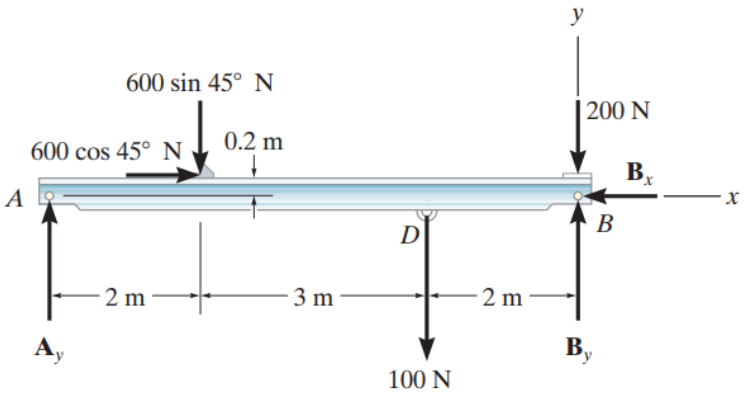
$$F_1 = 833.33 \text{ N} = 833 \text{ N} \qquad \textbf{Ans.}$$



Example 3

Determine the reaction at the supports A and B. Neglect the thickness of the beam.





Equations of Equilibrium. Summing forces in the x direction yields

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad 600\cos 45°\,\text{N} - B_x = 0$$

$$B_x = 424\,\text{N} \qquad \textit{Ans.}$$

A direct solution for $\mathbf{A}_y$ can be obtained by applying the moment equation $\Sigma M_B = 0$ about point B.

$$\circlearrowleft + \Sigma M_B = 0; \qquad 100\,\text{N}(2\,\text{m}) + (600\sin 45°\,\text{N})(5\,\text{m})$$

$$- (600\cos 45°\,\text{N})(0.2\,\text{m}) - A_y(7\,\text{m}) = 0$$

$$A_y = 319\,\text{N} \qquad \textit{Ans.}$$

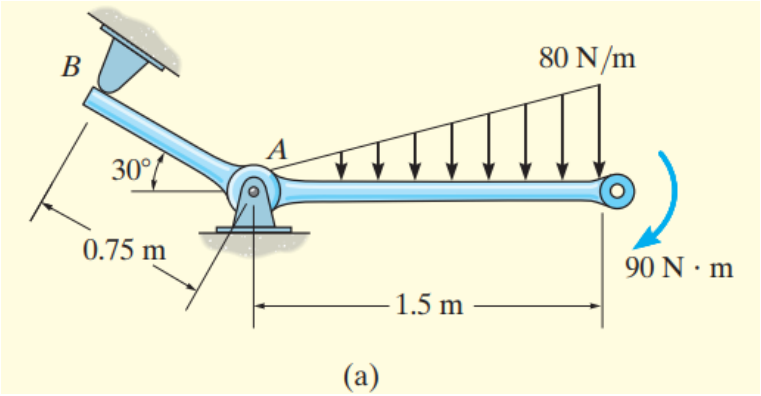
Summing forces in the y direction, using this result, gives

$$+\uparrow \Sigma F_y = 0; \qquad 319\,\text{N} - 600\sin 45°\,\text{N} - 100\,\text{N} - 200\,\text{N} + B_y = 0$$

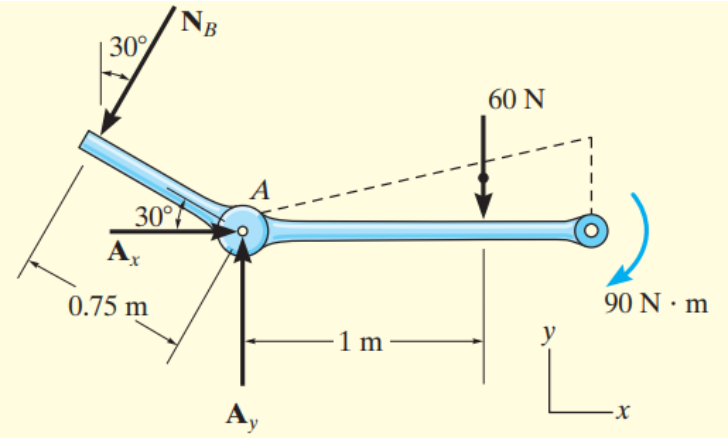
$$B_y = 405\,\text{N} \qquad \textit{Ans.}$$

Example 4

The member shown in Fig. 5–14a is pin connected at A and rests against a smooth support at B. Determine the horizontal and vertical components of reaction at the pin A.



(a)



(b)

Fig. 5–14

Equations of Equilibrium. Summing moments about A, we obtain a direct solution for N_B,

$$\circlearrowleft + \Sigma M_A = 0; \quad -90\ \text{N}\cdot\text{m} - 60\ \text{N}(1\ \text{m}) + N_B(0.75\ \text{m}) = 0$$

$$N_B = 200\ \text{N}$$

Using this result,

$$\pm \Sigma F_x = 0; \qquad A_x - 200 \sin 30°\ \text{N} = 0$$

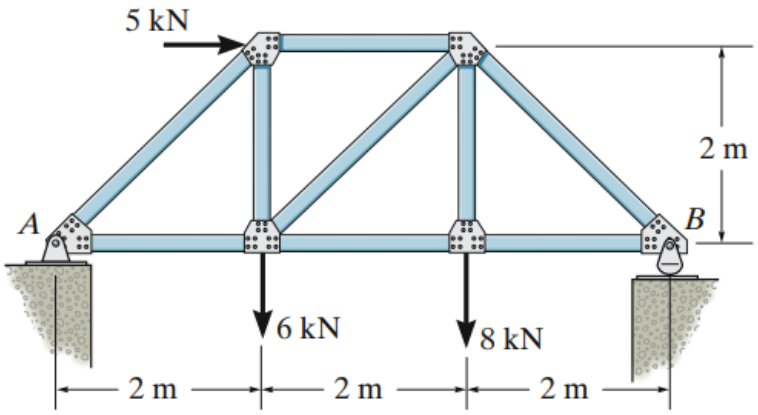
$$A_x = 100\ \text{N} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 200 \cos 30°\ \text{N} - 60\ \text{N} = 0$$

$$A_y = 233\ \text{N} \qquad \textit{Ans.}$$

Example 5

Determine the reactions at the supports.



SOLUTION

Equations of Equilibrium. $\mathbf{A}_y$ and $\mathbf{N}_B$ can be determined by writing the moment equations of equilibrium about points B and A, respectively, by referring to the *FBD* of the truss shown in Fig. *a*.

$\zeta+\Sigma M_B = 0; \quad 8(2) + 6(4) - 5(2) - A_y(6) = 0$

$A_y = 5.00\ \text{kN}$ **Ans.**

$\zeta+\Sigma M_A = 0; \quad N_B(6) - 8(4) - 6(2) - 5(2) = 0$

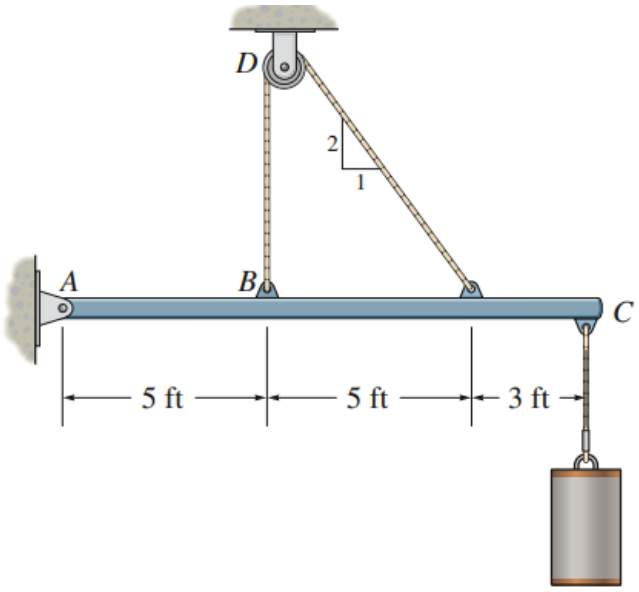
$N_B = 9.00\ \text{kN}$ **Ans.**

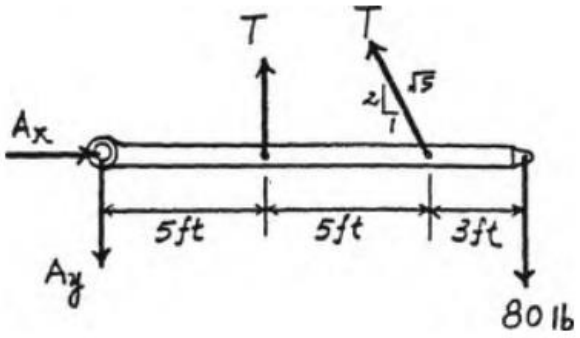
Also, $\mathbf{A}_x$ can be determined directly by writing the force equation of equilibrium along x axis.

$\pm\rightarrow \Sigma F_x = 0; \quad 5 - A_x = 0 \quad A_x = 5.00\ \text{kN}$ **Ans.**

Example 6

Determine the tension in the cable and the horizontal and vertical components of reaction of the pin A. The pulley at D is frictionless and the cylinder weighs 80 lb.





SOLUTION

Equations of Equilibrium: The tension force developed in the cable is the same throughout the whole cable. The force in the cable can be obtained directly by summing moments about point A.

$$\zeta + \Sigma M_A = 0; \quad T(5) + T\left(\frac{2}{\sqrt{5}}\right)(10) - 80(13) = 0$$

$$T = 74.583 \text{ lb} = 74.6 \text{ lb} \qquad \textbf{Ans.}$$

$$\pm\rightarrow \Sigma F_x = 0; \quad A_x - 74.583\left(\frac{1}{\sqrt{5}}\right) = 0$$

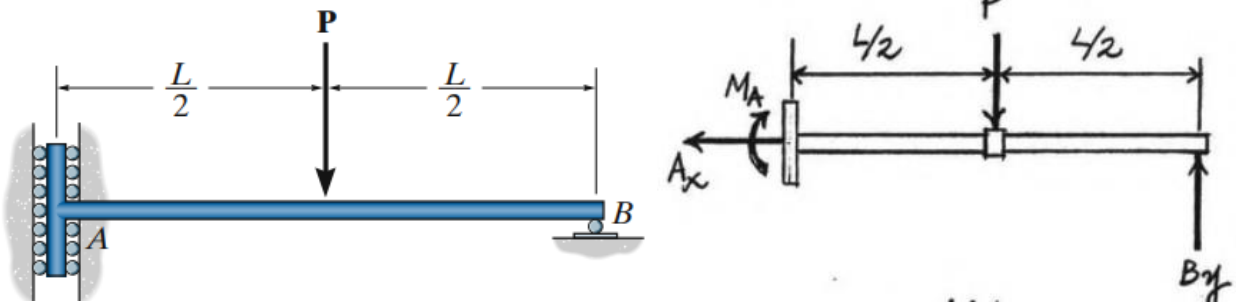
$$A_x = 33.4 \text{ lb} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 74.583 + 74.583\left(\frac{2}{\sqrt{5}}\right) - 80 - B_y = 0$$

$$A_y = 61.3 \text{ lb} \qquad \textbf{Ans.}$$

Example 7

Determine the components of reaction at the supports A and B on the rod.



$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - P = 0$$

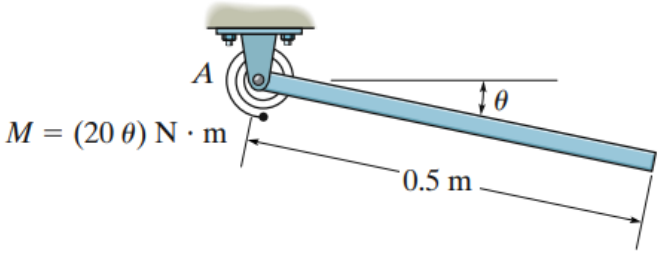
$$B_y = P$$

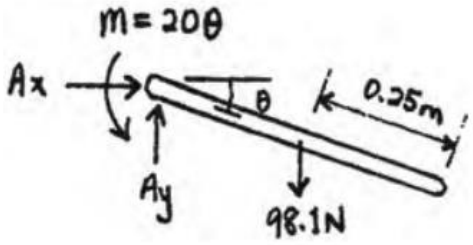
$$\zeta + \Sigma M_B = 0; \qquad P\left(\frac{L}{2}\right) - M_A = 0$$

$$M_A = \frac{PL}{2}$$

Example 7

A linear torsional spring deforms such that an applied couple moment M is related to the spring's rotation ɵ in radians by the equation M = (20 ɵ) N . m. If such a spring is attached to the end of a pin-connected uniform 10-kg rod, determine the angle ɵ for equilibrium. The spring is undeformed when ɵ = 0°.



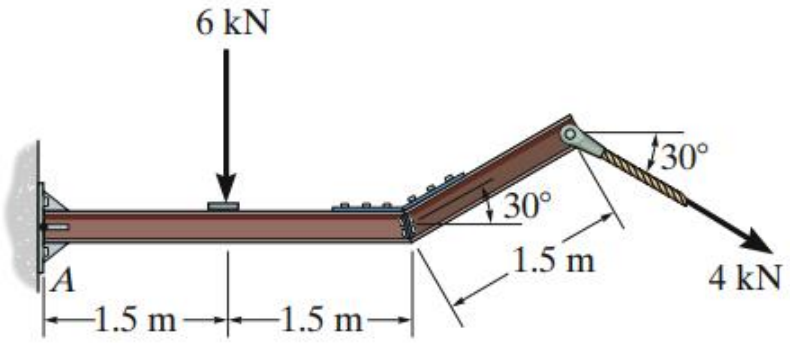


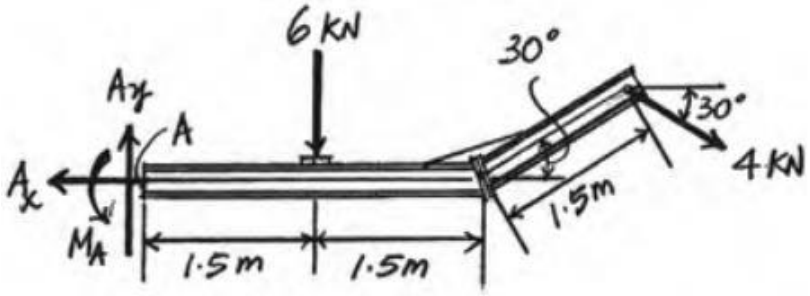
$$\zeta + \Sigma M_A = 0; \qquad -98.1\,(0.25\cos\theta) + 20(\theta) = 0$$

$$\text{ɵ} = 47.5°$$

Example 8

Determine the components of the support reactions at the fixed support A on the cantilevered beam.





Equations of Equilibrium: From the free-body diagram of the cantilever beam, Fig. *a*, A_x, A_y, and M_A can be obtained by writing the moment equation of equilibrium about point A.

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 4\cos 30° - A_x = 0$

$A_x = 3.46\ \text{kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 6 - 4\sin 30° = 0$

$A_y = 8\ \text{kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0; M_A - 6(1.5) - 4\cos 30°\,(1.5\sin 30°) - 4\sin 30°(3 + 1.5\cos 30°) = 0$

$M_A = 20.2\ \text{kN}\cdot\text{m}$ **Ans.**

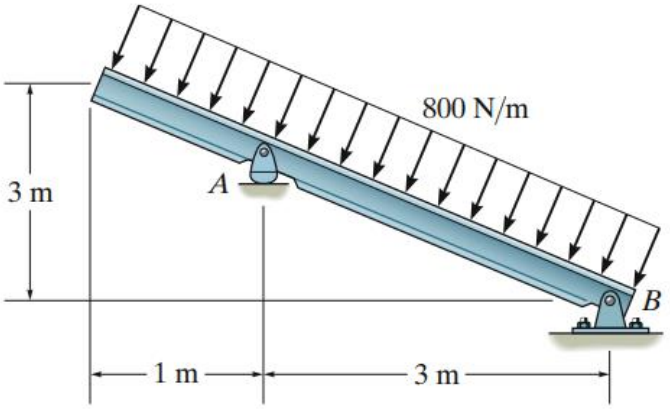


Assignments

All problem solutions must include an FBD.

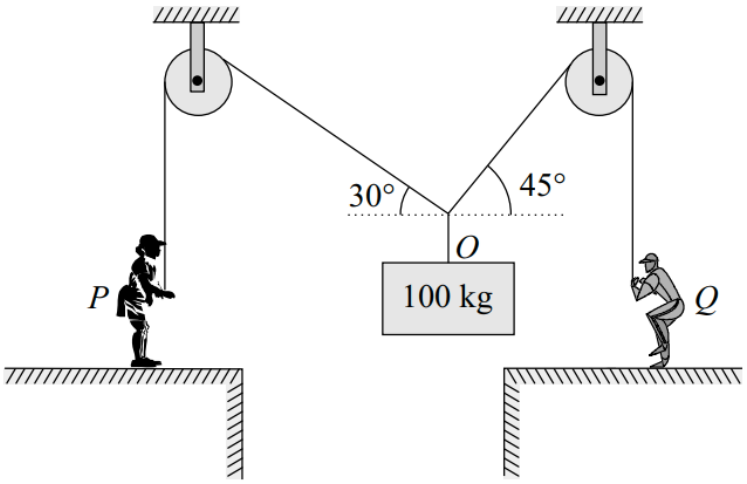
Assignment 1

Determine the reactions at the supports.



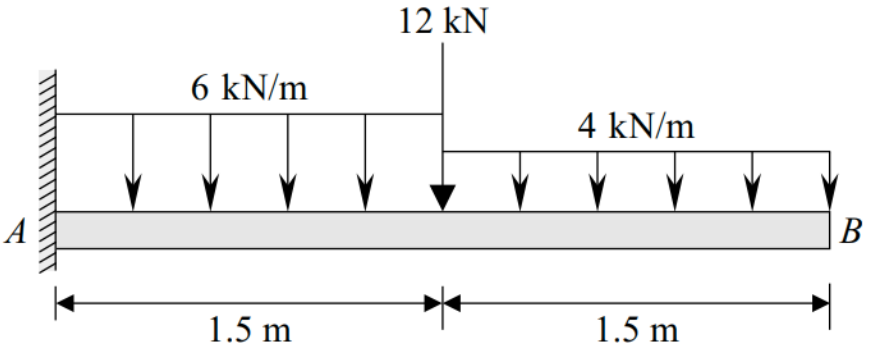
Assignment 2

Two persons lift a mass of 100 kg by cables passing over two pulleys as shown in the figure below. Determine the forces P and Q that must be applied by the two persons if the body is in equilibrium at the position shown.



Assignment 3

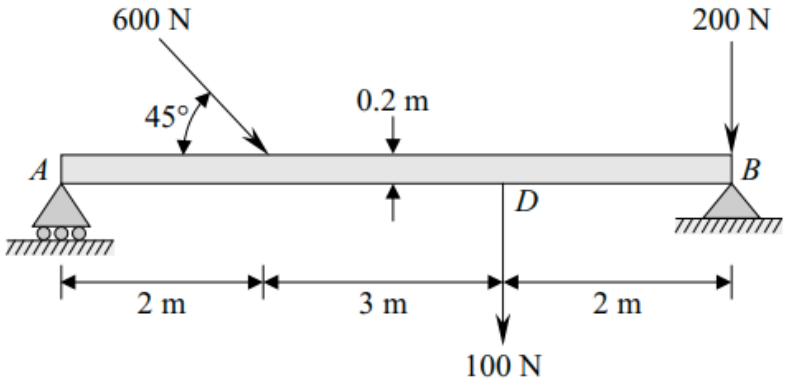
Determine the support reactions for the cantilever beam AB shown.



Assignment 4

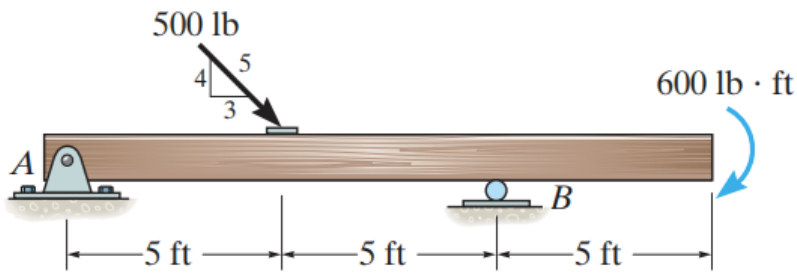
Determine the support reactions for the beam AB loaded as shown in the figure below. Neglect the weight of the beam.

319.5



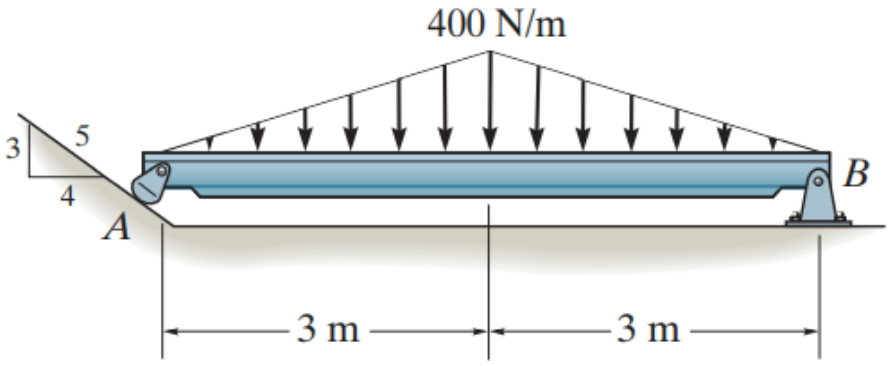
Assignment 5

Determine the horizontal and vertical components of reaction at the supports. Neglect the thickness of the beam.



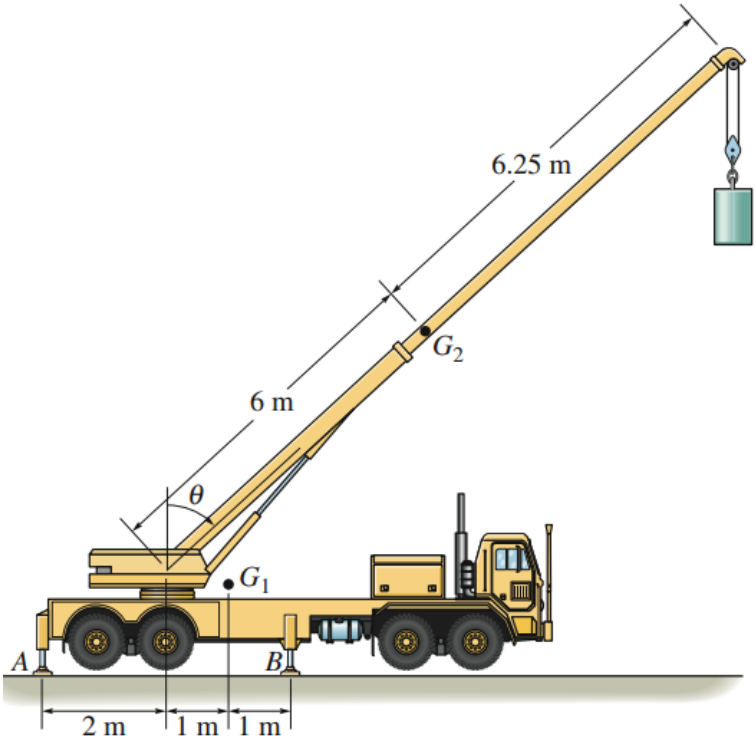
Assignment 6

Determine the reactions at the supports.



Assignment 7

The mobile crane is symmetrically supported by two outriggers at A and two at B in order to relieve the suspension of the truck upon which it rests and to provide greater stability. If the crane and truck have a mass of 18 Mg and center of mass at G1, and the boom has a mass of 1.8 Mg and a center of mass at G2, determine the vertical reactions at each of the four outriggers as a function of the boom angle ɵ when the boom is supporting a load having a mass of 1.2 Mg. Plot the results measured ɵ=0 from to the critical angle where tipping starts to occur.



Assignment 8

If the roller at A and the pin at B can support a load up to 4 kN and 8 kN, respectively, determine the maximum intensity of the distributed load w, measured in kN/m, so that failure of the supports does not occur.

