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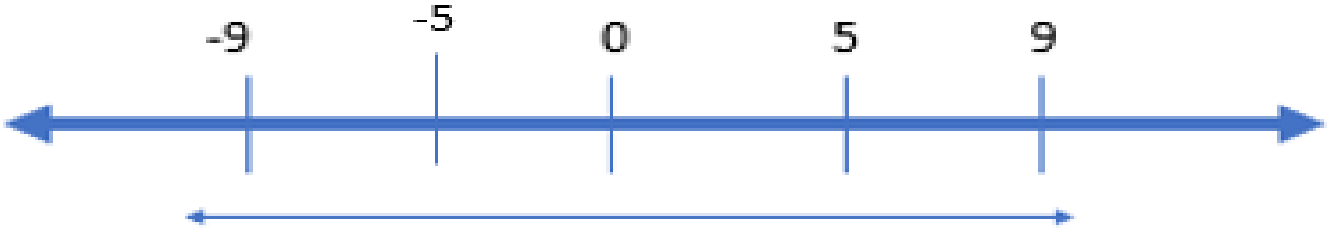
Mathematics

Absolute value of magnitude

- *Dr. MURTAJA ALI SAARE*

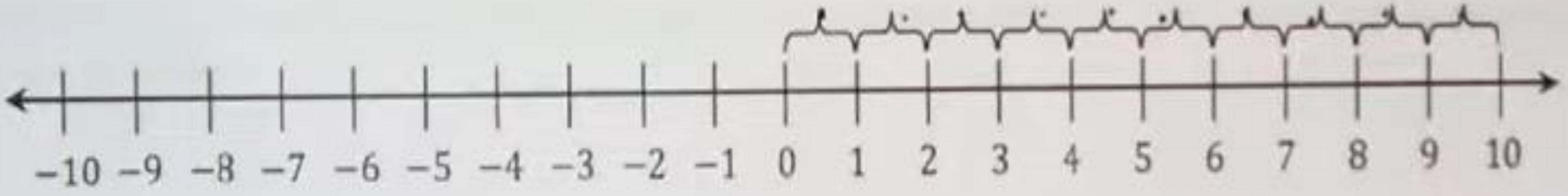


Distance=magnitude
(How long)
Opposite numbers are the same distance from zero in opposite directions

Example 1: The Absolute Value of a Number

The absolute value of ten is written as $|10|$. On the number line, count the number of units from 10 to 0. How many units is 10 from 0?

$|10| =$



What other number has an absolute value of 10? Why?

The absolute value of a number is the distance between the number and zero on the number line.

Example 2: Using Absolute Value to Find Magnitude

Mrs. Owens received a call from her bank because she had a checkbook balance of $-\$45$. What was the magnitude of the amount overdrawn?

The magnitude of a measurement is the absolute value of its measure.

Absolute value = a numbers distance from zero

Magnitude= The absolute value of a number.

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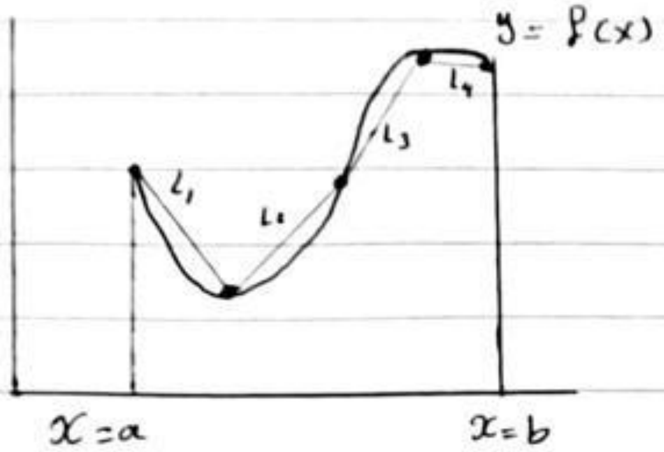
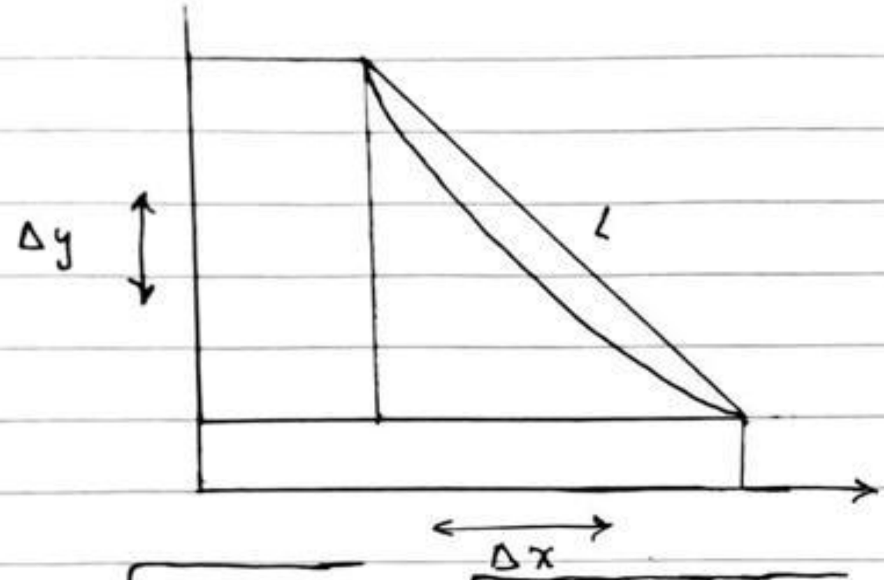
Application Length of Plan Curve

Dr. MURTAJA ALI SAARE

Application Length of Plan Curve





If $n \to \infty$

$\Delta x \to 0$

$\Rightarrow \Sigma \to \int$

$$L = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{\Delta x^2 \left(1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right)}$$

$$L = \Delta x \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}$$

$$\therefore L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \cdot dy$$

ex1 Find the area Length of $y = \frac{1}{3}\sqrt{x}\,(3-x)$ from $0 \leqslant x \leqslant 3$

Sol:-

$$y = \sqrt{x}\,(3-x) = x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}$$

$$\bar{y} = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{2}}$$

نطبق القانون

$$1 + \left(\frac{dx}{dy}\right)^2 = 1 + \left(\frac{1}{2\sqrt{x}} - \frac{\sqrt{x}}{2}\right)^2$$

$$1 + \left[\frac{1-x}{2\sqrt{x}}\right]^2$$

$$1 + \frac{(1-x)^2}{(2\sqrt{x})^2}$$

$$\frac{(2\sqrt{x})^2 + (1-x)^2}{(2\sqrt{x})^2}$$

$$\frac{4x + 1 - 2x + x^2}{(2\sqrt{x})^2} = \frac{2x + x^2 + 1}{(2\sqrt{x})^2}$$

نرتب

$$= \frac{x^2 + 2x + 1}{(2\sqrt{x})^2} = \frac{(x+1)(x+1)}{(2\sqrt{x})^2}$$

$$= \frac{(x+1)^2}{(2\sqrt{x})^2}$$

$$L = \int_0^3 \frac{x+1}{2\sqrt{x}}\,dx = \frac{1}{2}\int_0^3 \left(\sqrt{x} + x^{-\frac{1}{2}}\right)dx$$

$$\frac{1}{2}\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right)\Bigg|_0^3 = \frac{1}{2}\left(\frac{2x^{\frac{3}{2}}}{3} + 2x^{\frac{1}{2}}\right)\Bigg|_0^3$$

$$\frac{\sqrt{(3)^3}}{3} + \sqrt{3} = \frac{3\sqrt{3}}{3} + \sqrt{3}$$

$$= 2\sqrt{3} \text{ unit Length}$$

exe Find the area Length of $y = \frac{2}{3}(1+x^2)^{\frac{3}{2}}$ from $0 \leq x \leq 3$

Sol

$$\bar{y} = (1+x^2)^{\frac{1}{2}} \cdot 2x$$

$$1 + (\bar{y})^2 = 1 + \left[2x(1+x^2)^{\frac{1}{2}}\right]^2$$

$$1 + \left[4x^2(1+x^2)\right]$$

$$1 + \left[4x^2 + 4x^4\right] = 1 + 4x^2 + 4x^4$$

نرتب الحدود $4x^4 + 4x^2 + 1 \Rightarrow (2x^2+1)(2x^2+1)$

$$(2x^2+1)^2$$

نطبق القاعده $L = \int_0^3 (1+2x^2)\, dx$

$$x + \frac{2x^3}{3} \Big|_0^3 = 3 + \frac{2(3)^3}{3} \Big| - 0$$

$$= 3 + 18 = 21 \text{ unit Length}$$

ex3 Find the length of the arc of $24xy = x^4+48$ from $x=2$ to $x=4$

Sol

$$y = \frac{x^4+48}{24x} = \frac{1}{24}\left(x^3 + 48x^{-1}\right)$$

$$\bar{y} = \frac{1}{24}\left(3x^2 - \frac{48}{x^2}\right) = \frac{x^2}{8} - \frac{2}{x^2} = \frac{x^4-16}{8x^2}$$

$$1+(\bar{y})^2 = 1+\left(\frac{x^4-16}{8x^2}\right)^2$$

$$= \frac{(8x^2)^2 + (x^4-16)^2}{(8x^2)^2} = \frac{64x^4+x^8-32x^4+256}{(8x^2)^2}$$

$$\frac{32x^4+x^8+256}{(8x^2)^2}$$

نرتب $$\frac{x^8+32x^4+256}{(8x^2)^2} = \frac{(x^4+16)(x^4+16)}{(8x^2)^2}$$

$$= \frac{(x^4+16)^2}{(8x^2)^2}$$

$$\therefore L = \frac{1}{8}\int_2^4 \left(\frac{x^4+16}{x^2}\right) dx = \frac{1}{8}\int_2^4 \left(\frac{x^4}{x^2} + \frac{16}{x^2}\right) dx$$

$$= \frac{1}{8}\int_2^4 \left(x^2 + 16x^{-2}\right) dx$$

$$\frac{1}{8}\left[\frac{x^3}{3} + \left(\frac{-16}{x}\right)\Big|_2^4\right] \Rightarrow \frac{1}{8}\left[\left(\frac{(4)^3}{3} - \frac{16}{4}\right) - \left(\frac{(2)^3}{3} - \frac{16}{2}\right)\right]$$

$$\frac{1}{8}\left[\frac{64}{3} - 4 - \frac{8}{3} + 8\right] \Rightarrow \frac{1}{8}\left(\frac{56}{3} + 4\right)$$

$$\frac{1}{8}\left(\frac{56+12}{3}\right) \Rightarrow \frac{68}{24} = \frac{17}{6} \text{ unit length}$$

ex 4 Find the Length of the arc of the curve
$x = 3y^{3/2} - 1$ from $y = 0$ to $y = 4$

Sol

$$\bar{y} = \frac{9}{2} y^{\frac{1}{2}}$$

$$1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{81y}{4}$$

$$\therefore L = \int_{y=0}^{4} \left(1 + \frac{81y}{4}\right)^{\frac{1}{2}} dy = \int_0^4 \frac{(4 + 81y)^{\frac{1}{2}}}{(2^2)^{\frac{1}{2}}} dy$$

[illegible]

Let $u = 4 + 81y \Rightarrow du = 81\, dy$

$$\Rightarrow L = \frac{1}{2} \int_{4}^{328} \frac{1}{81} (u)^{\frac{1}{2}} du = \frac{1}{162} \left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right) \Bigg|_4^{328}$$

$u = 4$ ← [illegible]

$$\frac{2}{162} \left(\frac{u^{\frac{3}{2}}}{3} \right) \Bigg|_4^{32} = \frac{2}{162} \left(\frac{\sqrt{(328)^3}}{3} - \frac{(2^2)^{\frac{3}{2}}}{3} \right)$$

$$= \frac{2}{162} \left(\frac{\sqrt{(328)^3}}{3} - \frac{8}{3} \right)$$

$$= \frac{2}{162} * \frac{1}{3} \left(\sqrt{(328)^3} - 8 \right) = \frac{2}{486} \left(\sqrt{(328)^3} - 8 \right.$$

$$\Rightarrow \frac{1}{243} \left(\sqrt{(328)^3} - 8 \right) \text{ unit Length}$$

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Mathematics

Differentiation of Trigonometric functions

Dr. MURTAJA ALI SAARE

Differentiation of trigonometric functions

مشتقة الداله x مشتقة الزاوية .

① $\sin y = \cos y \frac{dy}{dx}$

② $\cos y = -\sin \frac{dy}{dx}$

③ $\tan y = \sec^2 y \frac{dy}{dx}$

④ $\cot y = -\csc^2 y \frac{dy}{dx}$

⑤ $\sec y = \sec y \tan y \frac{dy}{dx}$

⑥ $\csc y = -\csc y \cot y \frac{dy}{dx}$

Ex

$\sin(7x^2 + 2x + 1)$ $\frac{dy}{dx}$ جد

$\frac{dy}{dx} = \cos(7x^2 + 2x + 1)(14x + 2)$

$\frac{dy}{dx} = (14x + 2)\cos(7x^2 + 2x + 1)$

Ex $y = \csc(5x^3) \Rightarrow y' = -\csc(5x^3)\cot(5x^3)(15x^2)$

$y' = -15x^2 \csc(5x^3) \cot(5x^3)$

Ex $y = \tan\sqrt{x}$

$y = \tan x^{\frac{1}{2}} \Longrightarrow y' = \sec^2 x^{\frac{1}{2}} \left(\frac{1}{2} x^{-\frac{1}{2}}\right)$

$y' = \left(\frac{1}{2x^{\frac{1}{2}}}\right) \sec^2 x^{\frac{1}{2}} \Longrightarrow$

$y' = \frac{1}{2\sqrt{2}} \sec^2 \sqrt{x}$

Ex

$y = (\cos 5x)^3$

$y' = 3(\cos 5x)^2 (-\sin 5x)(5)$

$y' = -15 (\cos^2 5x)(\sin 5x)$

Ex $y = \sqrt{\tan 7x} \Longrightarrow y = (\tan 7x)^{\frac{1}{2}}$

$y' = \frac{1}{2} (\tan 7x)^{-\frac{1}{2}} (\sec^2 7x)(7)$

$y' = \frac{7}{2} \frac{\sec^2 7x}{(\tan 7x)^{\frac{1}{2}}}$

$y' = \frac{7}{2} \frac{\sec^2 7x}{\sqrt{\tan 7x}}$

٤) جد y''' $\quad f(x) = 4\sqrt{6-2x} \quad \forall x < 3$

٥) جد $f'''(x)$ $\quad f(x) = \sin \pi x$

٦) جد $f'''(x)$ $\quad f(x) = \dfrac{3}{2-x}$

Derivative of exponential functions:

$$\frac{d}{dx}\left(e^{F(x)}\right) = e^{F(x)} \cdot F'(x)$$

Ex: Find y' for $y = e^x$

sol/

$$\frac{dy}{dx} = e^x \cdot 1 = e^x$$

Ex: $y = 5^{\sqrt{1+5x^2}}$

$$y' = e^{\sqrt{1+5x^2}} * \frac{1}{2}(1+5x^2)^{-1/2} \cdot 10x$$

$$= 5x(1+5x^2)^{-1/2} \cdot e^{\sqrt{1+5x^2}} = \frac{5x}{\sqrt{1+5x^2}} \cdot e^{\sqrt{1+5x^2}}$$

Ex: $y = e^{x^2 \cdot \tan x}$

$$\frac{dy}{dx} = e^{x^2 \tan x} * \left[x^2 \cdot \sec^2 x + \tan x \cdot 2x\right]$$

The function (a^x) [Derivative]

$$\frac{d}{dx}\left(a^{F(x)}\right) = a^{F(x)} * F'(x) * \ln a$$

Ex: find $\frac{dy}{dx}$ for $y = 2^{5x}$

$$y' = 2^{5x} \cdot 5 \cdot \ln 2$$

Ex: $y = 7^{\cos\sqrt{2x+3}}$

$$y' = 7^{\cos\sqrt{2x+3}} \cdot -\sin\sqrt{2x+3} \cdot \frac{2}{2\sqrt{2x+3}} \cdot \ln 7$$

$$y' = \frac{-7^{\cos\sqrt{2x+3}}}{\sqrt{2x+3}} \cdot \sin\sqrt{2x+3} \cdot \ln 7$$

H.W $y = 5^{\csc x^3}$, $y = 3^{(x^2 - e^{6x^2})}$

The derivative of function "u^v"

Where u and v are differentiable function of x are found by Logarthimic differentation.

let $y = u^v \Rightarrow \ln y = v \cdot \ln u$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{v}{u} \cdot \frac{du}{dx} + \ln u \cdot \frac{dv}{dx}$$

$$\frac{dy}{dx} = y \left[\frac{v}{u} \cdot \frac{du}{dx} + \ln u \cdot \frac{dv}{dx} \right]$$

$$\therefore \frac{d}{dx} u^v = u^v \left[\frac{v}{u} \cdot \frac{du}{dx} + \ln u \cdot \frac{dv}{dx} \right]$$

Ex: find $\frac{dy}{dx}$ for: (a) $y = x^{\cos x}$ (b) $y = (\ln x + x)^{\tan x}$

(a) $y = x^{\cos x} \Rightarrow \ln y = \cos x . \ln x \Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{\cos x}{x} + \ln x(-\sin x)$

$\Rightarrow \frac{dy}{dx} = y\left[\frac{\cos x}{x} - \sin x \ln x\right]$

or by formula where $u = -x$ and $v = \cos x$

$\frac{dy}{dx} = y\left[\frac{\cos x}{x} - \sin x \ln x\right]$

(b) $y = (\ln x + x)^{\tan x} \Rightarrow \ln y = \tan x \ln(\ln x + x)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{\tan x}{\ln x + x}\left(\frac{1}{x} + 1\right) + \ln(\ln x + x)\sec^2 x$

$\frac{dy}{dx} = y\left[\frac{(x+1)\tan x}{x(\ln x + x)} + \ln(\ln x + x)\sec^2 x\right)$

or by formula $u = \ln x + x$, $v = \tan x$

$\frac{dy}{dx} = y\left[\frac{\tan x}{\ln x + x}\left(\frac{1}{x} + 1\right) + \ln(\ln x + x)\sec^2 x\right]$

Chain Rule

① IF y is a differentiable function of t and x is a differentiable function of t then $\frac{dy}{dx}$ will be :-

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

② IF y is a differentiable function of t and t is a differentiable function of x, then $\frac{dy}{dx}$ will be :-

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

Ex: ⓐ $y = \frac{t^2}{t^2+1}$ and $t = \sqrt{2x+1}$

ⓑ $y = \frac{1}{t^2+1}$ and $x = \sqrt{4t+1}$

sol: ⓐ

$$\frac{dy}{dt} = \frac{2t(t^2+1) - 2t \cdot t^2}{(t^2+1)^2} = \frac{2t}{(t^2+1)^2}$$

$$t = (2x+1)^{1/2} \Rightarrow \frac{dt}{dx} = \frac{1}{2}(2x+1)^{-1/2} \cdot 2 = \frac{1}{\sqrt{2x+1}}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{2t}{(t^2+1)^2} \cdot \frac{1}{\sqrt{2x+1}}$$

$\because t = \sqrt{2x+1}$

$$\therefore \frac{dy}{dx} = \frac{2\sqrt{2x+1}}{\left((\sqrt{2x+1})^2+1\right)^2} \cdot \frac{1}{\sqrt{2x+1}} = \frac{1}{2(x+1)^2}$$

(b) $y = (t^2+1)^{-1} \Rightarrow \frac{dy}{dt} = -2t(t^2+1)^{-2} = \frac{-2t}{(t^2+1)^2}$

$x = (4t+1)^{1/2} \Longrightarrow \frac{dx}{dt} = \frac{1}{2}(4t+1)^{-1/2} \cdot 4 = \frac{2}{\sqrt{4t+1}}$

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{-2t}{(t^2+1)^2} \div \frac{2}{\sqrt{4t+1}} = \frac{-t\sqrt{4t+1}}{(t^2+1)^2}$$

$$= -\frac{x^2-1}{4} \cdot x \cdot \frac{1}{y^2} = \frac{-x(x^2-1)}{4y^2}$$

(1) ... $x = \sqrt{4t+1}$
... $x^2 = 4t+1$
(2) ... $\therefore t = \frac{x^2-1}{4}$

H.W (1) $y = \left(\frac{t-1}{t+1}\right)^2$ and $x = \frac{1}{t^2} - 1$ at $t = 2$

(2) $y = 1 - \frac{1}{t}$ and $t = \frac{1}{1-x}$ at $x = 2$

Second and higher derivative

The derivative is called the second derivative of y with respect to x which can be written in many ways such as y'', $f''(x)$ or $\frac{d^2 f(x)}{dx^2}$

In the same manner, we can define the third and higher derivative by using similar rations. The n^{th} derivative may be written as:

$$y^{(n)}, \quad f^{(n)}(x), \quad \frac{d^n y}{dx^n}$$

Ex: $y = 3x^3 - 4x^2 + 7x + 10$

$y' = 9x^2 - 8x + 7$

$y'' = 18x - 8$

$y''' = 18$

$y'''' = y''''' = 0$

Ex: $y = \frac{1}{x} + \sqrt{x^3}$

Sol:

$$\frac{dy}{dx} = \frac{-1}{x^2} + \frac{3}{2} x^{1/2}$$

$$\frac{d^2y}{dx^2} = \frac{2}{x^3} + \frac{3}{4} x^{-1/2}$$

$$\frac{d^3y}{dx^3} = \frac{-6}{x^4} - \frac{3}{8} x^{-3/2} = \frac{-6}{x^4} - \frac{3}{8\sqrt{x^3}}$$

H.W / Find $\frac{d^5y}{dx^5}$ for $y = \sinh(4x^2) - \sqrt{\coth(6x^3)}$

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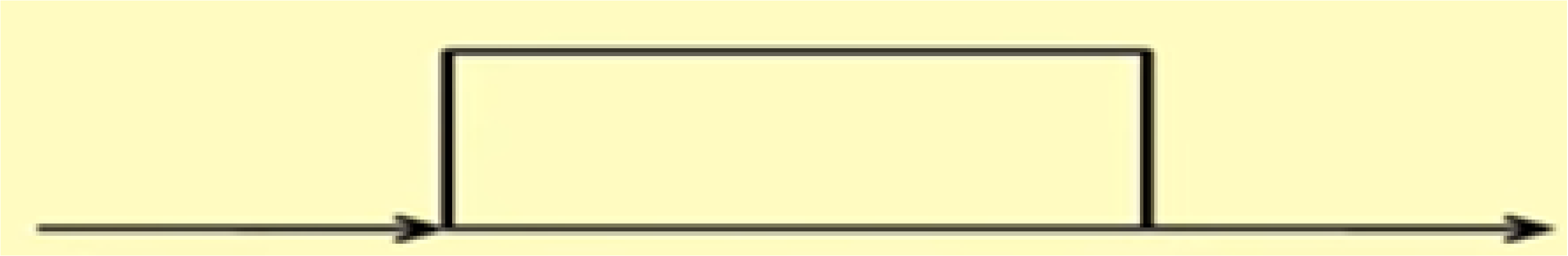
Mathematics

Functions and their graphs

• *Dr. MURTAJA ALI SAARE*

A function is a rule that for every input assigns a specific output. You can also think of a function as a machine in which each input produces one output.

For example , let's say you own a prepaid phone . Your monthly cost is a function of the number of minutes you use . The cost is $0.15 per minute .



The input, usually x, called the independent variable.
The output, usually y, called the dependent variable.

- The set of all possible inputs is called the DOMAIN.

- The DOMAIN is the set of all possible x - values .

- The set of all possible outputs is called the RANGE .

- • The RANGE is the set of all possible y - values .

Determine the domain and range.

Example

Y=4+3x

x	y
0	5
1	7
2	9
3	11
.	
.	
.	

Example

y=5+2x

f(x)=5+2x

- Domain: {0,1,2,3,......}
- Range:{5,7,9,1,....}

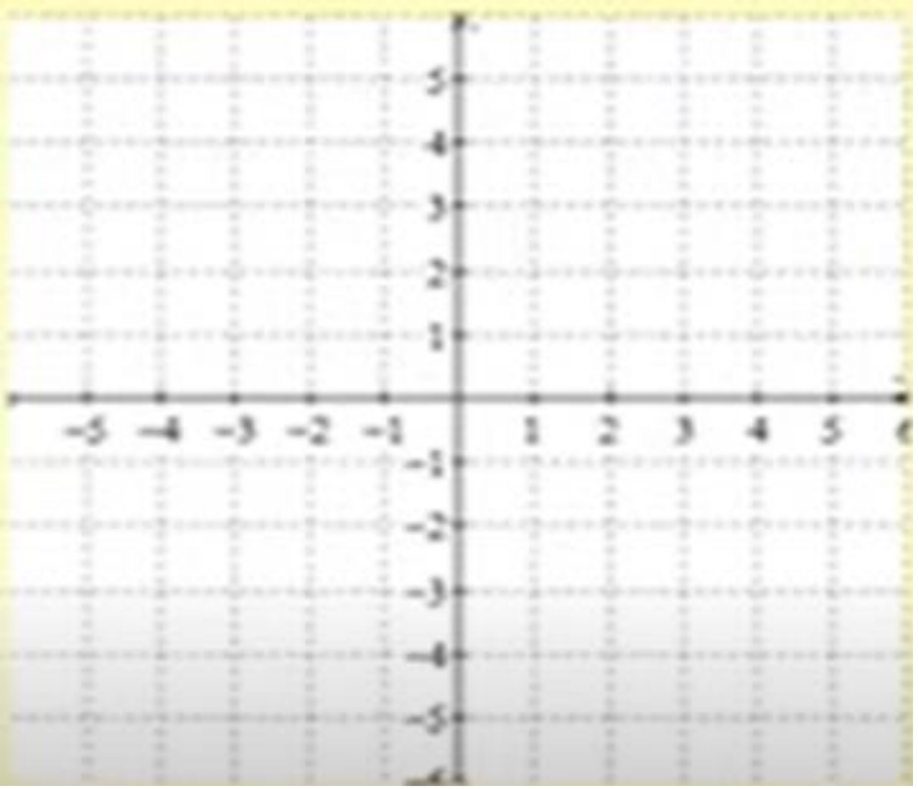
Determining function values and graphing functions.

Example 1:

Determine the domine and range

$f(x)= 4x-2$

x	y

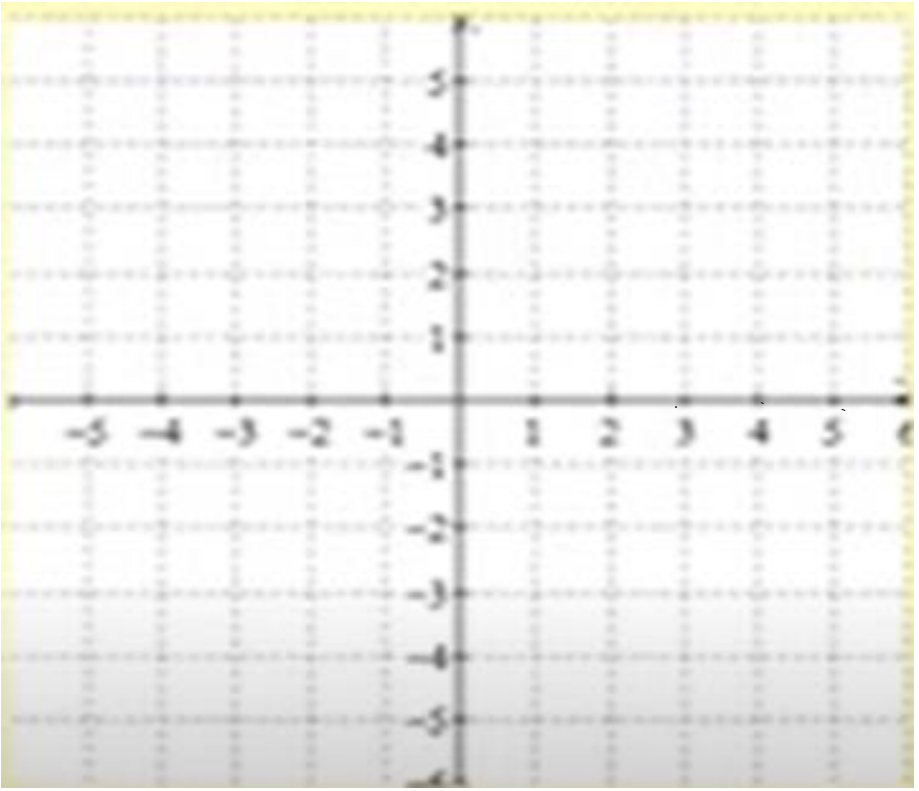


Example 2:

Determine the domine and range

f(x)= x^2

x	y

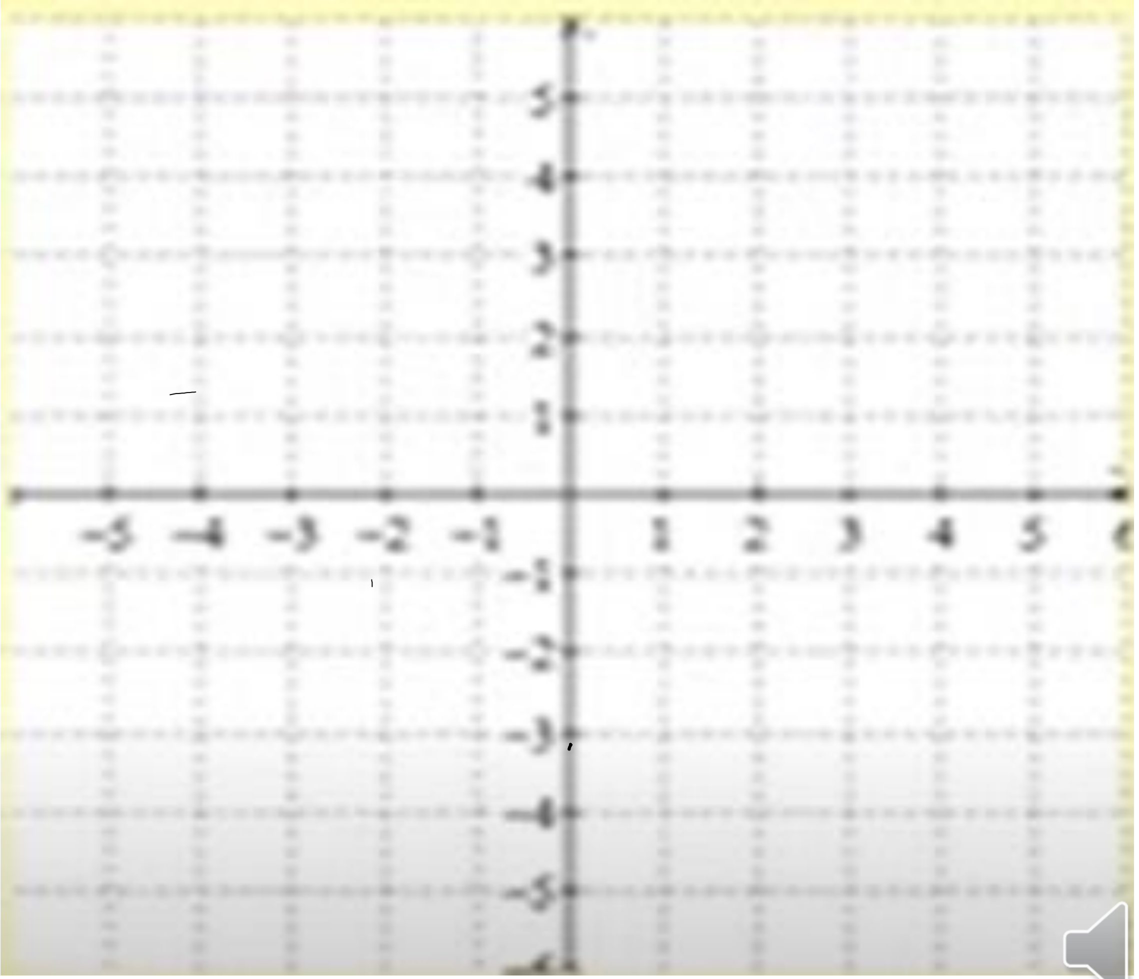


Example 3:

Determine the domine and range

f(x)= $|x| - 3$

x	y



How to determine if a relation or correspondence is a Function

Goal :
Given a relation , determine if is a function

Formal Definition of a Function

A function is a correspondence between a first set , called the domain , and a second set , called the range , such that each member of the domain corresponds to exactly one member of the range .

Domain: set of x-values
Range: set of y-values
Function: If every x-value is paired with exactly 1 y- value

Example :Determine whether or not each correspondence is a function.

Domain		Range
Sep 2006	→	8,729,000
Jan 2007	→	21,066,000
Mar 2007	→	10,549,000
Jun 2007	→	9,815,000

Example:

b) Squaring

Domain		Range
2	→	4
3	→	9
4	→	16
–4	→	16

Example :

c) Baseball Teams

Domain		Range
Arizona	→	Diamondbacks
Chicago	→	Cubs
Chicago	→	White Sox
New York	→	Yankees

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Example :

c) Baseball Teams

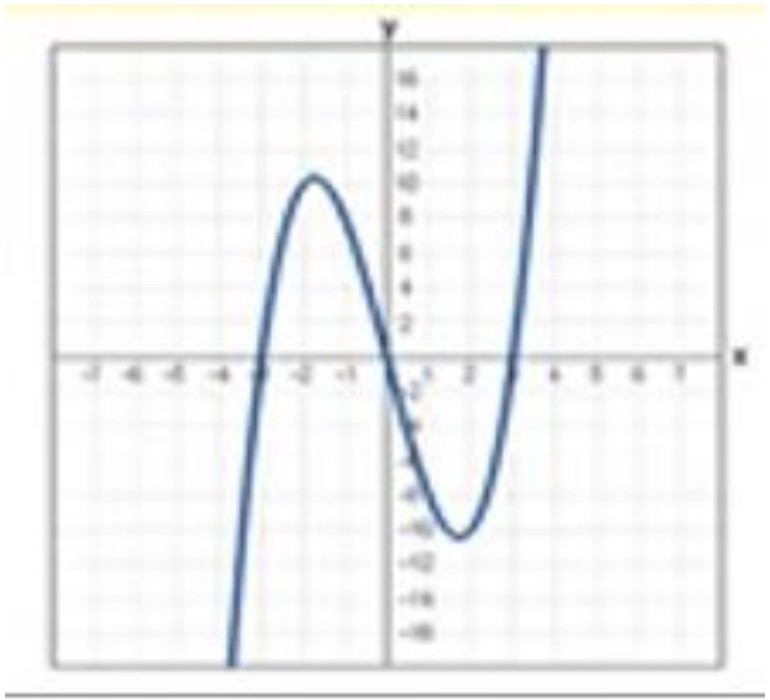
Domain		Range
Arizona	→	Diamondbacks
Chicago	→	Cubs
Chicago	→	White Sox
New York	→	Yankees

Graphs of Functions

Definition : The graph of a function f is a drawing that represents all the input - output pairs (x , f (x)) .
In cases where the function is given by an equation , the graph of a function is the graph of the equation y = f (x) .

Example : The graph of the cubic polynomial on the real line is { x^3 - 9x): x is a real number } .

f(x)= $x^2 - 9x$

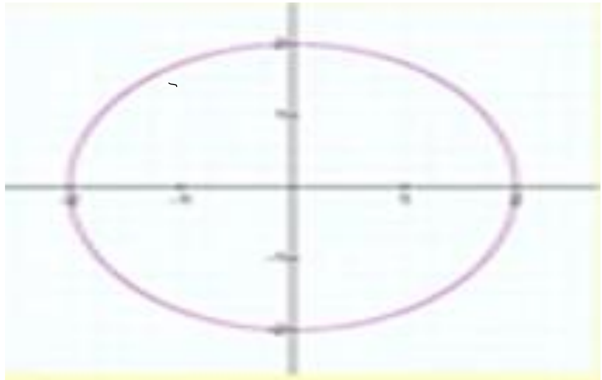


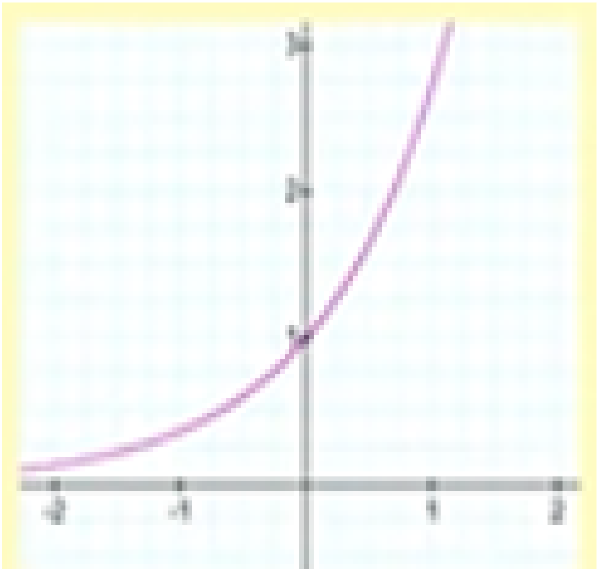
- Determining if the graph of a relation or correspondence is a function

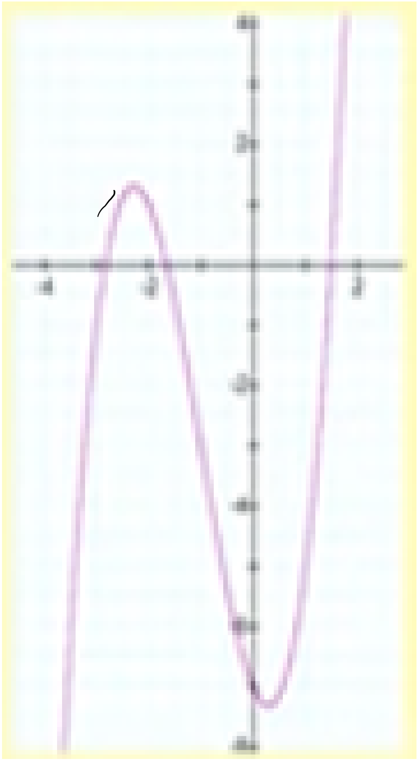
- The Vertical Line Test

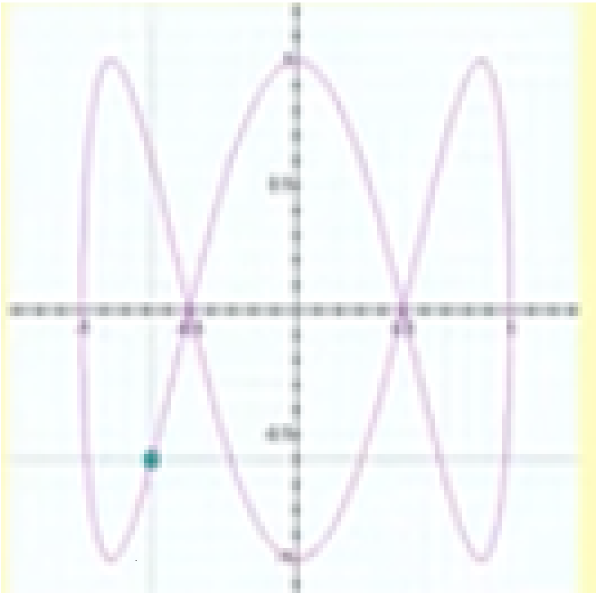
- A graph represents a function if it is impossible to draw a vertical line that intersects the graph more than once.

Example: Determine whether each of the following is the graph of a function.









Determining Domain and Range

The domain of the function is the set of all x - values , or inputs , of the points on the graph .

❖ The domain can be viewed as the curve's shadow onto the x - axis , or how it behaves from left to right

The range of the function is the set of all y - values , or outputs , of the points on the graph .

❖ The range can be viewed as the curve's shadow onto the y - axis or how it behaves up and down .

Domain :

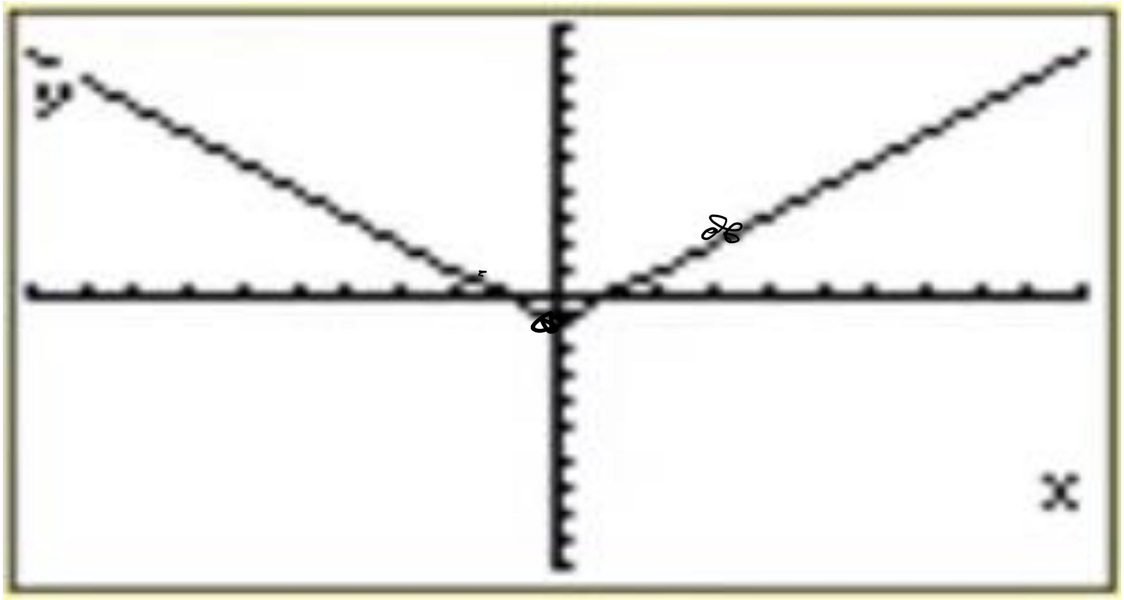
Range :

Example: State the domain and range of the following relation. Is the relation a function?

{(2,-3),(4,6),(3,-1),(6,6),(2,3)}

Example: Find the domain and range of given function

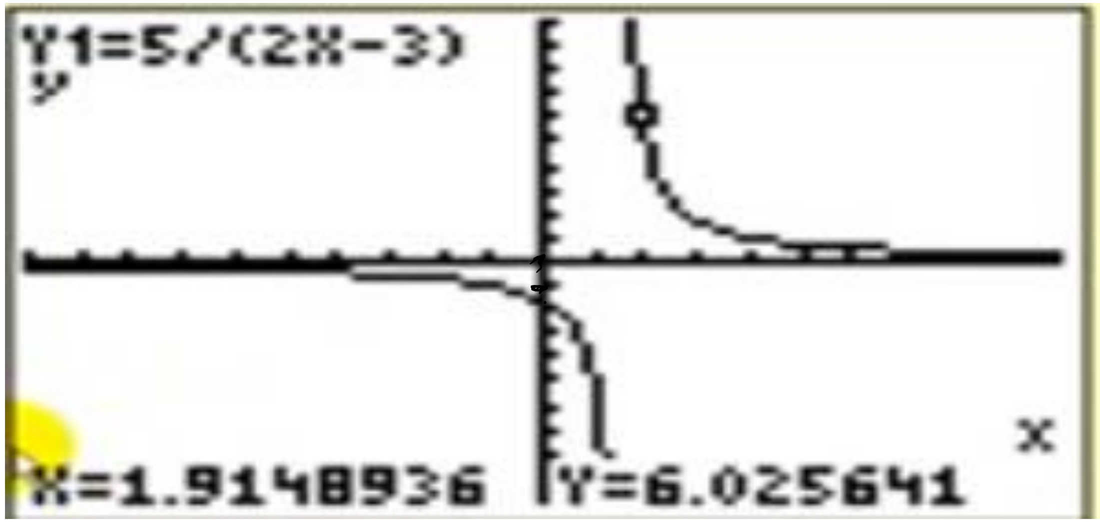
f(x)= $|x| - 1$



Example: Find the domain and range of given function.

f(x)=$\frac{5}{2x-3}$

Graphical method



Algebraic method

Domain:

Range:

Example: Find the domain and range of given function.

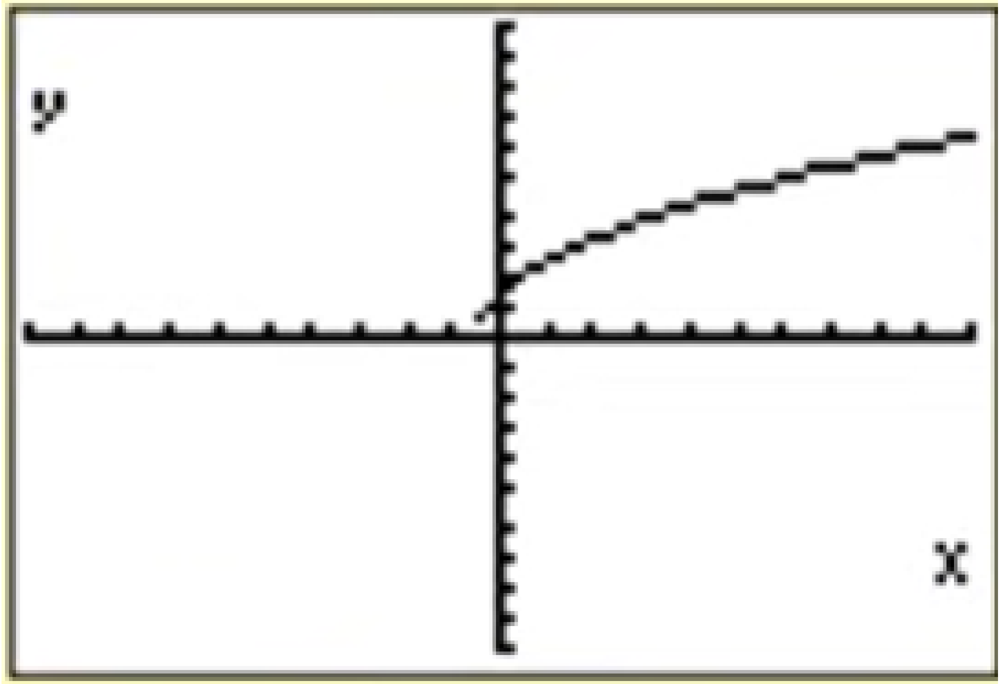
f(x)=$\sqrt{4x+2}$

Graphical method

Algebraic method

Domain:

Range:



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Mathematics

Limits and continuity
Part (1)

• *Dr. MURTAJA ALI SAARE*

1. إيجاد غاية لدالة متعددة حدود

(غير كسرية= لا يوجد متغير في المقام لايوجد متغير له اس سالب)

- $f(x) = x^2 + 3x + 1$
- $f(t) = 2t^2 + t + 1$
- $f(x) = \frac{1}{2}x^2 + 1$
- $f(x) = 1$

حل الغاية لدالة متعددة الحدود هو التعويض المباشر

$$\lim_{x\to 2} x^2 = (2)^2 = 4$$

$$\lim_{x\to 3} x^2 + x = (3)^2 + 3 = 12$$

$$\lim_{t\to 2} \frac{1}{4}t + 1 = \frac{1}{4}(2)+1 = \frac{1}{2} + 1 = \frac{3}{2}$$

$$\lim_{x\to 2} 1 = 1$$

2. إيجاد الغاية لدالة كسرية

الحالة الأولى :عند التعويض تظهر لدينا غاية (المقام لا يساوي صفرا)

$$\lim_{x \to 1} \frac{x^2 + x - 6}{x + 2} = \frac{(1)^2 + (1) - 6}{(1) + 2} = \frac{-4}{3}$$

إيجاد الغاية لدالة كسرية

الحالة الثانية: عند التعويض لا تظهر لدينا غاية (المقام يساوي صفرا)

$$\lim_{x\to 3}\frac{x^2+x-6}{x-3} = \frac{(3)^2+(3)-6}{(3)-3} = \frac{6}{0} = \text{غيرمعرف}$$

التحليل او التبسيط (التجربة او الفرق بين مربعين او فرق بين مكعبين او سحب عامل مشترك او توحيد المقامات او اخرى)

التجربة

- $\lim_{x\to 3}\frac{x^2+x-12}{x-3} = \lim_{x\to 3}\frac{(x+4)(x-3)}{(x-3)} = \lim_{x\to 3}(x+4) = 3+4=7$

- $\lim_{x \to 2} \frac{x^2-4}{x-2} =$

$$\lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)} =$$

$$\lim_{x \to 2} (x+2) = (2)+2=4$$

تحليل فرق بين مربعين

$$a^2 - b^2 = (a-b)(a+b)$$

$$w^2 - 3 = (w - \sqrt{3})(w + \sqrt{3})$$

$$\lim_{x\to 5}\frac{x^3-125}{x-5}=$$

$$\lim_{x\to 5}\frac{(x-5)(x^2+5x+25)}{(X-5)}=$$

$$\lim_{x\to 5}\left(x^2+5x+25\right)$$

$$=(5)^2+5(5)+25=75$$

تحليل الفرق بين مكعبين ومجموع مكعبين

$$a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$$

$$a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$$

سحب عامل مشترك

$$\lim_{x\to 0}\frac{x^2-4x}{x}=$$

$$\lim_{x\to 0}\frac{(x)(x-4)}{x}=$$

$$\lim_{x\to 0}(x-4)=(0)-4= -4$$

الضرب في العامل المرافق

$$\lim_{x\to 0}\frac{\sqrt{x+5}-\sqrt{5}}{x}=\lim_{x\to 0}\frac{\sqrt{x+5}-\sqrt{5}}{x}*\frac{\sqrt{x+5}+\sqrt{5}}{\sqrt{x+5}+\sqrt{5}}$$

$$=\lim_{x\to 0}\frac{x+5+\sqrt{5}*\sqrt{x+5}-\sqrt{5}*\sqrt{x+5}-5}{x(\sqrt{x+5}+\sqrt{5})}=\lim_{x\to 0}\frac{x}{x(\sqrt{x+5}+\sqrt{5})}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{x+5}+\sqrt{5}}=\frac{1}{\sqrt{(0)+5}+\sqrt{5}}=\frac{1}{\sqrt{(5)}+\sqrt{5}}=\frac{1}{2\sqrt{5}}$$

توحيد المقامات

$$\lim_{x\to 0} \frac{5}{x^2+x} - \frac{5}{x} = \lim_{x\to 0} \frac{5}{x(x+1)} - \frac{5}{x} =$$

$$\lim_{x\to 0} \frac{5-5(x+1)}{x(x+1)} =$$

$$\lim_{x\to 0} \frac{+5-5x-5}{x(x+1)}$$

$$= \lim_{x\to 0} \frac{-5x}{x(x+1)} = \lim_{x\to 0} \frac{-5}{(x+1)} = \frac{-5}{(0)+1} = -5$$

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Mathematics

Limits and continuity

Part (2)

• *Dr. MURTAJA ALI SAARE*

أولا: استخدام الغاية لايجاد مماس دالة في نقطة معينة

Ex1: Find the equation of the tangent line to the curve y = √ x at P(1, 1).

أوجد معادلة خط المماس للمنحنى y = √ x عند النقطة (1,1)

$f(x) = \sqrt{x}$

$f(1) = \sqrt{1} = 1$

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to 1} \frac{\sqrt{x} - \sqrt{1}}{x - 1} = \qquad \lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1} = \lim_{x \to 1} \frac{(\sqrt{x} - 1)}{(\sqrt{x} - 1)(\sqrt{x} + 1)}$$

$$= \lim_{x \to 1} \frac{1}{\sqrt{x} + 1} = \frac{1}{\sqrt{1} + 1} = \frac{1}{2}$$

Ex2: Find the equation of the tangent line to the graph of $f(x) = x^2 + 5x$ at the point (1, 6).

أوجد معادلة خط المماس بالرسم البياني لـ $f(x) = x^2 + 5x$ عند النقطة (1,6)

$f(x) = x^2 + 5x$

$f(1+h) = (1+h)^2 + 5(1+h)$

$f(1) = 1^2 + 5(1) = 6$

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

$$\lim_{h \to 0} \frac{[(1+h)^2 + 5(1+h)] - (6)}{h} = \lim_{h \to 0} \frac{[(1+2h+h^2)+5+5h]-(6)}{h}$$

$$= \lim_{h \to 0} \frac{h^2+7h}{h} = \lim_{h \to 0} \frac{h(h+7)}{h} = \lim_{h \to 0}(h+7) = 0 + 7 = 7$$

ثانيا: استخدام الغاية لايجاد مشتقة دالة لنقطة معينة

Ex3: Let f(x) = x^2 + 5x. Find f ` (a)

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

f(x) = x^2 + 5x

f(a) = a^2 + 5a

f(a+h) = $(a+h)^2$ + 5(a+h)

$$\lim_{h \to 0} \frac{[(a+h)^2 + 5(a+h)] - (a^2 + 5a)}{h}$$

$$= \lim_{h \to 0} \frac{[(a^2+2ah+h^2)+5a+5h] - a^2 - 5a}{h} \qquad = \lim_{h \to 0} \frac{2ah+h^2+5h}{h}$$

$$= \lim_{h \to 0} \frac{h(2a+h+5)}{h} \qquad = \lim_{h \to 0}(2a+h+5) = 2a+0+5 = 2a+5$$

ثانيا: استخدام الغاية لايجاد مشتقة دالة لنقطة معينة

Ex 4: Let f(x) = x^2 + 5x. Find f ` (2)

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

f(x) = x^2 + 5x

f(2) = 2^2 + 5(2)=14

f(2+h) = $(2+h)^2$ + 5(2+h)

$$\lim_{h \to 0} \frac{[(2+h)^2 + 5(2+h)] - (2^2 + (5 * 2))}{h}$$

$$= \lim_{h \to 0} \frac{[(4+4h+h^2)+10+5h]-4-10}{h}$$

$$= \lim_{h \to 0} \frac{h^2+9h}{h}$$

$$= \lim_{h \to 0} \frac{h(h+9)}{h}$$

$$= \lim_{h \to 0} (h+9) = 0 + 9 = 9$$

ثانيا: استخدام الغاية لايجاد مشتقة دالة لنقطة معينة

Ex5: Let $f(x) = x^2 + 5x$. Find $f`(-1)$

$f(x) = x^2 + 5x$

$f(-1) = (-1)^2 + 5(-1)=4$

$f(-1+h) = (-1+h)^2 + 5(-1+h)$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{h\to 0} \frac{[(-1+h)^2 + 5(-1+h)] - (-1^2 + (5 * -1))}{h}$$

$$= \lim_{h\to 0} \frac{[(1-2h+h^2) - 5 + 5h] - 1 + 5}{h} \qquad = \lim_{h\to 0} \frac{h^2+3h}{h}$$

$$= \lim_{h\to 0} \frac{h(h+3)}{h} \qquad = \lim_{h\to 0}(h+3) = 0 + 3 = 3$$

ثالثا: استخدام الغاية لايجاد السرعة اللحظية في زمن محدد

Ex6: The position function of a stone thrown from a bridge is given by s(t) = 10t − 16t² feet (below the bridge) after t seconds. (a) What is the average velocity of the stone between $t_1 = 1$ and $t_2 = 5$ seconds?

تم إعطاء دالة موضع حجر عند رميه من جسر بواسطة $s(t) = 10t - 16t^2$ قدم (أسفل الجسر) بعد t ثانية.
(أ) ما هو متوسط سرعة الحجر بين t1 = 1 و t2 = 5 ثوان؟ . (لاحظ أن السرعة = | السرعة |).

average velocity= $$\frac{f(t_2) - f(t_1)}{t_2 - t_1}.$$

$s(t) = 10t - 16t^2$

$s(t_1) = s(1) = 10(1) - 16(1)^2 = 10 - 16 = -6$

$s(t_2) = s(5) = 10(5) - 16(5)^2 = 50 - 400 = -350$

the average velocity= $$\frac{-350-(-6)}{5-1} = \frac{-344}{4} = |-86| = 86 feet/sec$$

The position function of a stone thrown from a bridge is given by $s(t) = 10t - 16t^2$ feet (below the bridge) after t seconds. (b) What is the instantaneous velocity of the stone at t = 1 second. (Note that speed = |Velocity|).

ب .ما هي السرعة اللحظية للحجر عند t = 1 ثانية

$$v(a) = \lim_{t \to a} \frac{f(t) - f(a)}{t - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\lim_{h \to 0} \frac{[10(1+h) - 16(1+h)^2] - (10(1) - 16(1)^2)}{h}$$

$s(t) = 10t - 16t^2$

$s(a) = s(1) = 10(1) - 16(1)^2 = 10 - 16 = -6$

$s(a+h) = s(1+h) = 10(1+h) - 16(1+h)^2$

$$= \lim_{h \to 0} \frac{[(10+10h) - 16(1+2h+h^2)] - (-6)}{h}$$

$$= \lim_{h \to 0} \frac{10 + 10h - 16 - 32h - 16h^2 + 6}{h}$$

$$= \lim_{h \to 0} \frac{-22h - 16h^2}{h} = \lim_{h \to 0} \frac{h(-22 - 16h)}{h} = \lim_{h \to 0} (-22 - 16h) = -22 - 16(0) = -22$$

4. استخدام الغاية لايجاد معدل التغير لدالة معين عند قيمة معينة

Ex7:The cost (in dollars) of producing x units of a certain commodity is C(x) = 50 + √ x.

(a) Find the average rate of change of C with respect to x when the production level is changed from x = 100 to x = 169.

تكلفة (بالدولار) لإنتاج وحدات x لسلعة معينة هي C (x) = 50 + √ x. (أ) أوجد متوسط معدل التغير لـ C فيما يتعلق بـ x عندما يتم تغيير مستوى الإنتاج من x = 100 إلى x = 169.

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{\Delta x}{\Delta y} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{50 + \sqrt{169} - (50 + \sqrt{100})}{169 - 100}$$

$$= \frac{13 - 10}{69} = \frac{3}{69} = .04347.$$

C(x) = 50 + √ x

C(100) = 50 + √ 100=50+10=60

C(169) = 50 + √ 169=50+13=63

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limits and continuity part3

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1. f(x) is defined

2. $\lim_{x \to a} f(x)$ → exists

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

3. $\lim_{x \to a} f(x) = f(a)$

Example

$f(x)=x^3+ux^2+5\ ,\ if\ x=1$ is it the function continuous or not

Solution

1. $f(1)=(1)^3+u(1)^2+5=10$

2. $\lim_{x\to1^+} f(x)=(1)^3+u(1)^2+5=10$

3. $f(x)=\lim_{x\to1} f(x)$ It does Continuous

Example

$$f(x) = \begin{cases} \sqrt{x+2}, & x < 2 \\ x^2 - 2, & 2 \le x < 3 \\ 2x + 5, & x \ge 3 \end{cases}$$

1. $f(x)=(2)^2-2 = 2$
2. $\lim_{x\to 2^-} f(x)=\sqrt{2+2}=2$
3. $\lim_{x\to 2^+} f(x)=(2)^2-2=2$

$$\lim_{x\to 2} f(x)=(2)=2$$

Example

$$f(x) = \begin{cases} \sqrt{x+2}, & x < 2 \\ x^2 - 2, & 2 \le x < 3 \\ 2x+5, & x \ge 3 \end{cases}$$

1. $f(3)$=2(3)+5 $= 11$

2. $\lim_{x\to 3^-} f(x)$= $(3)^2$-2=7

3. $\lim_{x\to 3^+} f(x)$=2(3)+5=11

$\lim_{x\to 3} f(x)$= It does not Continuous

Example

$$f(x) = \begin{cases} 2x + 5, & x < -1 \\ x^2 + 2, & x > -1 \end{cases}$$

1. $f(x)$=$2(-1) + 5$=3
2. $\lim_{x \to -1^-} f(x)$=$2(-1) + 5$=3
3. $\lim_{x \to -1^+} f(x)$=$(-1)^2$+1=3

$\lim_{x \to -1} f(x)$=(3)= It does Continuous

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Mathematics

Numerical Integration

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Numerical Integration

The numerical integration methods are approximate rules for evaluating definite integral. It used when we cannot compute the value of an integral exactly and specially useful for approximately integral of functions that are available only in graphical or tabular form.
In the present study, we will deal with three methods:

1. Trapezoidal method
2. Mid point method
3. Simpson's rule method.

A-Trapezoidal Method:

If we have the function $y = f(x)$ and we want to estimate $\int_a^b f(x)dx$

If we have n divisions

$$h = \frac{b-a}{n}$$

A = total area under the curve

$y = f(x)$ from $x = a$ to $x = b$

$$A \approx a_1 + a_2 + a_3 + \ldots\ldots a_n = \sum_{i=1}^{n} a_i$$

$$a_1 = \left(\frac{y_o+y_1}{2}\right)h,\ \ a_2 = \left(\frac{y_1+y_2}{2}\right)h,\ \ a_3 = \left(\frac{y_2+y_3}{2}\right)h,\ \ a_n = \left(\frac{y_{n-1}+y_n}{2}\right)h$$

$$A = \left(\frac{y_o+y_1}{2}\right)h + \left(\frac{y_1+y_2}{2}\right)h + \left(\frac{y_2+y_3}{2}\right)h + \ldots\ldots \left(\frac{y_{n-1}+y_n}{2}\right)h$$

$$A \approx h\left[\frac{y_0}{2} + \frac{y_1}{2} + \frac{y_1}{2} + \frac{y_2}{2} + \frac{y_2}{2} + \frac{y_3}{2} + \frac{y_3}{2} + \ldots\ldots \frac{y_{n-1}}{2} + \frac{y_{n-1}}{2} + \frac{y_n}{2}\right]$$

$$A \approx h\left[\frac{y_0}{2} + y_1 + y_2 + y_3 + \ldots\ldots y_{n-1} + \frac{y_n}{2}\right]$$

$$A = \frac{h}{2}[y_0 + 2y_1 + 2y_2 + \ldots\ldots\ldots + 2y_{n-1} + y_n]$$

Where,

$y_0 \longrightarrow y_n$: values of the function $f(x)$ at $X_0 \longrightarrow X_n$

$y_0 = f(a)$, $y_n = f(b)$

h = width of Trapezoid = (b – a)/n

n = number of divisions

Example 9: use Trapezoidal rule to approximate $\int_1^6 (x^3+3)dx$; n = 6, then compare the result with the exact value and find the percentage of error.

Solution:

$$h = \frac{6-1}{6} = \frac{5}{6}$$

$$A = \frac{h}{2}[y_0 + 2y_1 + 2y_2 + 2y_3 + 2y_4 + 2y_5 + y_6]$$

x	$y = f(x) = x^3 + 3$
$x_0 = a = 1$	$f(a) = y0 = 4$
$x_1 = 1 + (5/6) = 11/6$	$y_1 = 9.16$
$x_2 = 16/6$	$y_2 = 21.96$
$x_3 = 21/6$	$y_3 = 45.87$

$x_4 = 26/6$	$y_4 = 84.37$
$x_5 = 31/6$	$y_5 = 140.92$
$x_6 = 36/6 = 6$	$y_6 = 219$

$A = \frac{5}{12}[4 + 2(9.16) + 2(21.96) + 2(45.87) + 2(84.37) + 2(140.92) + 219]$

$= 344.82$

The exact value $= \int_1^6 (x^3 + 3)dx$

$= [\frac{x^4}{4} + 3x]_1^6 = 338.75$

The percentage of error $= \frac{344.82 - 338.75}{338.75} . 100 = 1.8\ \%$

Example 10: Find the upper bound error estimate from using the Trapezoidal method with n = 10 for the integral $\int_0^1 x \sin x$

Solution:

a = 0, b = 1, n = 10, h = (1 – 0)/10 = 1/10

$f(x) = x \sin x$

$f(x) = x \cos x + \sin x$

$f(x) = x(-\sin x) + \cos x + \cos x = 2\cos x - x \sin x$

$D = 2$

$$|E_T| \le \frac{b-a}{12} h^2 D$$

$$|E_T| \le \frac{1}{12}\left(\frac{1}{10}\right)^2 (2)$$

$$|E_T| \le \frac{1}{600}$$

Mid-Point Method

In this method, the area under the curve is divided into a number of rectangles. The curve intersects each rectangle at the mid-point of the top side.

$$A \approx M = \sum_{i=1}^{n} f(ck) \,.\, h$$

n = number of rectangles

C_k : X- coordinate for the midpoint.

The error estimate for midpoint method is:

$$|E_M| \le \frac{b-a}{24} h^2 D$$

Example 11: Estimate $\int_1^2 x^2 dx$ with n = 4 by Midpoint method.

Solution: a = 1, b = 2, n = 4

h = (2-1)/4 = ¼

C_k	f (C_k)
C1 = 1 + (1/4)/2 = 9/8	f(C1) = $(9/8)^2$ = 81/64
C2 = 9/8 + ¼ = 11/8	f(C2) = $(11/8)^2$ = 121/64
C3 = 11/8 + ¼ = 13/8	f(C3) = 169/64
C4 = 13/8 + ¼ = 15/8	f(C4) = 225/64

A ≈ ¼ [81/64 + 121/64 + 169/64 + 225/64] = 149/64 = 2.328125

$f(x) = x^2$, $f(x) = 2x$, $f(x) = 2$, $D = 2$

$$|E_M| \leq \frac{2-1}{24}\left(\frac{1}{4}\right)^2 (2)$$

$$|E_M| \leq \frac{1}{192}$$

$$|E_M| \leq 0.005208$$

Simpson's Rule

Simpson's rule is based on approximately curves with parabolas instead of line segments. Each three points are connected with a parabola.

The general equation of parabola is $y = Ax^2 + Bx + C$

$$da = y\,dx = \int_{-h}^{h} y\,dx$$

$$Ar = \int_{-h}^{h} (Ax^2 + Bx + C)dx$$

$$Ar = [\frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx]_{-h}^{h}$$

$$Ar = \frac{Ah^3}{3} + \frac{Bh^2}{2} + Ch - (\frac{-Ah^3}{3} + \frac{Bh^2}{2} - Ch)$$

$$= \frac{2Ah^3}{3} + 2Ch$$

$$Ar = \frac{1}{3}(2Ah^3 + 6Ch) = \frac{h}{3}(2Ah^2 + 6C) = \text{Exact Area}$$

Since the curve passes through the points $(-h, y_0)$, $(0, y_1)$, (h, y_2) then:

$$Y_0 = Ah^2 - Bh + C \text{ .... (1)}$$

$Y_1 = C$ … ....(2)

$Y_2 = Ah^2 + Bh + C$ ....(3)

$Y_0 - y_1 = Ah^2 - Bh$

$Y_2 - y_1 = Ah^2 + Bh$

$Y_0 - y_1 + y_2 - y_1 = 2Ah^2$ $\quad Ar = \frac{h}{3}[y_0 + y_2 - 2y_1 + 6C]$

$Ar = \frac{h}{3}[y_0 + y_2 - 2y_1 + 6C] = \frac{h}{3}[y_0 + 4y_1 + y_2]$

Now: $Ar1 = \frac{h}{3}[y_0 + 4y_1 + y_2]$

$$Ar2 = \frac{h}{3}[y_2 + 4y_3 + y_4]$$

Total area = Ar1 + Ar2 $= \frac{h}{3}[y_0 + 4y_1 + y_2 + y_2 + 4y_3 + y_4]$

$= \frac{h}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4]$

$$A = S = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots \ldots . +2y_{n-2} + 4y_{n-1} + y_n)$$

Example 12: Estimate $\int_1^2 x^2 dx$ with n = 4 by Simpson's rule.

Solution: a = 1, b = 2, n = 4

h = (2 – 1)/4 = ¼

$$A = S = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$$

x	y
$x_0 = 1$	$y_0 = 1$
$x_1 = 5/4$	$y_1 = 25/16$
$x_2 = 6/4$	$y_2 = 36/16$
$x_3 = 7/4$	$y_3 = 49/16$
$x_4 = 2$	$y_4 = 4$

$$A = \frac{1}{4*3}[1 + 4\left(\frac{25}{16}\right) + 2\left(\frac{36}{16}\right) + 4\left(\frac{49}{16}\right) + 4]$$

= 7/3 = 2.333333

The error estimate for Simpson's rule is :

$$|E_S| \le \frac{b-a}{180}h^4 D$$

Example 13: Determine n that will guarantee an accuracy of at least 10^{-7} for using:

1. **Trapezoidal rule**
2. **Simpson's rule**

To approximate $\int_2^4 x^4\,dx$

Solution:

1. **By Trapezoidal rule:**

$$|E_T| \le \frac{b-a}{12} h^2 D$$

a = 2, b = 4, h = (b – a)/n = 2/n

$f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

at x = 4 $\quad f(4) = (4)^2 * 12 = 192 \quad$ D = 192

$|E_T| \le \frac{4-2}{12}\left(\frac{2}{n}\right)^2 * 192$ $|E_T| \le \frac{128}{n^2}$

$\frac{128}{n^2} \le 10^{-7}$ $n^2 \ge 128 * 10^7$ $n \ge 35777.08$ $n \ge 35778$

2. By Simpson's rule

$$|E_S| \le \frac{b-a}{180} h^4 D$$

$f(x) = 12\,x^2$

$f(x) = 24\,x$

$f(x) = 24$ $D = 24$

$|E_S| \le \frac{2}{180}\left(\frac{2}{n}\right)^4 * 24$ $|E_S| \le \frac{64}{15n^4}$

$\frac{64}{15n^4} \le 10^{-7}$ $n \ge 80.82$ $n = 82$

Example 14: The table below shows the velocity of submarine with the travelling time. Use Simpson's rule to estimate the distance travelled during the 10 hours period.

t (hr)	*v (mph)*
0	12
1	14
2	17
3	21
4	22
5	21
6	15
7	11
8	11
9	14
10	17

Solution:

$v = \frac{ds}{dt}$ $\quad\quad ds = v.\, dt$

$$s = \int_0^{10} v(t).\, dt$$

$h = 1,\ n = 10$

S = 1/3 [12 + 4(14) + 2 (17) + 4(21) + 2(22) + 4(21) + 2(15) + 4(11) + 2(11) + 4(14) + 17]

S = 161 mile

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Mathematics

Rules of integral

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The integral of $k\,f(x)$ where k is a constant

A constant factor in an integral can be moved outside the integral sign as follows:

Key Point 1

$$\int k\,f(x)\,dx = k\int f(x)\,dx$$

Find the indefinite integral of $11x^2$: that is, find $\int 11x^2\,dx$

Solution

$$\int 11x^2\,dx = 11\int x^2\,dx = 11\left(\frac{x^3}{3}+c\right) = \frac{11x^3}{3} + K \quad \text{where } K \text{ is a constant.}$$

Find the indefinite integral of $-5\cos x$; that is, find $\int -5\cos x \, dx$

Solution

$$\int -5\cos x \, dx = -5\int \cos x \, dx = -5\left(\sin x + c\right) = -5\sin x + K \quad \text{where } K \text{ is a constant.}$$

The integral of $f(x) + g(x)$ and of $f(x) - g(x)$

When we wish to integrate the sum or difference of two functions, we integrate each term separately as follows:

Key Point 2

$$\int [\,f(x) + g(x)\,]\;dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int [\,f(x) - g(x)\,]\;dx = \int f(x)\,dx - \int g(x)\,dx$$

Find $\int (x^3 + \sin x)\, dx$

Solution

$$\int (x^3 + \sin x)\, dx = \int x^3\, dx + \int \sin x\, dx = \frac{1}{4}x^4 - \cos x + c$$

Note that only a single constant of integration is needed.

Find $\displaystyle\int (3t^4 + \sqrt{t})\, dt$

The hyperbolic sine and cosine functions, $\sinh x$ and $\cosh x$, are defined as follows:

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

Note that they are combinations of the exponential functions e^x and e^{-x}.
Find the indefinite integrals of $\sinh x$ and $\cosh x$.

$$\int \sinh x \, dx = \int \left(\frac{e^x - e^{-x}}{2} \right) dx =$$

Answer

$$\int \sinh x \, dx = \frac{1}{2} \int e^x \, dx - \frac{1}{2} \int e^{-x} \, dx = \frac{1}{2} e^x + \frac{1}{2} e^{-x} + c = \frac{1}{2} (e^x + e^{-x}) + c = \cosh x + c.$$

$$\int \cosh x \, dx = \int \left(\frac{e^x + e^{-x}}{2} \right) dx =$$

Exercises

1. Find $\int (2x - \mathrm{e}^x)\, dx$
2. Find $\int 3\mathrm{e}^{2x}\, dx$
3. Find $\int \frac{1}{3}(x + \cos 2x)\, dx$
4. Find $\int 7x^{-2}\, dx$
5. Find $\int (x+3)^2\, dx$, (be careful!)

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Mathematics

Triangulation Inverse

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$\tan^{-1} x$ $\cot^{-1} x$ (−) $\frac{1}{1+x^2}$	$\tanh^{-1} x$ $\frac{1}{1-x^2}$	$\coth^{-1} x$ $\frac{1}{1-x^2}$
$\sin^{-1} x$ $\cos^{-1} x$ (−) $\frac{1}{\sqrt{1-x^2}}$	$\sinh^{-1} x$ $\frac{1}{\sqrt{1+x^2}}$	$\cosh^{-1} x$ $\frac{1}{\sqrt{x^2-1}}$
$\sec^{-1} x$ $\csc^{-1} x$ (−) $\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	(−) $\operatorname{sech}^{-1} x$ $\frac{1}{x\sqrt{1-x^2}}$	$\operatorname{csch}^{-1} x$ $\frac{1}{\lvert x\rvert\sqrt{1+x^2}}$

Example 1

$$y = 5\, cos^{-1}\,(1 - x^2) \text{ find } y^-$$

sol

$$y^- = \frac{-1}{\sqrt{1-(1-x^2)^2}} \; *\text{-2x}$$

$$= \frac{10x}{\sqrt{1-(1-x^2)^2}}$$

Example 2

$$if\ f(r) = cot^{-1}\left(\frac{2}{r}\right) + tan^{-1}\left(\frac{r}{2}\right), \text{ prove that } f(x)^{-} = \frac{4}{r^2+4}$$

sol

$$= \frac{-1}{1+(\frac{2}{r})^2} * \frac{-2}{r^2} + \frac{1}{1+(\frac{r}{4})^2} * \frac{1}{2}$$

$$= \frac{1}{\frac{r^2+4}{r^2}} * \frac{2}{r^2} + \frac{2}{\frac{4+r^2}{4}} * \frac{1}{2}$$

$$= \frac{2}{r^2+4} + \frac{2}{r^2+4}$$

$$= \frac{2}{r^2+4} + \frac{2}{r^2+4} \Longrightarrow f(x)^{-} = \frac{4}{r^2+4}$$

Example 3

$$f(x) = x^2 cos^{-1}\left(\frac{1}{x}\right) - \sqrt{x^2 - 1} \text{ find f}(x)^{-}$$

sol

$$= x^2 * \frac{-1}{\sqrt{1-(\frac{1}{x})^2}} * (-\frac{1}{x^2}) + cos^{-1}\frac{1}{x} * (2x) - \frac{2x}{2\sqrt{(x)^2-1}}$$

$$\text{f}(x)^{-} = \frac{1}{\sqrt{\frac{x^2-1}{x^2}}} + 2x\, cos^{-1}\frac{1}{x} - \frac{x}{\sqrt{x^2-1}}$$

$$\text{f}(x)^{-} = \frac{x}{\sqrt{x^2-1}} + 2x\, cos^{-1}\frac{1}{x} - \frac{x}{\sqrt{x^2-1}}$$

$$\text{f}(x)^{-} = 2x\, cos^{-1}\frac{1}{x}$$

Example 2

$if\ f(k) = k\sqrt{a^2 - k^2} + a^2\ sin^{-1}\left(\frac{k}{a}\right)$, prove that f$(x)^{-}$=2$\sqrt{a^2 - k^2}$ $, a\ is\ cosistant$

sol

$$= k * \frac{-2k}{2\sqrt{a^2-k^2}} + \sqrt{a^2 - k^2} + a^2 * \frac{1}{\sqrt{\frac{1-k^2}{a^2}}} * \frac{1}{a}$$

$$= \frac{-k}{\sqrt{a^2 - k^2}} + \sqrt{a^2 - k^2} + \frac{a^2}{2\sqrt{a^2 - k^2}}$$

$$= \frac{-k^2 + a^2 - k^2 + a^2}{\sqrt{a^2 - k^2}}$$

$$= \frac{2\,a^2 - 2\,k^2}{\sqrt{a^2 - k^2}} \Longrightarrow = \frac{2\,(a^2 - k^2)}{\sqrt{a^2 - k^2}} = 2\sqrt{a^2 - k^2}$$

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Mathematics

Trigonometric Function

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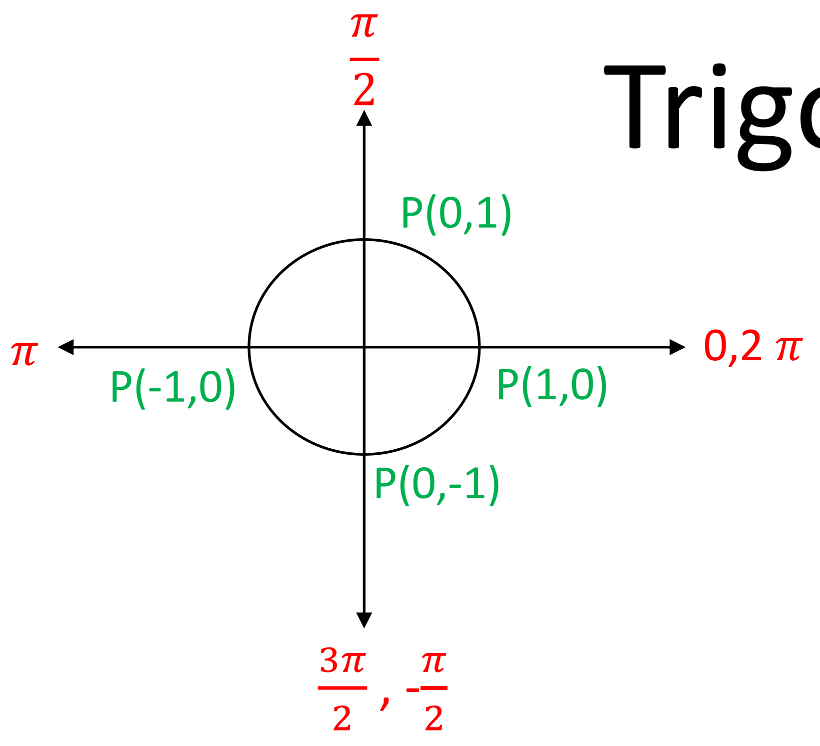
Trigonometric Function



P(Cos,Sin)

$\tan = \frac{Sin}{Cos}$

$Sec = \frac{1}{Cos}$

$Csc = \frac{1}{Sin}$

Example

$\text{Cos}(0)=1$

$\text{Sin}(\pi) = 0$

$\text{Cos}(\frac{3\pi}{2})=0$

$\text{Sin}(\frac{\pi}{2})=1$

$\tan(0)=\frac{Sin\ 0}{Cos\ 0} = \frac{0}{1} = 0$

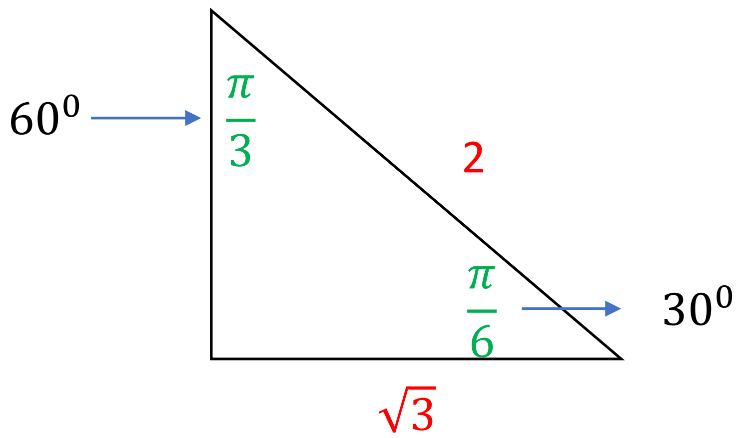
$\tan(\pi) = \frac{Sin\ \pi}{Cos\ \pi}= \frac{0}{-1}=0$

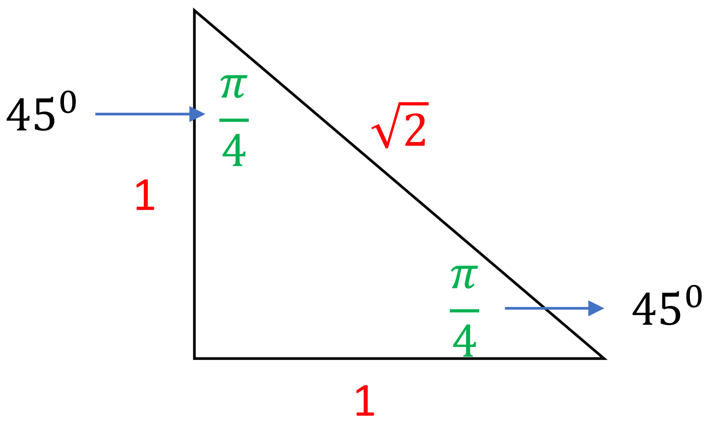
$\text{Sec}(\pi) = \frac{1}{Cos\ \pi}= \frac{1}{-1}$

$\text{Csc}(\frac{\pi}{2})= \frac{1}{Sin\frac{\pi}{2}} =\frac{1}{1} = 1$

$\text{Sec}(0)= \frac{1}{Cos\ (0)} = \frac{1}{1} =1$

$\text{Csc}(-\frac{\pi}{2})= \frac{1}{Sin\ (-\frac{\pi}{2})}= \frac{1}{-1}=-1$





Sin = $\frac{opposite}{hypotenuse}$ = $\frac{\text{المقابل}}{\text{الوتر}}$

Cos = $\frac{adjacent}{hypotenuse}$ = $\frac{\text{المجاور}}{\text{الوتر}}$

tan = $\frac{opposite}{adjacent}$ = $\frac{\text{المقابل}}{\text{المجاور}}$

Example:

$$Cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$Sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$$

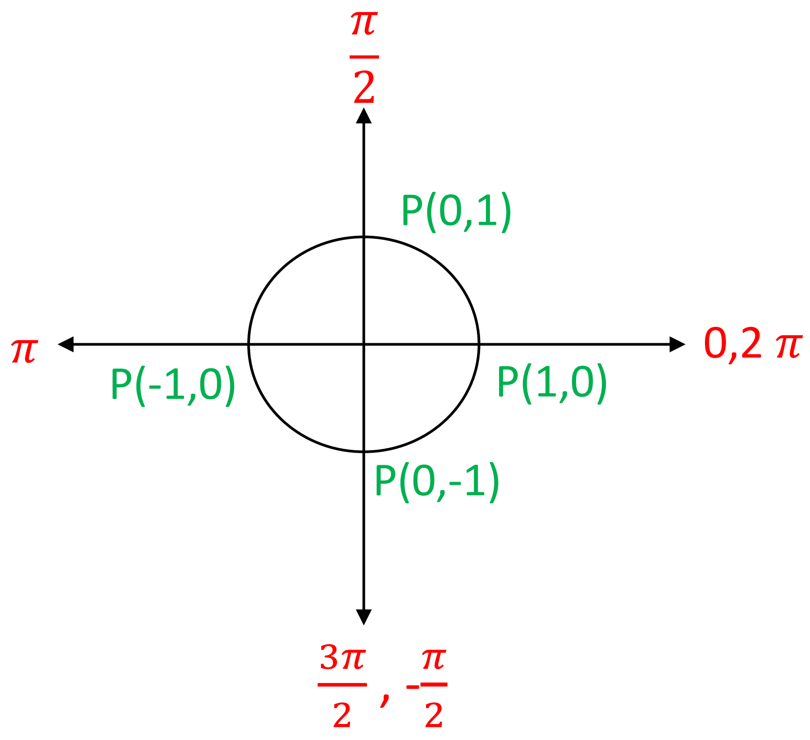
$$cot\left(\frac{\pi}{4}\right) = 1$$

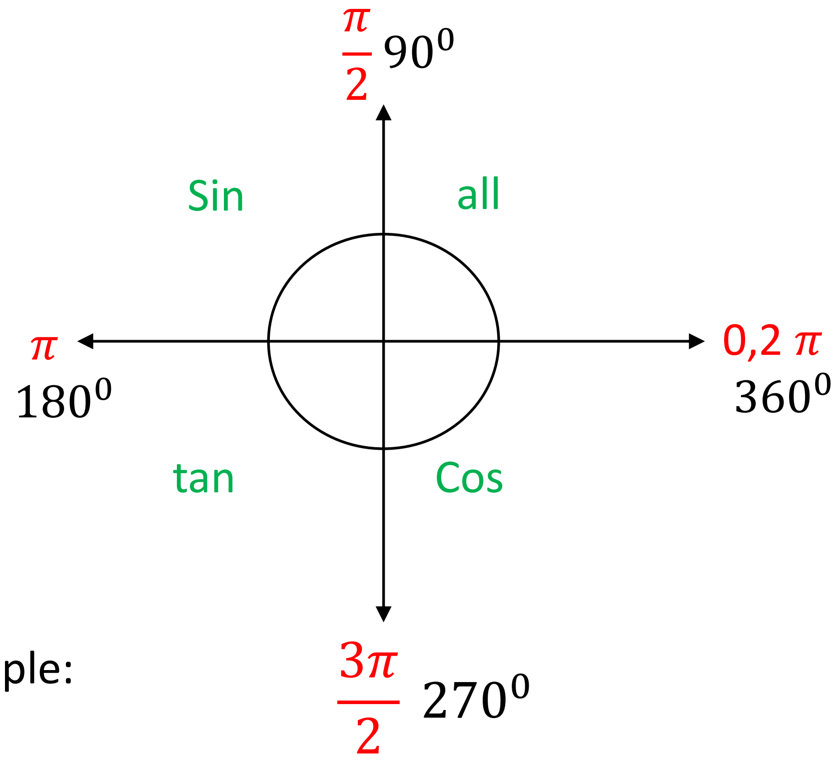
$$sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$$

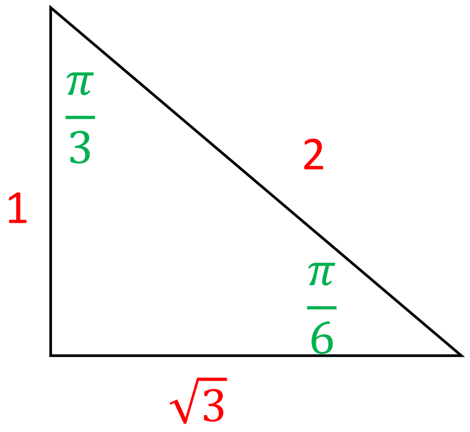
$$\text{Csc}\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$$

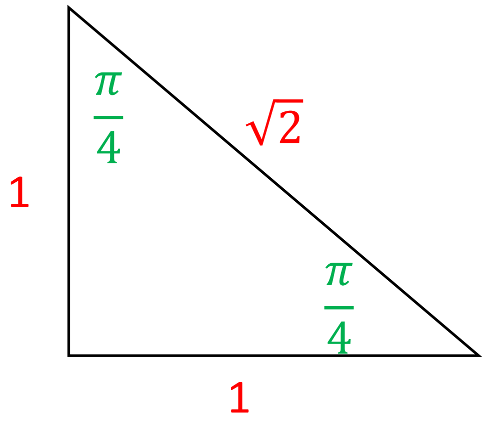
$$\tan\left(\frac{\pi}{4}\right) = 1$$

$$\text{Cos}\ (\pi) = -1$$









Example:

$$Cos\left(\frac{5\,\pi}{6}\right) = -Cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$Sin\left(\frac{3\pi}{4}\right) = Sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$tan\left(\frac{2\,\pi}{3}\right) = -tan\left(\frac{\pi}{3}\right) = -\sqrt{3}$$

$$sec\left(\frac{5\,\pi}{4}\right) = -sec\left(\frac{\pi}{4}\right) = -\sqrt{2}$$

$$\mathrm{Csc}\left(\frac{4\pi}{3}\right) = -\mathrm{Csc}\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$$

$$cot\left(\frac{7\pi}{6}\right) = cot\left(\frac{\pi}{6}\right) = \sqrt{3}$$

$$\mathrm{Cos}\left(\frac{7\pi}{4}\right) = \mathrm{Cos}\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\mathrm{Sin}\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Homework

(a) $\text{cosec}\left(\frac{5\pi}{3}\right)$

(b) $\sec\left(-\frac{7\pi}{2}\right)$

(c) $\cot\left(\frac{7\pi}{6}\right)$

(d) $\sec\left(\frac{3\pi}{4}\right)$

(e) $\cot(6\pi)$

(a) $\sin(\frac{5\pi}{3})$

(b) $\cos(\frac{5\pi}{4})$

(c) $\tan(\frac{7\pi}{6})$

(d) $\sin(2\pi)$

(e) $\cos(-\frac{7\pi}{2})$

(f) $\sin(-\frac{7\pi}{4})$

(g) $\tan(-\frac{17\pi}{6})$