SHATT AL-ARAB UNIVERSITY COLLEGE

DEPARTMENT CIVIL ENG.

FLUID MECHANICS-I

SECOND YEAR

DR. JASIM AL-BATTAT

The Syllabus

1. Introductory Concepts of Fluid Mechanics
2. Properties of a Fluid
3. Fluid Static
4. Fluid Dynamics

References

1. Fluid Mechanics by Streeter and Wylie.
2. Fluid Mechanics for Engineers by Albertson, Barton, and Simons.
3. Fluid Mechanics by Hydraulics (Schaum's Series) by Griles
4. ميكانيك الموائع – دز نزار علي سبتي
5. مبادئ ميكانيك الموائع – د. جميل الملائكة
6. Fluid Mechanics with Engineering Applications by Daugherty, Franzini, and Finnemore.
7. Elementary Fluid Mechanics by Vennard and Street.

Chapter One

Introductory Concepts of Fluid Mechanics

1.1. The Concept of a Fluid and Fluid Mechanics

Mechanics is the oldest physical science that deals with both stationary and moving bodies under the influence of forces. The branch of mechanics that deals with bodies at rest is called statics, while the branch that deals with bodies in motion is called dynamics. The subcategory fluid mechanics is defined as the science that deals with the behavior of fluids at rest (fluid statics) or in motion (fluid dynamics), and the interaction of fluids with solids or other fluids at the boundaries. Fluid mechanics is also referred to as fluid dynamics by considering fluids at rest as a special case of motion with zero velocity. Fluid is a substance that deforms continuously when subjected to shear stress, no matter how small that shear stress may be. Fluids may be either liquids or gases. Solids, as compared to fluids, cannot be deformed permanently (plastic deformation) unless a certain value of shear stress (called the yield stress) is exerted on it. Figure 1.1 illustrates a solid block resting on a rigid plane and stressed by its own weight. The solid sags into a static deflection, shown as a highly exaggerated dashed line, resisting shear without flow. A free-body diagram of element A on the side of the block shows that there is shear in the block along a plane cut at an angle θ through A. Since the block sides are unsupported, element A has zero stress on the left and right sides and compression stress $\sigma = -p$ on the top and bottom. Mohr's circle does not reduce to a point, and there is nonzero shear stress in the block.

Prior to fluid mechanics, statics, and dynamics, was taken, involve solid mechanics. Mechanics is the field of science focused on the motion of material bodies. Mechanics involves force, energy, motion, deformation, and material properties. When mechanics applies to material bodies in the solid phase, the discipline is called solid mechanics. When the material body is in the gas or liquid phase, the discipline is called fluid mechanics. In contrast to a solid, a fluid is a substance whose molecules move freely past each other. More specifically, a fluid is a substance that will continuously deform [that is, flow under the action of a shear stress]. Alternatively, a solid will deform under the action of a shear stress but will not flow like a fluid. Both liquids and gases are classified as fluids.

This lecture notes introduces fluid mechanics by describing gases, liquids, and the continuum assumption. This lecture notes also presents an approach for using units and primary dimensions in fluid mechanics calculations.

1.2 Liquids and Gases

Liquids and gases differ because of forces between the molecules. As shown in the figure 1.1, a liquid will take the shape of a container whereas a gas will expand to fill a closed container. The behavior of the liquid is produced by strong attractive force between the molecules. This strong attractive force also explains why the density of a liquid is much higher than the density of gas. A ***gas*** is a phase of material in which molecules are widely spaced, molecules move about freely, and forces between molecules are minuscule, except during collisions. Alternatively, a ***liquid*** is a phase of material in which molecules are closely spaced, molecules move about freely, and there are strong attractive forces between molecules.

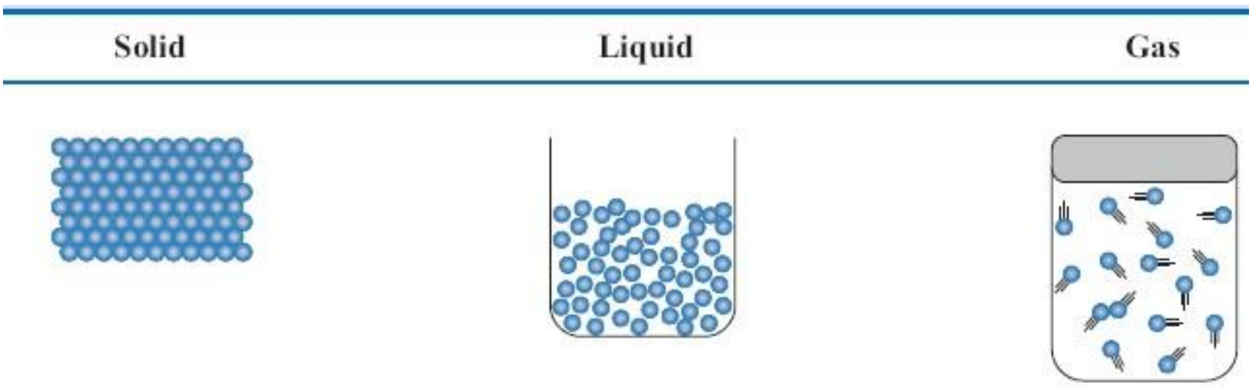


Figure 1.1

1.3 Application Areas of Fluid Mechanics

Why are we studying fluid mechanics on a Civil Engineering course? The provision of adequate water services such as the supply of potable water, drainage, sewerage is essential for the development of industrial society. It is these services which civil engineers provide. Fluid mechanics is involved in nearly all areas of Civil Engineering either directly or indirectly. Some examples of direct involvement are those where we are concerned with manipulating the fluid:

- Sea and river (flood) defenses;
- Water distribution / sewerage (sanitation) networks;
- Hydraulic design of water/sewage treatment works;
- Dams;
- Irrigation;
- Pumps and Turbines;
- Water retaining structures.

And some examples where the primary object is construction - yet analysis of the fluid mechanics is essential:

- Flow of air around buildings;
- Bridge piers in rivers;
- Ground-water flow.

1.4 Dimensions and Units,

A dimension is the measure by which a physical variable is expressed quantitatively. A unit is a particular way of attaching a number to the quantitative dimension. In fluid mechanics there are only four primary dimensions from which all other dimensions can be derived: mass, length, time, and temperature. These dimensions and their units in both systems are given in Table 1.1. Note that the kelvin unit uses no degree symbol. The braces around a symbol like [M] mean “the dimension” of mass. All other variables in fluid mechanics can be expressed in terms of [M], [L], [T], and [Θ]. For example, acceleration has the dimensions [LT^2].

Table 1.1: Primary Dimensions in SI and BG Systems.

Primary dimension	SI unit	BG unit	Conversion factor
Mass $\{M\}$	Kilogram (kg)	Slug	1 slug = 14.5939 kg
Length $\{L\}$	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Time $\{T\}$	Second (s)	Second (s)	1 s = 1 s
Temperature $\{\Theta\}$	Kelvin (K)	Rankine (°R)	1 K = 1.8°R

A list of some important secondary variables in fluid mechanics, with dimensions derived as combinations of the four primary dimensions, is given in Table 1.2.

Table 1.2: Secondary Dimensions in Fluid Mechanics.

Secondary dimension	SI unit	BG unit	Conversion factor
Area $\{L^2\}$	m^2	ft^2	$1\ m^2 = 10.764\ ft^2$
Volume $\{L^3\}$	m^3	ft^3	$1\ m^3 = 35.315\ ft^3$
Velocity $\{LT^{-1}\}$	m/s	ft/s	1 ft/s = 0.3048 m/s
Acceleration $\{LT^{-2}\}$	m/s^2	ft/s^2	$1\ ft/s^2 = 0.3048\ m/s^2$
Pressure or stress $\{ML^{-1}T^{-2}\}$	$Pa = N/m^2$	lbf/ft^2	$1\ lbf/ft^2 = 47.88\ Pa$
Angular velocity $\{T^{-1}\}$	s^{-1}	s^{-1}	$1\ s^{-1} = 1\ s^{-1}$
Energy, heat, work $\{ML^2T^{-2}\}$	$J = N \cdot m$	$ft \cdot lbf$	$1\ ft \cdot lbf = 1.3558\ J$
Power $\{ML^2T^{-3}\}$	W = J/s	$ft \cdot lbf/s$	$1\ ft \cdot lbf/s = 1.3558\ W$
Density $\{ML^{-3}\}$	kg/m^3	$slugs/ft^3$	$1\ slug/ft^3 = 515.4\ kg/m^3$
Viscosity $\{ML^{-1}T^{-1}\}$	$kg/(m \cdot s)$	$slugs/(ft \cdot s)$	$1\ slug/(ft \cdot s) = 47.88\ kg/(m \cdot s)$
Specific heat $\{L^2T^{-2}\Theta^{-1}\}$	$m^2/(s^2 \cdot K)$	$ft^2/(s^2 \cdot °R)$	$1\ m^2/(s^2 \cdot K) = 5.980\ ft^2/(s^2 \cdot °R)$

Example 1:

A body weighs 1000 lbf when exposed to a standard earth gravity $g = 32.174\ ft/s^2$.
(*a*) What is its mass in kg?
(*b*) What will the weight of this body be in N if it is exposed to the moon's standard acceleration $g_{moon} = 1.62\ m/s^2$?
(*c*) How fast will the body accelerate if a net force of 400 lbf is applied to it on the moon or on the earth?

Solution:

Part (a) We can express Eq. (1) dimensionally, using braces by entering the dimensions of each term from Table 1.2:

$$\{ML^{-1}T^{-2}\} = \{ML^{-1}T^{-2}\} + \{ML^{-3}\}\{L^2T^{-2}\} + \{ML^{-3}\}\{LT^{-2}\}\{L\}$$

$$= \{ML^{-1}T^{-2}\} \text{ for all terms} \qquad \textit{Ans. (a)}$$

Part (b) Enter the SI units for each quantity from Table 1.2:

$$\{\text{N/m}^2\} = \{\text{N/m}^2\} + \{\text{kg/m}^3\}\{\text{m}^2/\text{s}^2\} + \{\text{kg/m}^3\}\{\text{m/s}^2\}\{\text{m}\}$$

$$= \{\text{N/m}^2\} + \{\text{kg/(m}\cdot\text{s}^2)\}$$

The right-hand side looks bad until we remember from Eq. (1.3) that 1 kg = 1 N $\cdot$ s^2/m.

$$\{\text{kg/(m}\cdot\text{s}^2)\} = \frac{\{\text{N}\cdot\text{s}^2/\text{m}\}}{\{\text{m}\cdot\text{s}^2\}} = \{\text{N/m}^2\} \qquad \textit{Ans. (b)}$$

Thus all terms in Bernoulli's equation will have units of pascals, or newtons per square meter, when SI units are used. No conversion factors are needed, which is true of all theoretical equations in fluid mechanics.

Part (c) Introducing BG units for each term, we have

$$\{\text{lbf/ft}^2\} = \{\text{lbf/ft}^2\} + \{\text{slugs/ft}^3\}\{\text{ft}^2/\text{s}^2\} + \{\text{slugs/ft}^3\}\{\text{ft/s}^2\}\{\text{ft}\}$$

$$= \{\text{lbf/ft}^2\} + \{\text{slugs/(ft}\cdot\text{s}^2)\}$$

But, from Eq. (1.3), 1 slug = 1 lbf $\cdot$ s^2/ft, so that

$$\{\text{slugs/(ft}\cdot\text{s}^2)\} = \frac{\{\text{lbf}\cdot\text{s}^2/\text{ft}\}}{\{\text{ft}\cdot\text{s}^2\}} = \{\text{lbf/ft}^2\} \qquad \textit{Ans. (c)}$$

1 (Ibf) = 4.45 (N)

1 (ft) = 0.305 (m)

Chapter Two

Properties of a Fluid

-Density RHO

1- Density :- The density (ρ) of a fluid is its mass per unit volume. In SI units (ρ) will be in Kg/m^3 which may also be expressed as units of N. sec^2/m^4. ($N = Kg \frac{m}{sec^2}$)

$$\therefore \rho = \frac{mass}{Volume} = \frac{m}{V} \quad ; \quad \rho_{water \text{ at } 4^\circ c} = 1000 \, kg/m^3$$

$$\rho_{Hg \text{ at } 20^\circ c} = 13.6 \times 10^3 \, kg/m^3$$

Gamma

- Specific weight (γ) : Is the weight of a fluid per unit volume & it represents the force exerted by gravity on a unit volume of fluid & therefore must have the of force per unit volume, such as N/m^3.

$$\gamma = \frac{Weight}{Volume} = \frac{W}{V}$$

By Newton's second law; $F = m \cdot a \Rightarrow W = m \cdot g$

$$\therefore \gamma = \frac{m \cdot g}{\forall} \Rightarrow \gamma = \rho g$$

for example; $\gamma_w = \rho_w g = 1000 * 9.81 = 9810 \ N/m^3$

- Specific Gravity (s.): Is the ratio of a fluid (liquid) density to that of pure water at a standard temperature ($4^\circ c$). ($\rho_{water \, 4^\circ c} = 1000 \ kg/m^3$)

$$S. = \frac{\gamma_{fluid}}{\gamma_{water \, 4^\circ c}} = \frac{\rho_{fluid} * g}{\rho_{water \, 4^\circ c} \, g} \quad (\text{or } \gamma_{water \, 4^\circ c} = 9810 \ N/m^3)$$

$$\therefore S. = \frac{\rho_{fluid}}{\rho_{water}}$$

2- Compressibility: Is defined as the change in volume of a fluid due to change in pressure & its inversely proportional to the Bulk modulus of Elasticity (E).

pressure unit (N/m^2) (or Pa.)

$$E = \frac{\text{Change of pressure}}{\text{Volumetric change}} = \frac{dP}{-\frac{d\forall}{\forall}} \quad \text{—} \ (1)$$

, the negative sign means that the volume of a fluid decreases with increasing in pressure

for a unit mass; $\rho = \frac{1}{\forall} \xrightarrow[\text{differentiation}]{\text{By}} d\rho = \frac{-d\forall}{\forall^2}$

$$\therefore -d\forall = d\rho \, \forall^2 \quad \text{—} \ (2)$$

substitute eq. (2) into eq. (1):

$$E = \frac{dP}{\frac{d\rho \forall^2}{\forall}} = \frac{dP}{d\rho \, \forall} =$$

But $\frac{1}{\forall} = \rho$

$$\therefore \boxed{E = \rho \frac{dP}{d\rho}}$$

Liquids are ordinary considered incompressible fluid since the change in volume (or density per unit mass) is so small & can be neglected ⇒ (E – constant) (ρ – constant). When (E) change, this means that the fluid is compressible as in air (gases in general).

3 - Viscosity: The viscosity of a fluid is a measure of its resistance to shear on any deformation. The friction forces in fluid flow result from cohesion & momentum interchange between molecules in the fluid. As the temperature increases, the viscosities of all liquids decrease, while the viscosities of all gases increase. This is because the force of cohesion, which diminishes with temperature, predominates with liquids, while with gases the predominating factor is the momentum interchange of molecules between layers of different velocities.

- μ (mu) (dynamic viscosity) ($\frac{N.s}{m^2}$ = Pascal.s = Pa.s.)

$$\mu_{\substack{water \\ at\ 20^\circ c}} = 1.005 * 10^{-3} \text{ Pa.s.}$$

$$\mu_{\substack{air \\ at\ 20^\circ}} = 1.8 * 10^{-5} \text{ Pa.s.}$$

- ν (nu) (Kinematic viscosity) ($\frac{m^2}{s.}$)

$$\nu = \frac{\mu}{\rho} \qquad \nu = \frac{\frac{N.s}{m^2}}{\frac{N.s^2}{m^4}} = \frac{\cancel{N}.\cancel{s}}{\cancel{m^2}} \cdot \frac{\cancel{m^4}\,m^2}{\cancel{N}.\cancel{s^2}\,1}$$

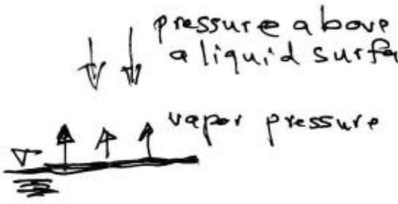
$$\therefore \nu = m^2/s.$$

4- Vapor Pressure

: All liquids tend to evaporate or vaporize, which they do by projecting molecules into the space above their surface. Molecular activity increases with temperature, & hence vapor pressure increases with temperature. The phenomena of vaporization & boiling are differentiated as follows:

* Vaporization if (Vapor pressure < pressure above a liquid surface at a temperature)

* Boiling if (Vapor pressure = pressure above a liquid surface at a temperature)

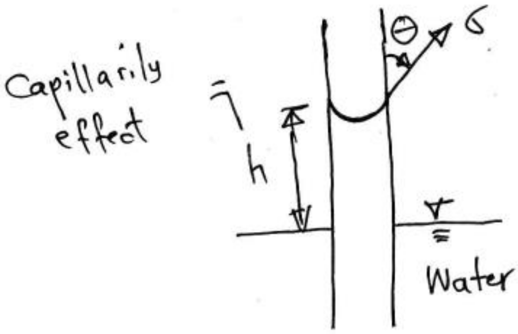


5- Surface Tension (σ) (Sigma) $\left(\frac{F}{length}\ \text{ex: } \frac{N}{m}\right)$

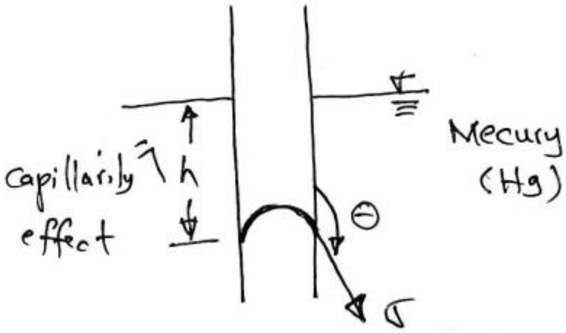
Liquids have cohesion & adhesion, both of which are forms of molecular attraction (تجاذب). Cohesion enables a liquid to resist tensile stress, while adhesion enables it to adhere to another body. The attraction between molecules forms an imaginary film capable of resisting tension at the interface between two immiscible (غير قابل للامتزاج) liquids or at the interface between a liquid & a gas. The liquid property that creates this capability is known as surface Tension.

$\sigma_{water} = 0.0736\ N/m$; $\sigma_{Hg} = 0.51\ N/m$
at $20^{\circ}c$ at $20^{\circ}c$

6- Capillarity : Is a liquid property that happened due to both cohesion & adhesion. When the former "cohesion" is of less effect than the latter "adhesion", the liquid will wet a solid surface with which it is in contact & rise at the point of contact. If cohesion predominates, the liquid surface will be depressed at the point of contact. For example, Capillarity makes water rise in a glass tube, while mercury is depressed below true level.



adhesion > cohesion



cohesion > adhesion

Derivation of Capillarity effect in Tube

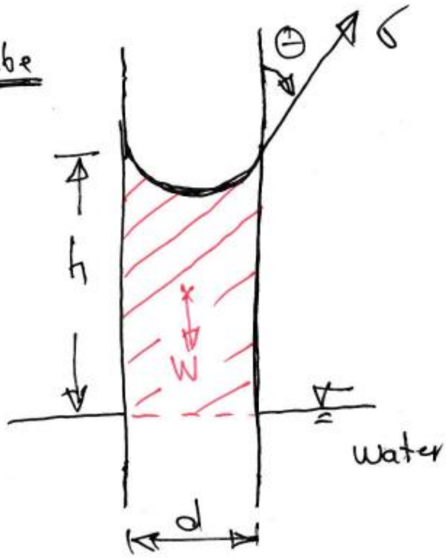
$\Sigma F_y = 0.$

$\sigma \cos\theta * \pi d = W$ — ①

since, $W = m \cdot g = \rho \forall \cdot g$

$\therefore W = \rho g \forall = \gamma \forall$

$= \gamma \frac{\pi}{4} d^2 h$



From eq.(1): $\sigma \cos\theta . \cancel{\pi} \cancel{d} = \gamma \frac{\cancel{\pi}}{4} d^{\cancel{2}} h$

$$\therefore h = \frac{4 \sigma \cos\theta}{\gamma d}$$

where, h = Capillarity effect height (m)

σ = surface tension (N/m)

γ = liquid specific weight (N/m^3) = ρg

ρ = liquid density (Kg/m^3)

g = gravity acceleration (m/sec^2)

d = diameter of tube (m)

for very clean glass tube & very smooth

$\theta = 0^\circ$ for water

& $\theta = 130^\circ$ for mercury (Hg)

Ex-1: Given oil weight = 1.9 KN, $\forall$ = 200 litter, find ρ, γ, & s.

Solution: $\rho = \frac{m}{\forall}$; $W = m \cdot g \Rightarrow m = \frac{W}{g} = \frac{1.9 * 10^3 \text{ N}}{9.81 \frac{m}{sec^2}}$

$\therefore m = 193.68$ Kg

$$\therefore \rho = \frac{193.68 \text{ kg}}{200 * 10^{-3} \text{ m}^3} = 968.4 \text{ Kg/m}^3$$

since, $\gamma = \rho g \Rightarrow \gamma = 968.4 * 9.81 = 9500 \text{ N/m}^3$

$= 9.5 \text{ KN/m}^3$

$$s. = \frac{\gamma_{liquid}}{\gamma_w} = \frac{9500 \text{ N/m}^3}{9810 \text{ N/m}^3} = 0.9684$$

Ex.2: A- Determine the change in volume of ($1 m^3$) water in $27^\circ c$ when exerted to change of pressure equal to (20 bar). "Assume E = 2.24 GN/m^2 or GPa"

B- Calculate the Bulk modulus of Elasticity for ($1 m^3$) water under a pressure of 35 bar when this volume becomes $0.99 m^3$ under 240 bar.

Note: (1 bar = 10^5 N/m^2 = 10^5 Pa. ≈ 10 m of water).

Sol.: **A:**

$$E = \frac{dP}{-\frac{d\forall}{\forall}} \Rightarrow d\forall = \frac{-\forall\, dP}{E}$$

$$\therefore d\forall = \frac{-(1)^{m^3}(20 * 10^5 \text{ Pa.})}{2.24 * 10^9 \text{ Pa.}} = -8.93 * 10^{-4}\ m^3$$

B:

$$E = \frac{dP}{-\frac{d\forall}{\forall}} = \frac{P_2 - P_1}{-\left(\frac{\forall_2 - \forall_1}{\forall_1}\right)} = \frac{(240-35) * 10^5}{-\left(\frac{0.99-1}{1}\right)}$$

$$= 2.05 * 10^9\ N/m^2 = 2.05\ GPa.$$

Ex.3: Given d = 4mm for clean glass tube,
Temperature of liquid = $20^\circ c$

Find the capillarity effect (h) of:

a- Water (σ_w = 0.0736 N/m)

b- Mercury (σ_{Hg} = 0.51 N/m).

Solution: a- since the glass is clean $\Rightarrow \theta_w = 0$

$$h = \frac{4\sigma\cos\theta}{\gamma_w d} = \frac{4\sigma\cos 0^\circ}{\gamma d} = \frac{4\sigma}{\gamma d}$$

$$h = \frac{4 * 0.0736}{9810 * 4 * 10^{-3}} = 0.0075\,m = 7.5\,mm \text{ rise of water}$$

b- for clean glass tube $\Rightarrow \theta_{Hg} = 130^\circ$

$$h = \frac{4 * 0.51 * \cos 130^\circ}{13.6 * 9810 * 4 * 10^{-3}} = -0.00246\,m$$

$$= -2.46\,mm \text{ depression of mercury}$$

7. Temperature Dependency

The effect of temperature on viscosity is different for liquids and gases. The viscosity of liquids decreases as the temperature increases, whereas the viscosity of gases increases with increasing temperature. To understand the mechanisms responsible for an increase in temperature that causes a decrease in viscosity in a liquid, it is helpful to rely on an approximate theory that has been developed to explained the observed trends (1). The molecules in a liquid form a structure with “holes” where there are no molecules, as shown in Fig. 2.2. Even when the liquid is at rest, the molecules are in constant motion, but confined to cells. The cell structure is caused by attractive forces between the molecules. The cells may be thought of as energy barriers. When the liquid is subjected to a rate of strain and thus caused to move, as shown in Fig. 2.2, there is a shear stress, τ, imposed by one layer on another in the fluid. This force/area assists a molecule in overcoming the energy barrier, and it can move into the next hole. The magnitude of these energy barriers is related to viscosity, or resistance to shear deformation. At a higher temperature the size of the energy barrier is smaller, and it is easier for molecules to make the jump, so that the net effect is less resistance to deformation under shear. Thus, an increase in temperature causes a decrease in viscosity for liquids. An equation for the variation of liquid viscosity with temperature is

$$\mu = Ce^{b/T} \tag{2.9}$$

where C and b are empirical constants that require viscosity data at two temperatures for evaluation.

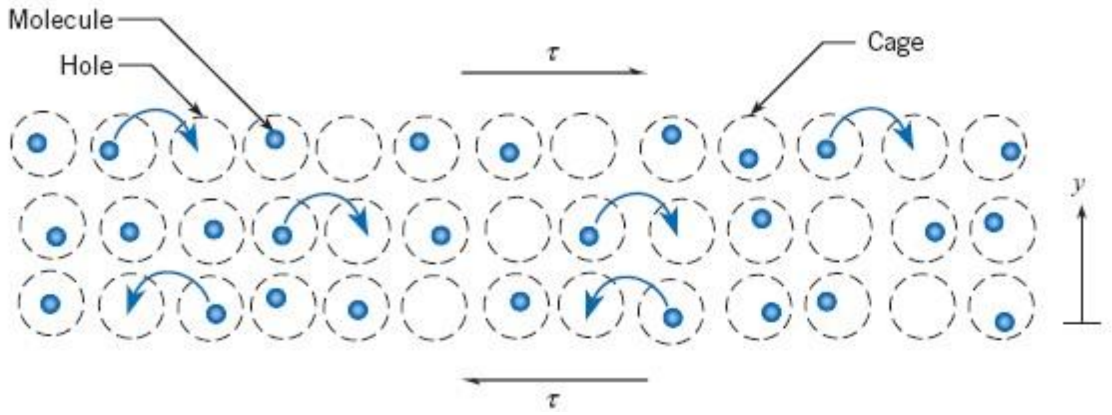


Fig. 2.2

As compared to liquids, gases do not have zones to which molecules are confined by intermolecular bonding. Gas molecules are always undergoing random motion. If this random motion of molecules is superimposed upon two layers of gas, where the top layer is moving faster than the bottom layer, periodically a gas molecule will randomly move from one layer to the other. As the gas temperature increases, more of the molecules will be making random jumps. Just highly mobile gas molecules have momentum, which must be resisted by the layer to which the molecules jump. Therefore, as the temperature increases, the viscosity, or resistance to shear, also increases.

Example 4

CALCULATING VISCOSITY OF LIQUID AS A FUNCTION OF TEMPERATURE

The dynamic viscosity of water at 20°C is $1.00 \times 10^{-3}\ \text{N} \cdot \text{s/m}^2$ and the viscosity at 40°C is $6.53 \times 10^{-4}\ \text{N} \cdot \text{s/m}^2$. Estimate the viscosity at 30°C**.** Viscosity of water is specified at two temperatures. Find The viscosity at 30°C by interpolation.

a) Water at 20°C, $\mu = 1.00 \times 10^{-3}\ \text{N} \cdot \text{s/m}^2$.

b) Water at 40°C, $\mu = 6.53 \times 10^{-4}\ \text{N} \cdot \text{s/m}^2$

Solution

1. Logarithm of Eq. (2.9)

$$\ln \mu = \ln C + b/T$$

2. Interpolation

$$-6.908 = \ln C + 0.00341 b$$
$$-7.334 = \ln C + 0.00319 b$$

3. Solution for and *b*

$$\ln C = -13.51 \qquad b = 1936\ (\text{K})$$
$$C = e^{-13.51} = 1.357 \times 10^{-6}$$

4. Substitution back in exponential equation

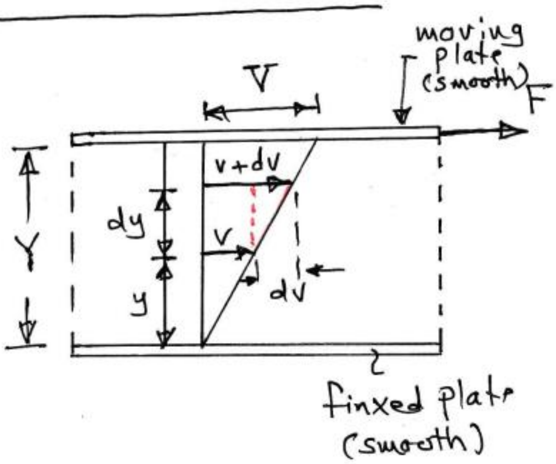
$$\mu = 1.357 \times 10^{-6} e^{1936/T}$$

At 30°C

$$\mu = \boxed{8.08 \times 10^{-4}\ \text{N} \cdot \text{s/m}^2}$$

Measurement of Viscosity

1- Measurement of Viscosity between two Parallel Plates



By experiment, it was found that:

$$F \propto \frac{AV}{Y} \quad \text{— ①}$$

where; F = applied force

A = contact area of the moving plate.

V : Velocity of the moving plate.

Y : distance between the two parallel plates

$$\therefore \frac{F}{A} \propto \frac{V}{Y} \Rightarrow \frac{F}{A} = \mu \frac{V}{Y}$$

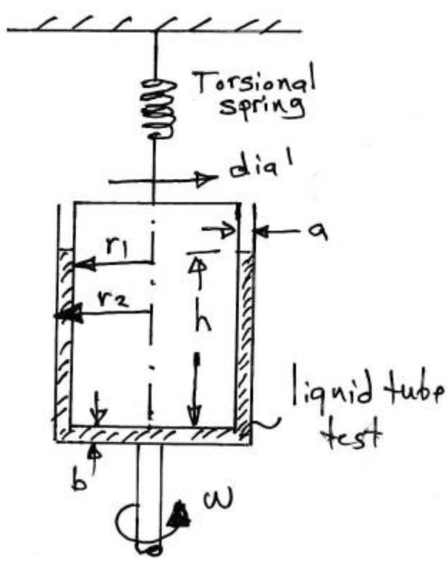
since; $\frac{F}{A} = \tau$: shear stress

For similar triangles & when the flow is laminar

$$\frac{V}{Y} = \frac{dv}{dy}$$

$$\therefore \boxed{\tau = \mu \frac{dv}{dy}}$$ (Newton's Law of viscosity)

2- Viscometer



$$\mu = \frac{2ab\,T}{\pi r_1^2 \omega (4 r_2 b h + r_1^2 a)}$$

where; μ = viscosity

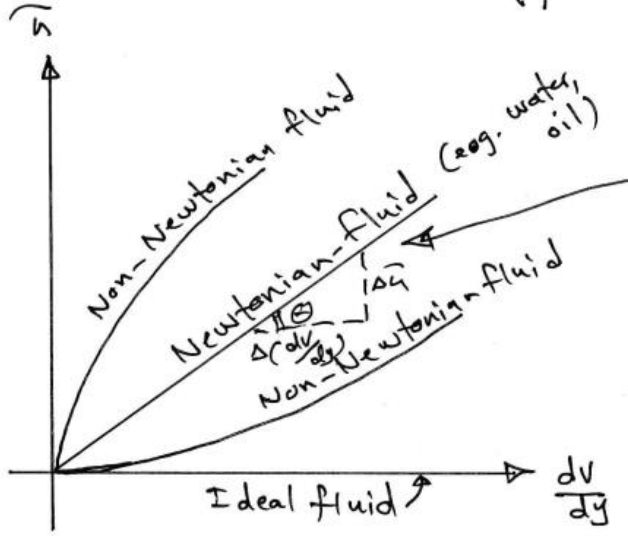
T = Torque in the torsional spring.

ω = constant angular velocity (rad./s.) = $\frac{2\pi N}{60}$

N = angular velocity (r.p.m.) (revolution per minute).

* There are two types of fluids depending on the existing of μ

Fluid
- → Ideal (Non-Viscous) ⇒ $\mu = 0$.
- → Real (Viscous) ⇒ $\mu \neq 0$.
  - → Newtonian Fluid (μ – const.)
  - → Non-Newtonian (μ – Varies



$\tan\theta = \frac{\Delta\tau}{\Delta(dv/dy)} = \mu$

since θ is costant ⇒ mu is constant

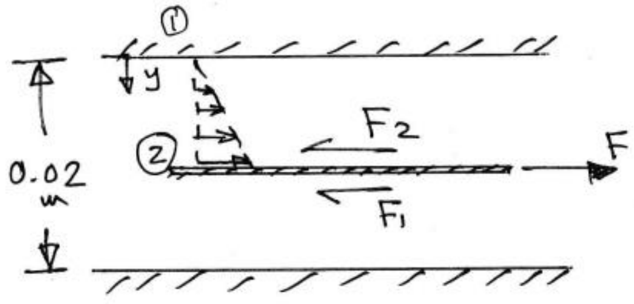
So, Newtonian fluid

* for Non-Newtonian fluid ⇒ μ is varied

Ex.1 : Find the required force (F) to pull a thin plate at the middle point between two large plates, which the distance between them (0.02m). The fluid between plates has ($\mu = 0.862$ Pa. s.) & the surface area of the thin plate is ($0.465\ m^2$) for each face. The constant velocity of the thin plate is (0.152 m/s.)

Note : Assume linear velocity distribution.

Solution :



For constant velocity of the thin plate $\Rightarrow a = 0$.

$F = m \cdot a \Rightarrow F = 0$.

$\therefore \sum F_x = 0$.

$F = F_1 + F_2$

$F_1 = F_2 = \bar{\tau} A$

$\Rightarrow F = 2F_1 = 2\bar{\tau} \cdot A$

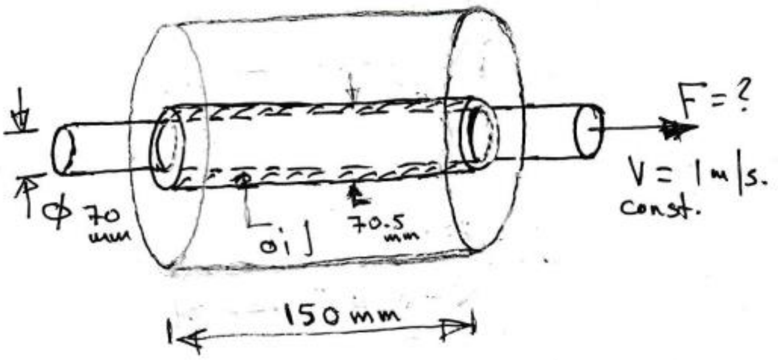
since, $\bar{\tau} = \mu \dfrac{dV}{dy}$; $\dfrac{dV}{dy} = \dfrac{V_2 - V_1}{y_2 - y_1} = \dfrac{0.152 - 0}{\frac{0.02}{2} - 0} = 15.2\ s^{-1}$

$\therefore \bar{\tau} = 0.862\,(15.2) = 13.1$ Pa.

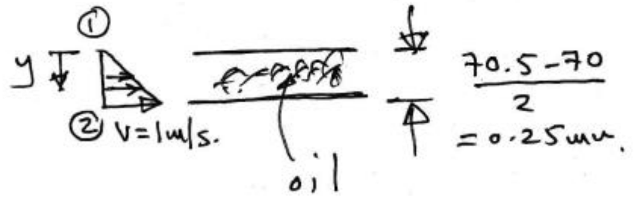
$\therefore F = 2\,(13.1) * 0.465 = 12.183$ N

Ex.2: A long circular rod of (70 mm) diameter slides concentrically in (150 mm) long fixed tube, shown below, of (70.5 mm) internal diameter. The annular space between the rod & the tube is filled with oil of viscosity (0.193 Pa.s.). What force is required to slide the rod through the tube with a velocity of (1 m/s.)?

Note: Assume the velocity of oil changes linearily.



Solution:



$$\tau = \mu \frac{dV}{dy}$$

$$\frac{F}{A} = \mu \frac{dV}{dy} \Rightarrow F = \mu A \frac{dV}{dy}$$

$$\frac{dV}{dy} = \frac{\Delta V}{\Delta y} = \frac{V_2 - V_1}{y_2 - y_1} = \frac{1 - 0.}{0.25 * 10^{-3} - 0.} = \frac{1}{0.25 * 10^{-3}} = 4000 \ s^{-1}$$

A = surface area of the moving rod which in contact with oil

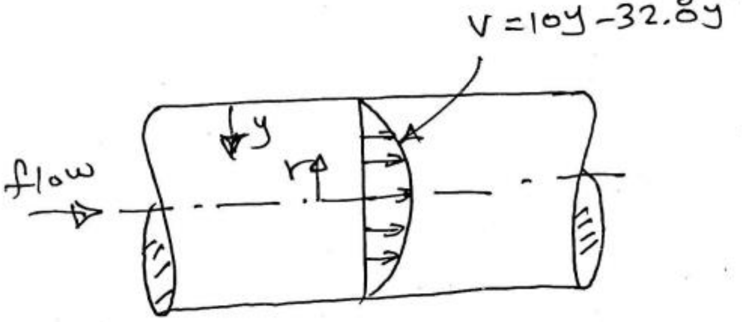
$$= \pi d_{rod} * L = \pi (70 * 10^{-3}) * 150 * 10^{-3}$$

$$\therefore A = 0.033 \ m^2$$

$$\therefore F = 0.193 (0.033)(4000) = 25.47 \ N$$

Ex.3: Water flows in a long pipe of dia. (0.305 m). The velocity profile has a parabolic shape, ($v = 10y - 32.8y^2$) where (y) is the distance measured from pipe wall toward the center. Find the equation of shear stress distribution, then calculate the shear stress at the wall & at the center of the pipe. ($\mu_{water} = 1.307 * 10^{-3}$ Pa.s.)

Solution:



$$\tau = \mu \frac{dv}{dy}$$

$$\frac{dv}{dy} = 10 - 32.8(2)y$$
$$= (10 - 65.6\,y)\ s^{-1}$$

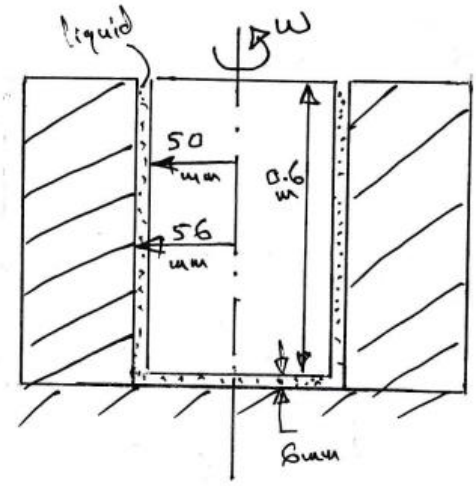
$$\therefore \tau = 1.307 * 10^{-3}(10 - 65.6y) \quad \text{Pa.}$$

at the pipe wall $\Rightarrow y = 0. \Rightarrow \tau = 1.307 * 10^{-3}(10) = 1.307 * 10^{-2}$ Pa.

at the pipe center $\Rightarrow y = r = \frac{D}{2} = \frac{0.305}{2} = 0.1525$ m

$$\tau = 1.307 * 10^{-3}(10 - 65.6 * 0.1525) = 0.$$

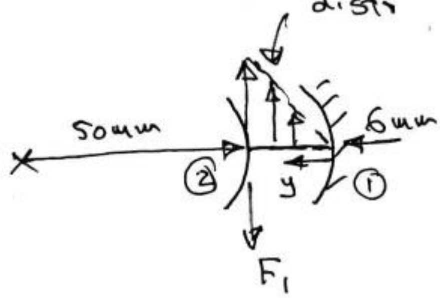
Ex. 4 A cylinder (50 mm) in radius & (0.6m) in length, rotates coaxially inside a fixed cylinder of radius (56 mm), as shown in figure below. Liquid of ($\mu = 1.48$ Pa.s.) fills the space between the two cylinders & between the inner cylinder & the base. Calculate the torque required to rotate the inner cylinder at a constant angular velocity of (20 r.p.m). (Take end effects into consideration).



Sol.: $T = T_1 + T_2$

where, T_1 = torque exerted on the wall of the inner cylinder.

T_2 = torque exerted on the base of the inner cylinder



$$T = F_1 \cdot r = F_1 (0.05)$$

$$F_1 = \tau_1 \cdot A_1 = \mu \frac{dv}{dy} A_1$$

$$\frac{dv}{dy} = \frac{V_2 - V_1}{y_2 - y_1} = \frac{V_2}{0.006} = \frac{\omega \cdot r}{0.006} = \frac{\omega (0.05)}{0.006} = 8.334\,\omega$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi (20)}{60} = 2.09 \text{ rad./s.}$$

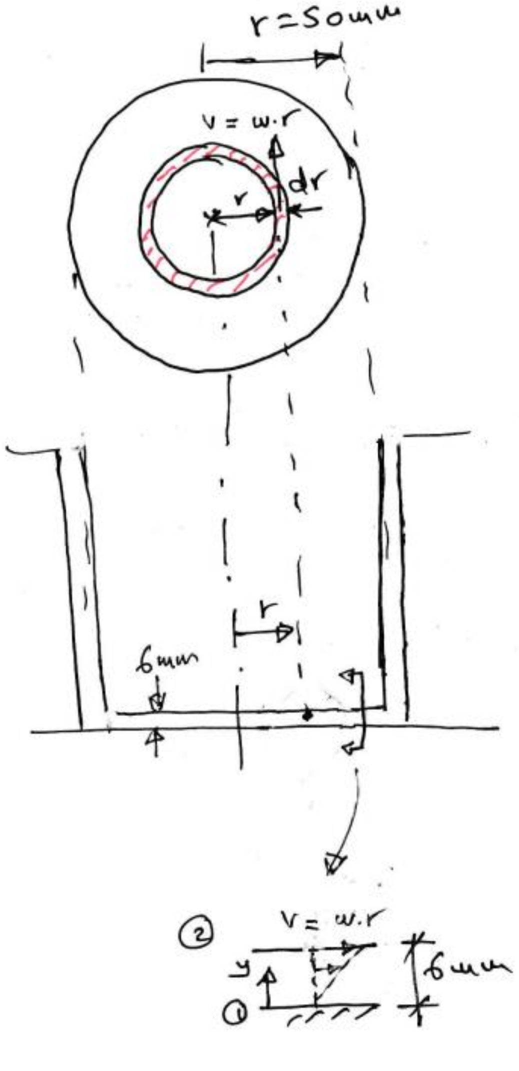
$\therefore \frac{dv}{dy} = 8.334\,(2.09) = 17.42\ s^{-1}$

$A_1 =$ surface area of inner cylinder $= 2\pi(0.05)(0.6)$

$= 0.188\ m^2$

$\therefore F_1 = 1.48\,(17.42)(0.188) = 4.85\ N$

$\therefore T_1 = 4.85\,(0.05) = 0.243\ N.m$



$T_2 = F_2 . r$

$F_2 = \tau_2 . A_2 = \mu \frac{dv}{dy} . A_2$

$\frac{dv}{dy} = \frac{v_2 - v_1}{y_2 - y_1} = \frac{w.r}{0.006}$

$A_2 = 2\pi r . dr$

$\therefore F_2 = \mu \frac{w.r}{0.006}\,(2\pi r . dr)$

$\therefore T_2 = \frac{\mu . w}{0.006}\,(2\pi . r^2 dr)$

$\therefore T_2 = \frac{2\pi(1.48)(2.09)}{0.006} \int_0^{0.05} r^3\,dr$

$T_2 = 5.06 * 10^{-3}\ N.m$

$\therefore T = 0.243 + 5.06 * 10^{-3} = 0.248\ N.m$

Equation of state for Perfect Gas

The perfect gas is defined as a substance that satifies the perfect gas law & has a constant specific heats. The equation of state for perfect gas is;

$$\boxed{P\nu = \frac{P}{\rho} = RT}$$

or

$$\boxed{P = \rho RT}$$

where;

P = absolute pressure (N/m^2 or $Pa.$)

ρ = gas density (Kg/m^3)

ν = specific volume $= \frac{1}{\rho}$ (m^3/Kg)

= volume for 1 unit mass of the gas

R = gas constant (which depends on gas type).

T = absolute temperature (Kelvin or K)

where; $K = {}^{\circ}C + 273$

The units of R can be determined from the equation when the other units are known.

$$R = \frac{P}{\rho T} = \frac{N/m^2}{Kg/m^3 \cdot K} = \frac{N \cdot m^{\cancel{3}}}{\cancel{m^2} \cdot Kg \cdot K} = \frac{N \cdot m}{Kg \cdot K} = \frac{Joule(J.)}{Kg \cdot K}$$

There are two types of specific heats, c_v is defined as the number of units of heat added per unit mass to raise the temperature of the gas one degree when the volume is held constant.

c_p is defined as the number of heat units added per unit mass to raise the temperature of the gas one degree when the pressure is held constant.

The perfect gas must be carefully distinguished from ideal fluid. An ideal fluid is frictionless ($\mu = 0.$) ($\bar{n} = 0.$) & incompressible (E- const.) (ρ - const.). The perfect gas has viscosity & can therefore develop shear stresses, & it is compressible according to the equation of state described above.

For Perfect gas, it is found;

$$\boxed{P v^n = P_1 v_1^n = \text{constant}}$$

where; n = any non-negative value depending upon the process to which the gas is subjected.

Ex.1: How many Kilograms of Carbon monoxide gas at $20^\circ c$ & 200 KPa (abs.) is contained in a volume of (100 l) if $R = 297 \frac{J.}{Kg.k}$

$M = ?$, $P = ✓$, $T = ✓$, $\forall = ✓ = 1\ l$

Solution: mass = ? $\Rightarrow \rho = ✓$ (where $\rho = \frac{m}{\forall} \Rightarrow m = \rho\forall$)

since $P = \rho RT$

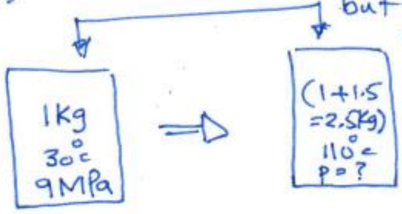
$$\therefore \rho = \frac{P}{RT} = \frac{200 * 10^3}{297\ (20+273)} = 2.298\ Kg/m^3$$

$$\text{since, } \rho = \frac{m}{\forall} \Rightarrow m = \rho\forall = 2.298 * 100 * 10^{-3}$$

$$\therefore m = 0.23\ Kg.$$

Ex.2: A container holds (1 Kg) air at $30^\circ c$ & (9 MPa abs.). If (1.5 Kg) air is added and the final temperature is $110^\circ c$, determine the final absolute pressure. ($R_{(air)} = 287 \frac{J.}{Kg.k}$)

Same volume of the container but the mass increase $\Rightarrow \rho$ increased



Solution:

Since; $P = \rho RT$

$$P_{final} = \rho_{final}\ R\ T_{final}$$

$$P_{final} = \frac{2.5}{\forall}\ R\ (110+273)$$

$$\therefore P_{final} = \frac{2.5}{\forall}(287)(383) \quad — ①$$

to find $\forall$;

since $P_{in.} = \rho_{init.} R T_{init.}$

$$9 * 10^{6} = \frac{1}{\forall}(287)(30+273)$$

$$\therefore \forall = 9.66 * 10^{-3} \ m^{3}$$

From eq. ①: $P_{final} = \frac{2.5}{9.66*10^{-3}}(287)(383)$

$$\therefore P_{final} = 28.45 * 10^{6} \ Pa.$$

$$= 28.45 \ MPa.$$

H.W.: show that $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$

Chapter Three

Fluid Static

This unit begins mechanics of fluids in depth by introducing many concepts related to pressure and by describing how to calculate forces associated with distributions of pressure. This chapter is restricted to fluids that are in hydrostatic equilibrium.

Pressure is defined as the ratio of normal force to area at a point. The pressure often varies from point to point. For example, pressure acting on the water tank wall will vary at different locations on the wall. Pressure is a scalar quantity; that is, it has magnitude only. Units for pressure give a ratio of force to area. Newtons per square meter of area, or Pascal (Pa), is the SI unit. The USC units include psi, which is pounds-force per square inch, and psf, which is poundsforce per square foot.

3.1 Absolute Pressure, Gage Pressure, and Vacuum Pressure

Absolute pressure is referenced to regions such as outer space, where the pressure is essentially zero because the region is devoid of gas. The pressure in a perfect vacuum is called absolute zero, and pressure measured relative to this zero pressure is termed ***absolute pressure***. When pressure is measured relative to prevailing local atmospheric pressure, the pressure value is called ***gage pressure***. For example, when a tire pressure gage gives a value of 300 kPa (44 psi), this means that the absolute pressure in the tire is 300 kPa greater than local atmospheric pressure. To convert gage pressure to absolute pressure, add the local atmospheric pressure. When pressure is less than atmospheric, the pressure can be described using vacuum pressure. ***Vacuum pressure*** is defined as the difference between atmospheric pressure and actual pressure.

Figure 3.1 provides a visual description of the three pressure scales. Gage, absolute, and vacuum pressure can be related using equations labeled as the “pressure equations.”

$$p_{\text{abs}} = p_{\text{atm}} + p_{\text{gage}}$$

$$p_{\text{abs}} = p_{\text{atm}} - p_{\text{vacuum}}$$

$$p_{\text{vacuum}} = -p_{\text{gage}}$$

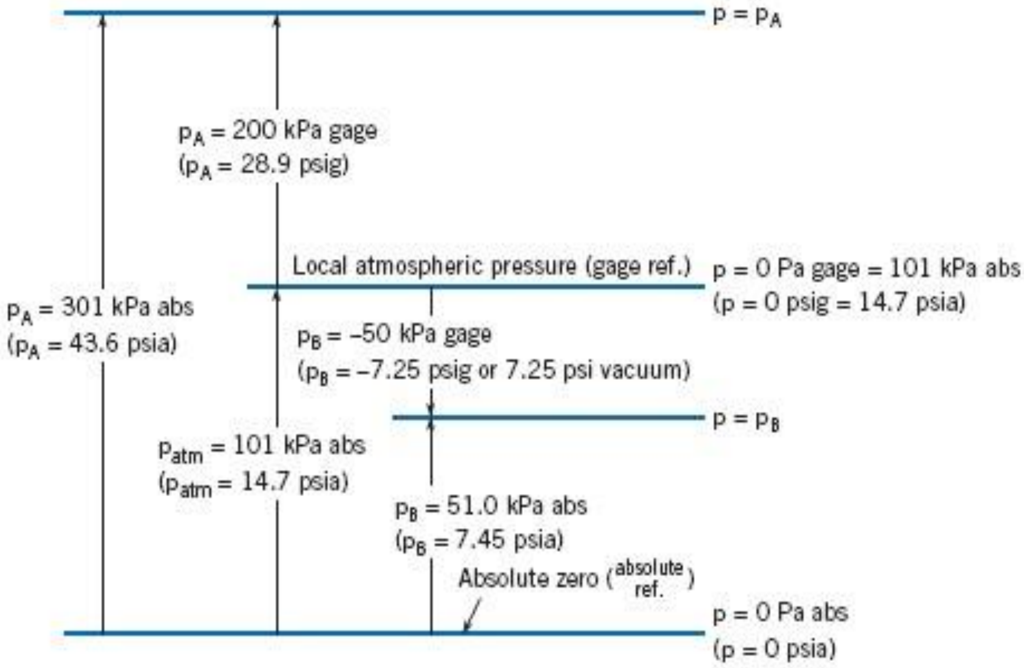


Fig. 3.1

3.2 Pressure Variation with Elevation

3.2.1 Hydrostatic Differential Equation

The hydrostatic differential equation is derived by applying force equilibrium to a static body of fluid. To begin the derivation, isolate a small cylindrical body, and then sketch a free-body diagram (FBD) as shown in Fig. 3.2. The cylindrical body is oriented so that its longitudinal axis is parallel to an arbitrary l direction. The body is Δl long, ΔA in cross-sectional area, and inclined at an angle α with the horizontal.

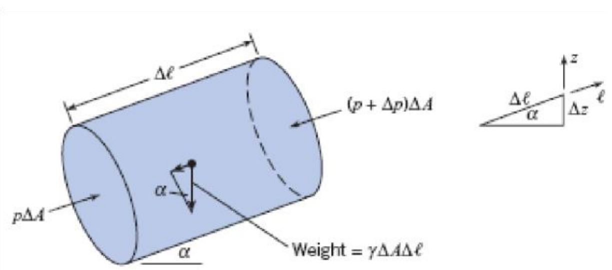


Fig. 3.2

Apply force equilibrium in the l direction:

$$\sum F_\ell = 0$$

$$F_{\text{Pressure}} - F_{\text{Weight}} = 0$$

$$p\Delta A - (p + \Delta p)\Delta A - \gamma \Delta A \Delta \ell \sin\alpha = 0$$

Simplify and divide by the volume of the body $\Delta\ell\,\Delta A$ to give $\frac{\Delta p}{\Delta \ell} = -\gamma \sin\alpha$

From Fig. 3.2, the sine of the angle is given by $\sin\alpha = \frac{\Delta z}{\Delta \ell}$
Combining the previous two equations and letting approach zero gives

$$\lim_{\Delta z \to 0} \frac{\Delta p}{\Delta z} = -\gamma$$

The final result is $\frac{dp}{dz} = -\gamma$ (hydrostatic differential equation) (3.1)

Equation (3.1) means that changes in pressure correspond to changes in elevation.

3.2.2 Hydrostatic Equation

The hydrostatic equation is used to predict pressure variation in a fluid with constant specific weight (constant density). Integrating Eq. (3.1) will give

$$p + \gamma z = p_z = \text{constant} \qquad (3.2)$$

Where, *z:* is elevation, (vertical distance above a fixed reference point - datum),

p_z; *piezometric pressure*.

Dividing Eq. (3.2) by γ gives

$$\frac{p_z}{\gamma} = \left(\frac{p}{\gamma} + z\right) = h = \text{constant} \qquad (3.3)$$

Where, *h* : *piezometric head*. Since *h* is constant in Eq. (3.3),

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 \qquad (3.4a)$$

Multiplying Eq. (3.4a) by γ gives

$$p_1 + \gamma z_1 = p_2 + \gamma z_2 \qquad (3.4b)$$

In Eq. (3.4b), letting $\Delta p = p_2 - p_1$ and $\Delta z = z_2 - z_1$ letting gives

$$\Delta p = -\gamma \Delta z \qquad (3.4c)$$

The hydrostatic equation is given by either Eq. (3.4a), (3.4b), or (3. c). These three equations are equivalent because any one of the equations can be used to derive the other two. Piezometric pressure and head are related by

$$p_z = h\gamma \qquad (3.5)$$

When hydrostatic equilibrium prevails in a body of fluid of constant density, then h will be constant at all locations.

EXAMPLE 3.2 PRESSURE IN TANK WITH TWO FLUIDS

Oil with a specific gravity of 0.80 forms a layer 0.90 m deep in an open tank that is otherwise filled with water. The total depth of water and oil is 3 m. What is the gage pressure at the bottom of the tank?

Problem Definition

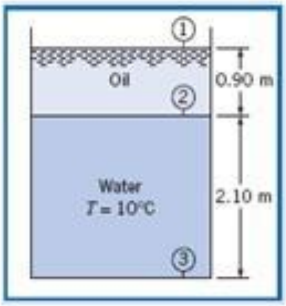
Situation: Oil and water are contained in a tank.

Find: Pressure (kPa gage) at the bottom of the tank.

Properties:

1. Oil (10°C), $S = 0.8$.
2. Water (10°C), $\gamma = 9810\ \text{N/m}^3$.

Sketch:



Plan

Use the idea that piezometric head is constant in a body of fluid with constant density. Recognize that pressure across the interface at elevation 2 is constant. The steps are

1. Find p_2 by applying the hydrostatic equation given in Eq. (3.4a).
2. Equate pressures across the oil-water interface.
3. Find p_3 by applying the hydrostatic equation given in Eq. (3.4a).

Solution

1. Hydrostatic equation (oil)

$$\frac{p_1}{\gamma_{oil}} + z_1 = \frac{p_2}{\gamma_{oil}} + z_2$$

$$\frac{0\ \text{Pa}}{\gamma_{oil}} + 3\ \text{m} = \frac{p_2}{0.8 \times 9810\ \text{N/m}^3} + 2.1\ \text{m}$$

$$p_2 = 7.063\ \text{kPa}$$

2. Oil-water interface

$$p_2|_{oil} = p_2|_{water} = 7.063\ \text{kPa}$$

3. Hydrostatic equation (water)

$$\frac{p_2}{\gamma_{water}} + z_2 = \frac{p_3}{\gamma_{water}} + z_3$$

$$\frac{7.063 \times 10^3\ \text{Pa}}{9810\ \text{N/m}^3} + 2.1\ \text{m} = \frac{p_3}{9810\ \text{N/m}^3} + 0\ \text{m}$$

$$\boxed{p_3 = 27.7\ \text{kPa gage}}$$

Review

Validation: Since oil is less dense than water, the answer should be slightly smaller than the pressure corresponding to a water column of 3 m. From Table F.1, a water column of $10\ \text{m} \approx 1$ atm. Thus, a 3 m water column should produce a pressure of about 0.3 atm = 30 kPa. The calculated value appears correct.

3.3 Pressure Measurements

Four scientific instruments for measuring pressure: the barometer, Bourdontube gage, piezometer and manometer, transducer will described.

Measurement Devices

a- Tube gauge → Piezometer (column) } used for liquids
→ Manometer

b- Bourdon gauge.

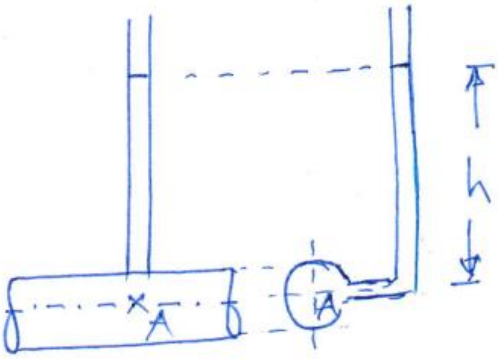
a- Tube Gauge

1- Piezometer (Column): it is a simple device which using for moderate (+ve) pressures of liquids. It consists of a tube in which the liquid can freely rise without overflowing.

$P_A = P_{atm.} + \gamma h$

since $P_{atm.} = 0$. (zero gauge press.)

$\therefore P_A = \gamma h$



2 - Manometer

i - Simple Manometer.

ii - Micro Manometer.

iii - Differential Manometer

iv - Inverted Differential Manometer.

i - Simple Manometer: It is a U-tube device that contains a liquid of high specific gravity (s.). It is convenient for measuring +ve or -ve pressures at a point.

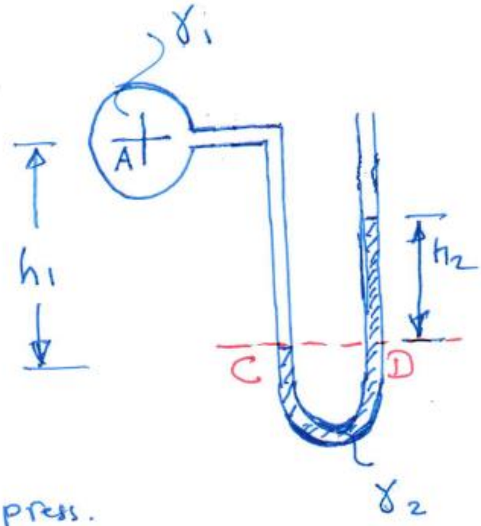
Case A:

In general; hydrostatic pressures at any horizontal level are the same;

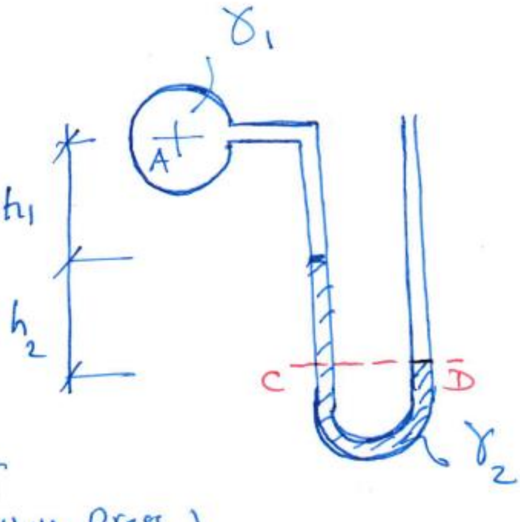
$P_C = P_D$

$$P_A + \gamma_1 h_1 = \underbrace{P_{atm.}}_{\text{zero gauge}} + \gamma_2 h_2$$

$\therefore P_A = \gamma_2 h_2 - \gamma_1 h_1$ +ve press.



Case B:



$P_C = P_D$

$$P_A + \gamma_1 h_1 + \gamma_2 h_2 = \overset{\text{zero gauge}}{P_{atm}}.$$

$$\therefore P_A = -(\gamma_1 h_1 + \gamma_2 h_2)$$

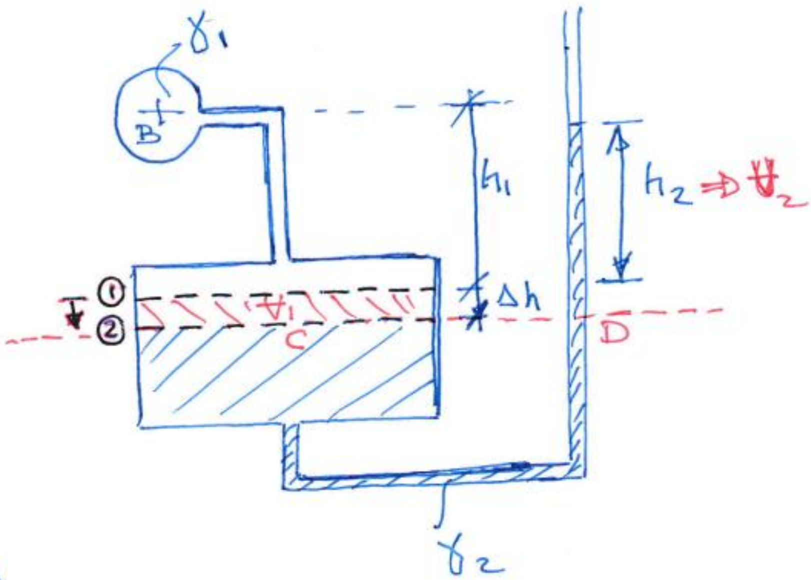
or $P_A = \gamma_1 h_1 + \gamma_2 h_2$ (-ve or Vacuum press.)

ii- **Micro Manometer**: Used to measure low +ve or -ve pressures at a point with a high accuracy. It is a U-tube device with a basin at one of its limbs.

let;

a: cross-sectional area of tube.

A: cross-sectional area of the basin.



$P_C = P_D$

$$P_B + \gamma_1 h_1 + \gamma_1 \Delta h = \overset{0}{P_{atm.}} + \gamma_2 h_2 + \gamma_2 \Delta h$$

$$\therefore P_B = \gamma_2 h_2 - \gamma_1 h_1 + \Delta h(\gamma_2 - \gamma_1)$$

since; $\forall_1 = \forall_2$

$$A \cdot \Delta h = a \cdot h_2$$

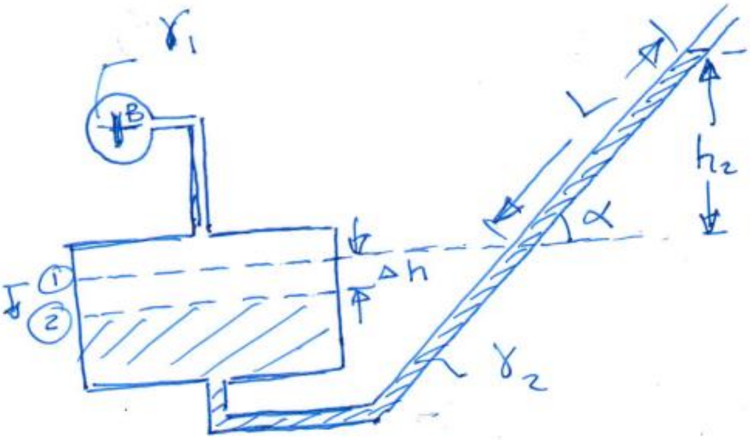
$$\therefore \Delta h = \frac{a}{A} \cdot h_2$$

$$\therefore P_B = \gamma_2 h_2 - \gamma_1 h_1 + \frac{a}{A} h_2 (\gamma_2 - \gamma_1)$$

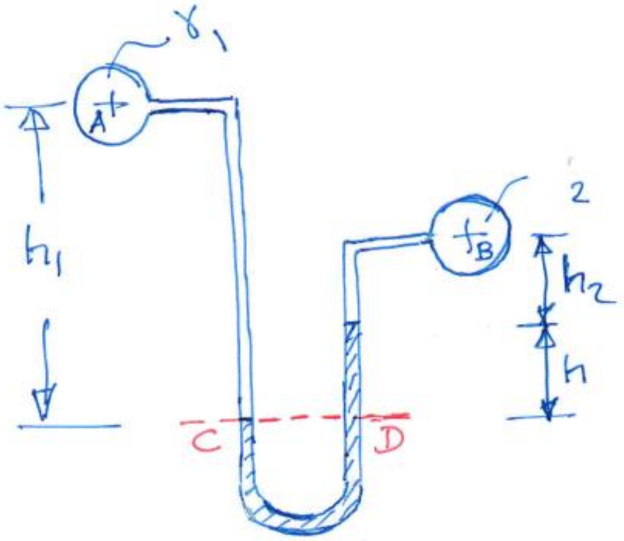
Inclined Micro Manometer:

$$\sin\alpha = \frac{h_2}{L}$$

$$\therefore h_2 = L \cdot \sin\alpha$$



iii _ Differential Manometer: It is a U-tube manometer that used to measure the pressure difference between two connected points.

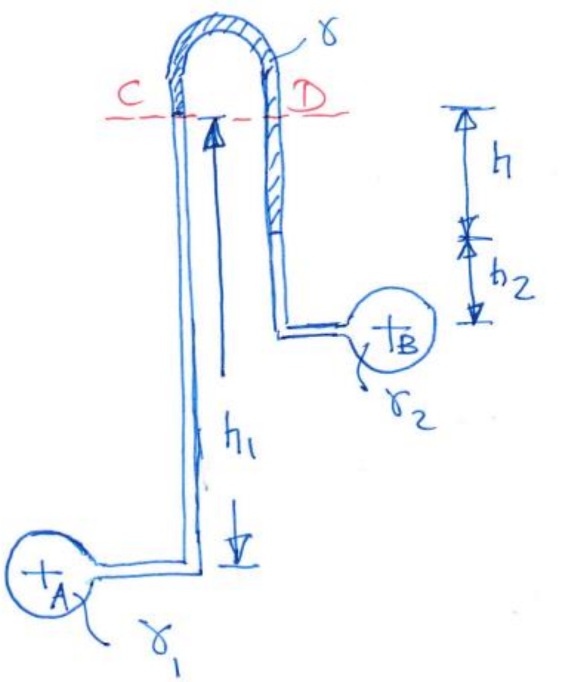


$$P_C = P_D$$

$$P_A + \gamma_1 h_1 = P_B + \gamma_2 h_2 + \gamma h$$

$$\therefore P_A - P_B = \gamma_2 h_2 - \gamma_1 h_1 + \gamma h$$

iv- Inverted Differential Manometer: It is an inverted U_tube device used to measure small pressure difference between two connected points.



$$P_C = P_D$$

$$P_A - \gamma_1 h_1 = P_B - \gamma_2 h_2 - \gamma h$$

$$\therefore P_A - P_B = \gamma_1 h_1 - \gamma_2 h_2 - \gamma h$$

b. Bourdon-Tube Gage

A *Bourdon-tube* gage measures pressure by sensing the deflection of a coiled tube. The tube has an elliptical cross section and is bent into a circular arc, as shown in Fig. 3.4.When atmospheric pressure (zero gage pressure) prevails, the tube is undeflected. When pressure is applied to the gage, the curved tube tends to straighten, thereby actuating the pointer to read a positive gage pressure.

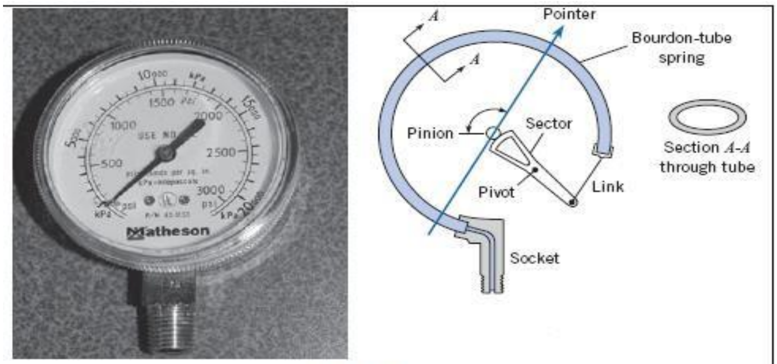
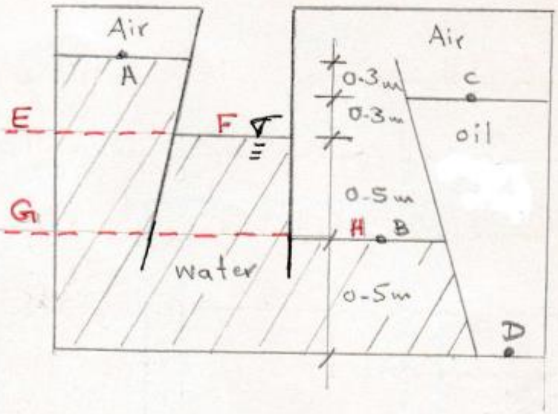


Fig.3.4

Ex① Calculate the gauge pressure at A, B, C & D.



Solution: $P_E = P_F$

$P_A + \gamma_w (0.6) = \overset{=0.}{P_{atm.}}$

$\therefore P_A = -9810 * 0.6$

$= -5886 \ N/m^2$

$= 5886 \ Pa. \ \text{(Vacuum)}$

$P_G = P_H \Rightarrow \overset{=0.}{P_{atm.}} + \gamma_w (0.5) = P_B$

$\therefore P_B = 4905 \ N/m^2$

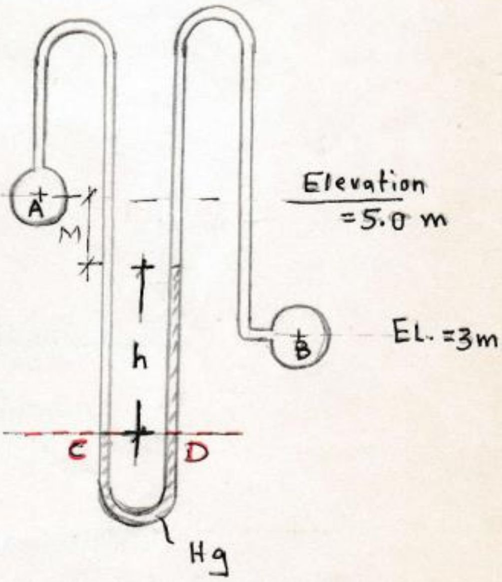
$P_C = P_B = 4905 \ N/m^2$

$P_D = P_C + \gamma_{oil} (1.3) = 4905 + 0.9 * 9810 * (1.3)$

$\therefore P_D = 16383 \ N/m^2$

Ex.②: Vessels A & B contain water under pressures of 2.76 bar, 1.38 bar, respectively. What is the deflection of the mercury in the differential gage (manometer)?

Note: $1 \text{bar} = 10^5 \ N/m^2 = 10^5 \ Pa.$



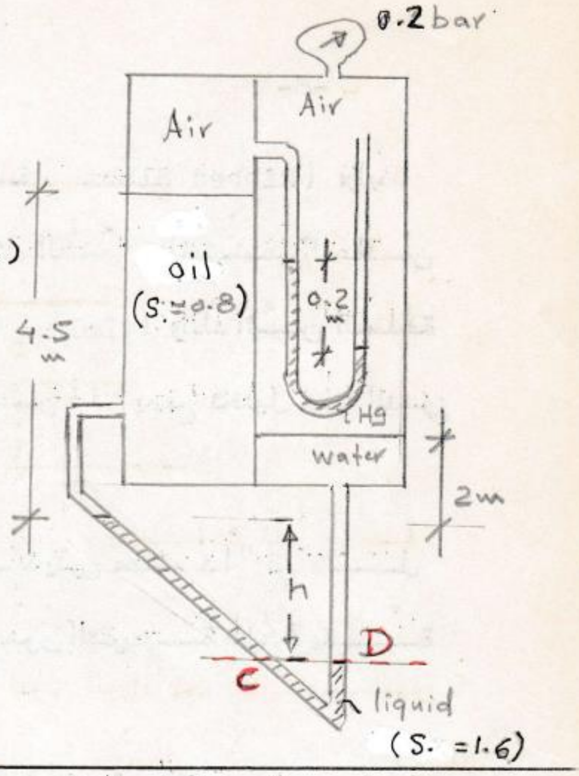
Solution: $P_C = P_D$

$P_A + \cancel{\gamma_w M} + \gamma_w h = P_B - \gamma_w (2) + \cancel{\gamma_w M} + \gamma_{Hg} h$

$2.76 * 10^5 + 9810 h = 1.38 * 10^5 - 9810 (2) + 13.6 * 9810 * h$

$\therefore h = 1.275 m$

Ex. ③: Calculate the difference head (h) in the manometer shown in figure below.



Solution:

$P_C = P_D$

$\cancel{0.2*10^5} - \gamma_{Hg}(0.2) + \gamma_{oil}(4.5) + \gamma_{liquid}(h) = \cancel{0.2*10^5} + \gamma_w(2) + \gamma_w(h)$

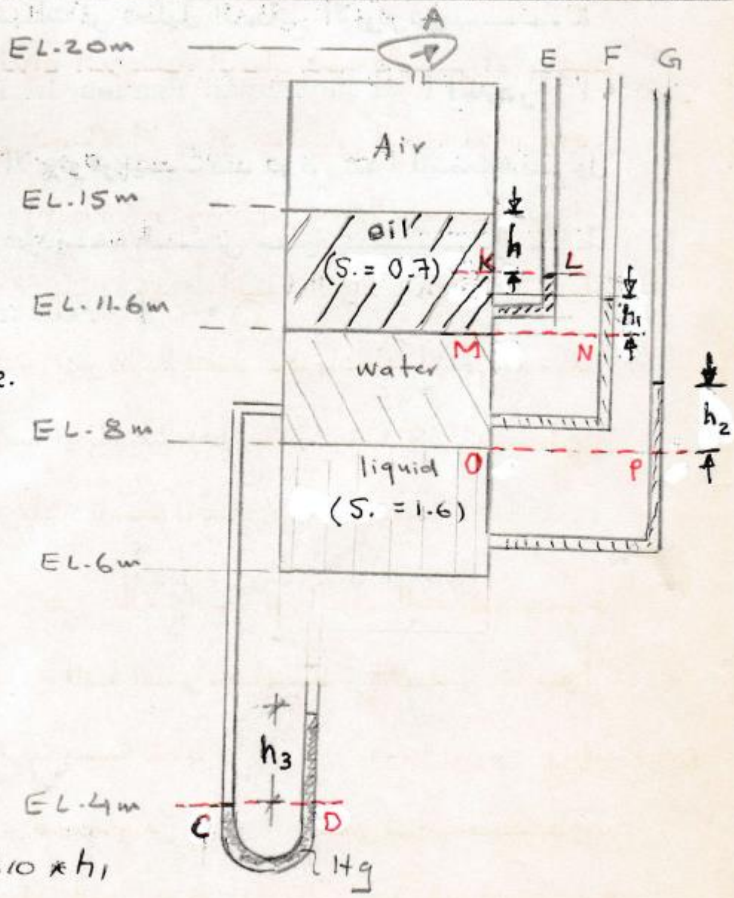
$-13.6*9810*0.2 + 0.8*9810(4.5) + 1.6*9810\,h = 9810(2) + 9810\,h$

$\therefore h = 1.866\,m$

Ex. ④: For a gauge reading at A of (17200 Pa) "Vacuum", determine:

a- the elevation of the liquids in open piezometer Columns E, F & G

b- the deflection of the mercury in the U-tube gauge.



Solution: ⓐ $P_K = P_L$

$P_A + \gamma_{oil}(h) = P_{atm.} = 0.$

$\therefore h = \dfrac{17200}{0.7*9810} = 2.5\,m$

$\therefore EL_E = 15 - 2.5 = 12.5\,m$

$P_M = P_N$

$-17200 + 0.7*9810*(15-11.6) = P_{atm.} + 9810*h_1$ ($P_{atm.} = 0.$)

$\therefore h_1 = 0.63\,m$

$\therefore EL_F = 11.6 + 0.63 = 12.23\,m$

$P_o = P_P$

$$-17200 + 0.7 * 9810\,(15-11.6) + 9810\,(11.6-8)$$
$$= 1.6 * 9810 * h_2 + \overset{\text{Fo.}}{P_{atm.}}$$

$\therefore h_2 = 2.64\text{m}$

$\therefore EL_G = 8 + 2.64 = 10.64\text{m}$

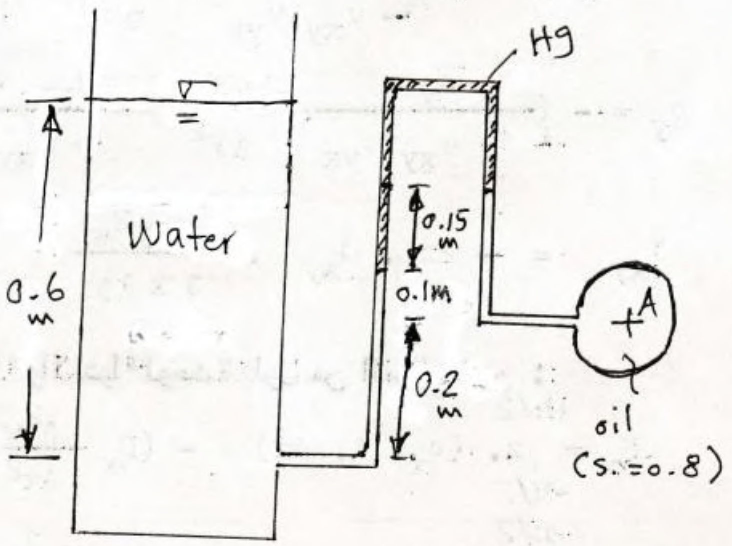
b_ $P_C = P_D$

$$-17200 + 0.7 * 9810\,(15-11.6) + 9810\,(11.6-8) + 9810(4)$$
$$= 13.6 * 9810 * h_3 + \overset{\text{Fo.}}{P_{atm.}}$$

$\therefore h_3 = 0.605\text{m}$

H.W.: As shown in figure below, determine the pressure at point A.

Ans.: 12.164 KPa Vacuum



3.4 Forces on Plane Surfaces (Panels)

This section explains how to represent hydrostatic pressure distributions on one face of a panel with a resultant force that passes through a point called the center of pressure. This information is relevant to applications such as dams, gates and water tanks.

1/ Hydrostatic force on a Submerged Horizontal Plane Surface

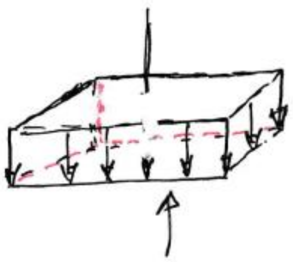
A plane surface in a horizontal position in a fluid at rest is subjected to a constant pressure. The magnitude of the force acting on one side of the surface is:

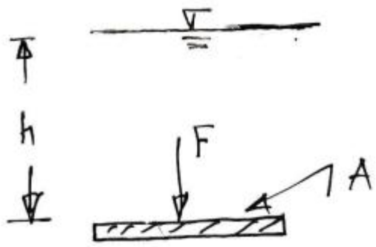
$$F = \int p \cdot dA = p \int dA$$

$$F = p \cdot A$$

since; $p = \gamma h$

$$\therefore \quad F = \gamma h A \leftarrow \text{(volume of the press. dist.)}$$

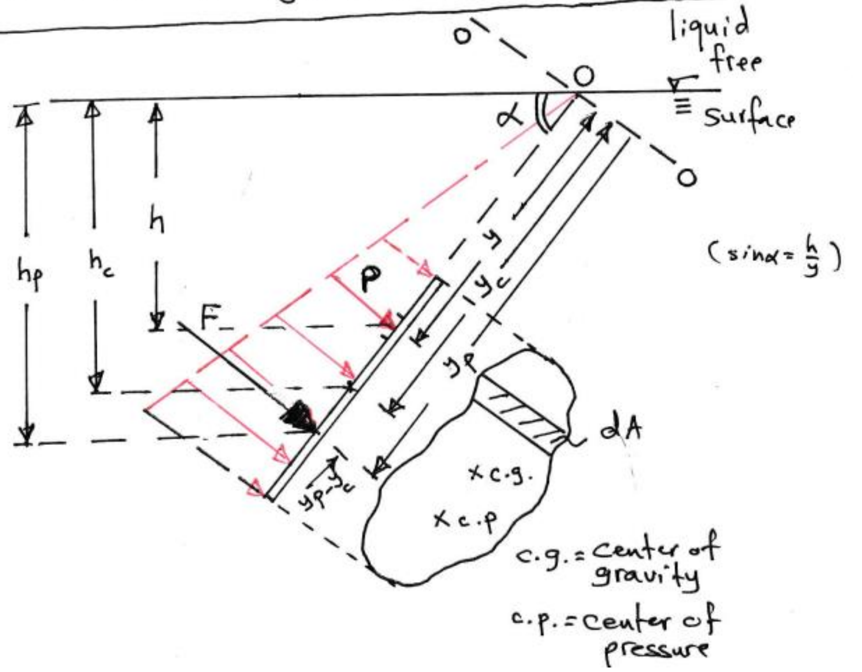




F is acting a the centroid of the plane surface.

2/ Hydrostatic force on a Submerged Inclined Plane Surface

Let a plane surface "A" with its centroid at c.g. be placed at an angle (α) with respect to liquid free surface.



Consider an elementary shaded strip of area dA & at a depth h.

$$F = \int_A p \cdot dA = \int_A \gamma h \cdot dA = \int_A \gamma\, y \sin\alpha \cdot dA$$

$$\therefore F = \gamma \sin\alpha \int y \cdot dA \quad — ①$$

since; $\int_A y \cdot dA = y_c \cdot A$ — ② $\quad \left(y_c = \frac{\int_A y \cdot dA}{\int dA} = \frac{\int_A y \cdot dA}{A} \right)$

where; y_c: centroid of plane surface about o-o axis

substitute eq. ② into eq. ①:

$$\therefore F = \gamma\, y_c \cdot \sin\alpha \cdot A \quad — ③$$

since; $y_c \cdot \sin\alpha = h_c$

$$\therefore \boxed{F = \gamma h_c A} \quad — ④$$

where; F = total hydr. force (N).
h_c = vertical depth from centroid of the plane surface to the liquid free surface (m).
A: area of the inclined plane surface (m^2).

Location of the Hydrostatic force: The point of action of the total hydrostatic force on the surface is called the center of pressure (c.p.). This point is calculated by equating the moment of the total hydr. force acting at c.p. to the summation of the moments due to the elementary forces acting on the elementary strips;

By applying Varignon's theorem; ($M_R = \Sigma M_{\text{components}}$)

$$F \cdot y_p = \int dF \cdot y = \int p \cdot dA \cdot y = \int \gamma h \cdot dA \cdot y$$

$$\therefore F \cdot y_p = \int \gamma y^2 \sin\alpha \cdot dA = \gamma \sin\alpha \int_A y^2 dA$$

$$\therefore y_p = \frac{\gamma \sin\alpha \int_A y^2 dA}{F}$$

From eq. ①: $F = \gamma \sin\alpha \int_A y \cdot dA$

$$\therefore y_p = \frac{\gamma \sin\alpha \int_A y^2 dA}{\gamma \sin\alpha \int_A y \, dA} \Rightarrow y_p = \frac{\int_A y^2 dA}{\int_A y \cdot dA} \quad \text{—} (5)$$

$\int_A y^2 \cdot dA$ = second moment of area of the plane surface about 0–0 axis.

= moment of inertia of the plane surface about o–o axis.

$= I_{o-o}$

$\therefore$ Eq. 5 becomes : $$y_p = \frac{I_{o-o}}{y_c \cdot A} \quad \text{— (6)}$$

since; $I_{o-o} = I_c + y_c^2 \cdot A$

where; I_c: moment of inertia of the plane surface about its centroid.

$$\therefore y_p = \frac{I_c + y_c^2 \cdot A}{y_c \cdot A} \Rightarrow \boxed{y_p = y_c + \frac{I_c}{y_c \cdot A}}$$

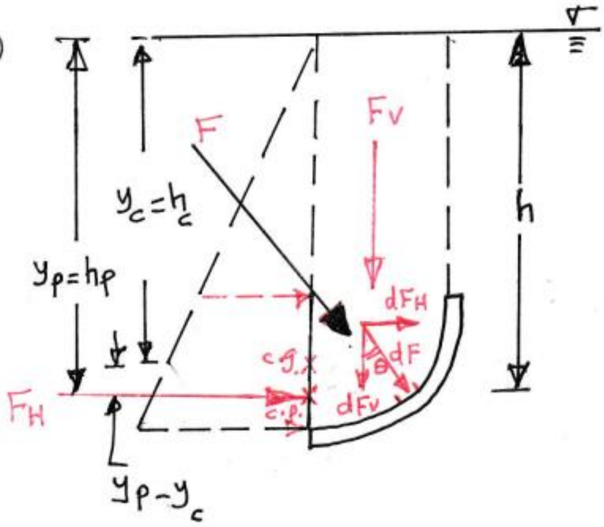
3/ Hydrostatic force on a Submerged Curved Surface

On a curved surface, as shown below, the element force (dF) varies both in magnitude & in direction. The x & y- components of total force (F) can be evaluated by the summation of elemental force components.

since, $F = \int dF = \int_A p \cdot dA$ —①

$\therefore dF_H = p \cdot dA \sin\theta$ —②

& $dF_V = p \cdot dA \cdot \cos\theta$ —③



Subs. eq.② into eq.①:

$$F_H = \int_A p \cdot dA \sin\theta = \int_{A_V} \gamma h (dA)_V \quad —④$$

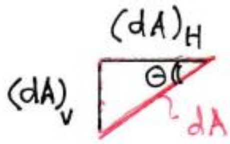
since; $h_c = \dfrac{\int h (dA)_V}{\int (dA)_V} = \dfrac{\int_{A_V} h (dA)_V}{A_V}$

$$\therefore \int_{A_V} h (dA)_V = h_c \cdot A_V$$

$\therefore$ Eq.④ becomes;

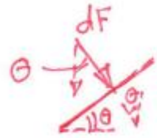
$$\boxed{F_H = \gamma h_c \cdot A_V}$$

$\sin\theta = \dfrac{(dA)_V}{dA}$



$(dA)_V = dA \sin\theta$

$\cos\theta = \dfrac{(dA)_H}{dA}$



$\therefore (dA)_H = dA \cdot \cos\theta$

subs. eq. ③ into eq. ①:

$$F_V = \int_A p \cdot dA \cdot \cos\theta = \int_{A_H} \gamma h \cdot (dA)_H = \int_{\forall} \gamma \, d\forall$$

$$\therefore \boxed{F_V = \gamma \forall}$$

$$F = \sqrt{F_H^2 + F_V^2}$$

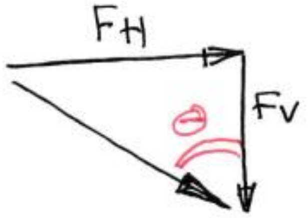
F : Total hydr. force (N).

where; $(dA)_V$: the projection of the element area normal to the x-axis.

$(dA)_H$: the projection of the element area normal to the y-axis.

Direction of the total hydr. force

$$\tan\theta = \frac{F_H}{F_V}$$

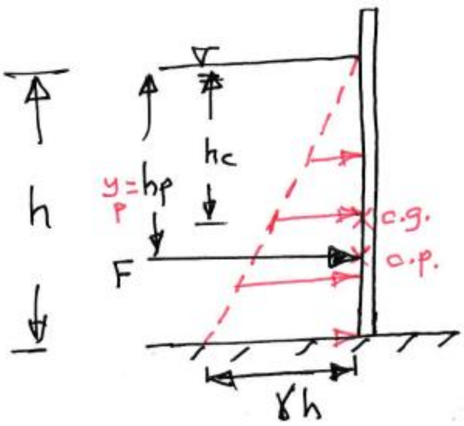


$$\theta = \tan^{-1}\left(\frac{F_H}{F_V}\right)$$ with the vertical.

Pressure Diagram:

The pressure diagram is used when the width of a submerged plane surface is constant.

width = 1 unit



For a unit width normal to the sketch;

since; $F = \gamma h_c A$

$\therefore F = \gamma \cdot \frac{h}{2} \cdot h$

$\therefore F = \frac{1}{2} \gamma h^2$

$\frac{1}{2} \gamma h^2$ = Area of the pressure diagram

$= \frac{1}{2} \cdot \gamma h \cdot h = \frac{1}{2} \times$ base $\times$ height

In general; $\boxed{F = \frac{1}{2} \gamma h^2 \cdot b}$

where; b: width of a submerged plane surface.

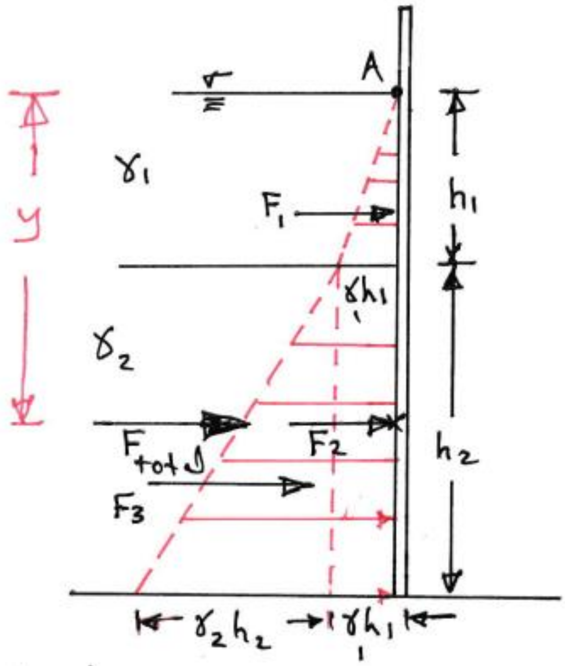
→ F is located at centroid of pressure diagram

$\therefore \; y_p = \frac{2}{3} h$

H.W.: prove $y_p = \frac{2}{3} h$

$b = 1$ unit

For the sketch shown;



$$F_1 = \frac{1}{2}\gamma_1 \cdot h_1 \cdot h_1 (1) = \frac{1}{2}\gamma h_1^2$$

$$F_2 = \gamma_1 \cdot h_1 \cdot h_2 (1) = \gamma_1 h_1 h_2$$

$$F_3 = \frac{1}{2}\gamma_2 h_2 \cdot h_2 (1) = \frac{1}{2}\gamma_2 h_2^2$$

$$F_{total} = F_1 + F_2 + F_3$$

To find the location;

By applying Varignon's theorem about point A

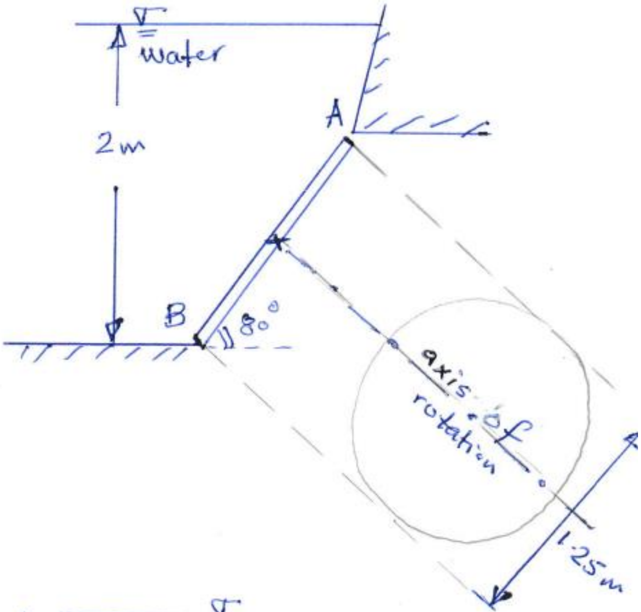
$$F \cdot (y) = F_1\left(\frac{2}{3}h_1\right) + F_2\left(\frac{h_2}{2} + h_1\right) + F_3\left(\frac{2}{3}h_2 + h_1\right)$$

$$\therefore\ y = \checkmark$$

Ex.1: For the gate (AB) shown in the figure below, calculate:

1- Hydrostatic force on the gate.

2- Turning moment about the axis of rotation.



Solution:

Since $F = \gamma h_c A$

$\gamma = \gamma_w = 9810 \ N/m^3$

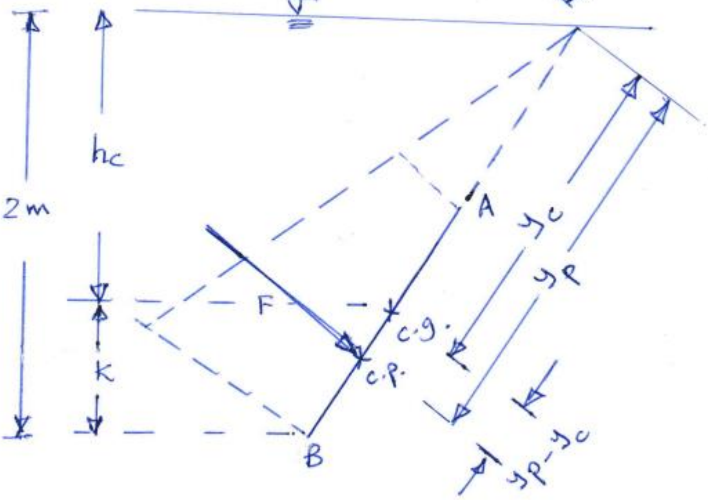
$= 9.81 \ KN/m^3$

From the sketch:

$h_c = 2 - K$

$= 2 - \frac{1.25}{2} \sin 80°$

$= 1.384 \ m$



$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (1.25)^2$

$= 1.227 \ m^2$

$F = 9.81 * 1.384 * 1.227$

$= 16.66 \ KN$

Since $y_p = y_c + \frac{I_c}{y_c A}$

$\therefore \ y_p - y_c = \frac{I_c}{y_c A} = \frac{\frac{\pi}{64} D^4}{\frac{h_c}{\sin 80°} * A}$

$y_p - y_c = \frac{\frac{\pi}{64} (1.25)^4}{\frac{1.384}{\sin 80°} * 1.227} = 0.07 m$

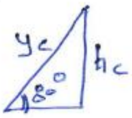
$\therefore$ Turning moment about the axis of rotation $= F(y_p - y_c) = 16.66 * 0.07 = 1.17 \ KN.m$

$\sin 80° = \frac{K}{\frac{1.25}{2}}$

$\therefore K = \frac{1.25}{2} \sin 80°$

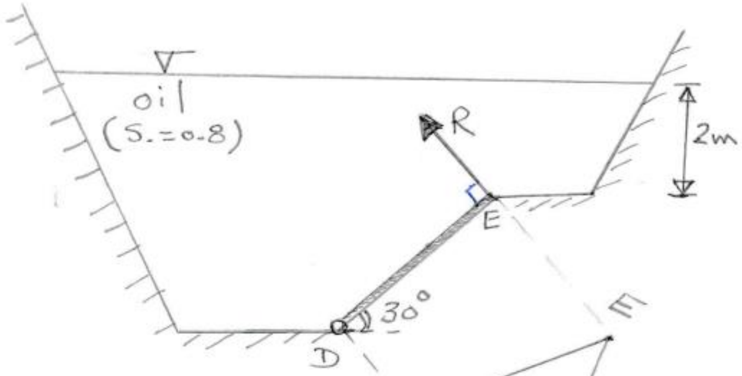
$K = 0.616 \ m$

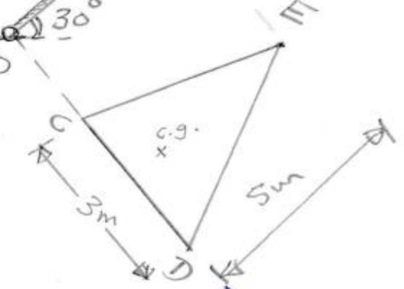
$\sin 80° = \frac{h_c}{y_c}$



$\therefore y_c = \frac{h_c}{\sin 80°}$

Ex.2: The triangular gate CDE is hinged along CD and is opened by a normal force R applied at E. It holds oil (s. = 0.8) above it and is open to atmosphere on its lower side. The gate weighs 20kN. Find (a) the magnitude of the hydrostatic force, (b) the location of pressure center, & (c) the force R needed to open the gate.





Solution:

(a) since $F = \gamma h_c A$

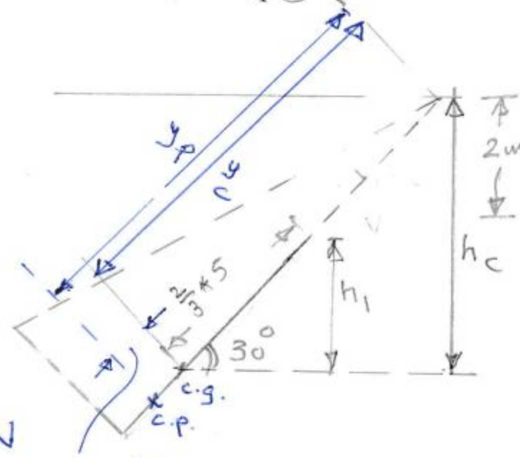
$$\gamma_{oil} = 0.8 * \gamma_{water} = 0.8 * 9810$$
$$= 7848\ N/m^3$$
$$= 7.848\ KN/m^3$$

$$h_c = h_1 + 2$$

$$h_c = \frac{2}{3} * 5 * \sin 30^\circ + 2 = 1.667 + 2$$
$$= 3.667\ m$$

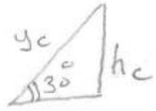
$$A = \frac{1}{2} bh = \frac{1}{2} * 3 * 5 = 7.5\ m^2$$

$$\therefore F = 7.848 * 3.667 * 7.5 = 215.84\ KN$$



$$y_p - y_c = \frac{I_c}{y_c . A}$$

(b) $y_p = y_c + \frac{I_c}{y_c . A}$



$$\sin 30^\circ = \frac{h_c}{y_c}$$

$$y_c = \frac{h_c}{\sin 30^\circ} = \frac{3.667}{\sin 30^\circ}$$

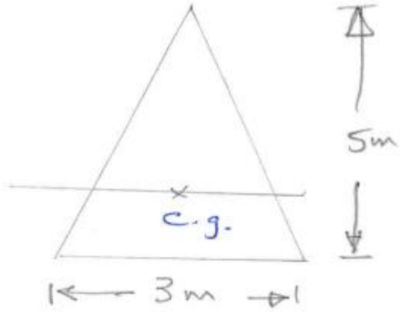
$$\therefore y_c = 7.334\ m$$

$$I_c = \frac{bh^3}{36}$$

$$\therefore I_c = \frac{3(5)^3}{36} = 10.417 \; m^4$$

$$\therefore \frac{I_c}{y_c A} = \frac{10.417}{7.334 * 7.5} = 0.189 \; m$$

$$\therefore y_p = 7.334 + \qquad = 7.523 m$$



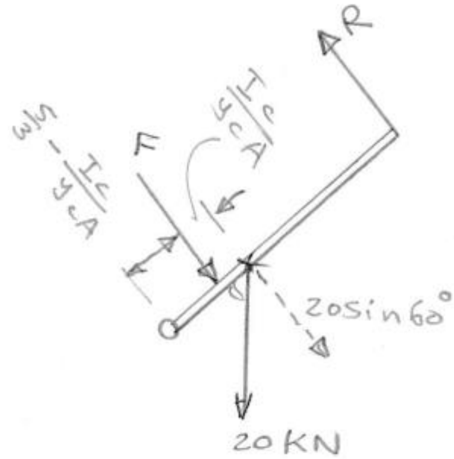
(c) $\sum M_{CD} = 0.$

$$R * 5 = F * \left(\frac{5}{3} - \frac{I_c}{y_c A}\right) + 20 \sin 60^\circ * \frac{5}{3}$$

$$5R = 215.84\left(\frac{5}{3} - 0.189\right) + 28.87$$

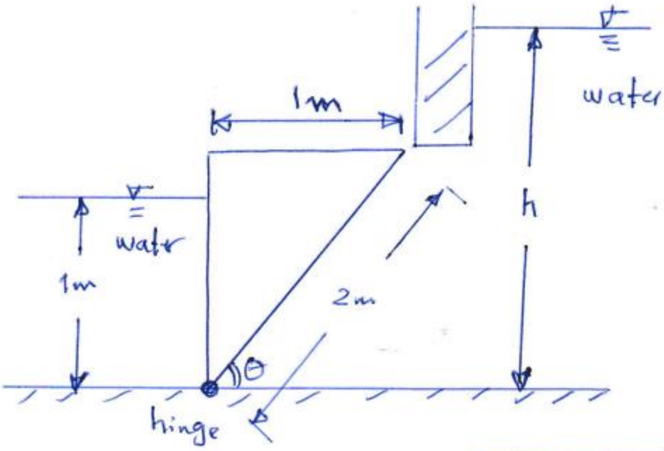
$$5R = 318.94 + 28.87$$

$$\therefore R = 69.56 \; KN$$



F.B.D of the gate

Ex.3: How long will the water on the right (h) has to rise to open the gate shown below. The gate is 2m wide, and is constructed of material with s. = 4.5.



Solution:

For F1

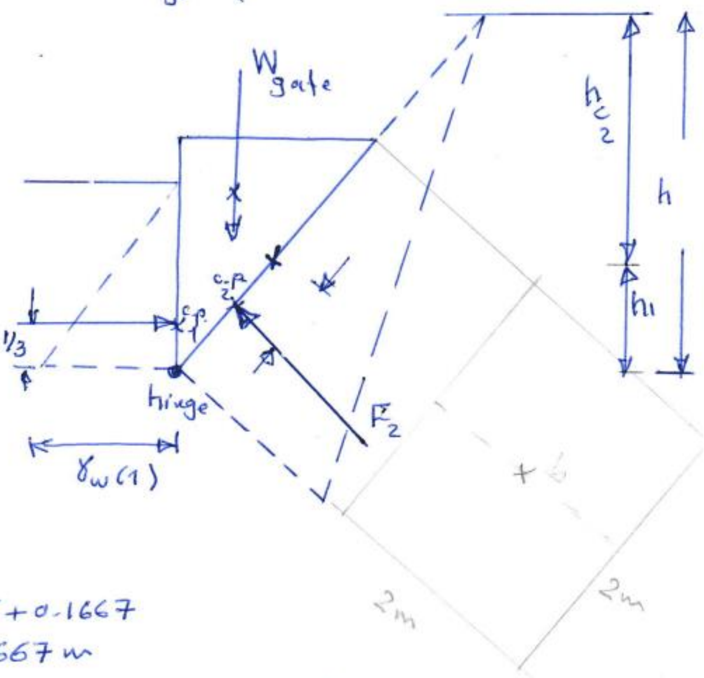
By using press. dist. diagram

$F_1 = \frac{1}{2}(\text{base}) * (\text{height}) * b$

$\text{base} = \gamma_w (1) = 9.81 \text{ KN}$

$F_1 = \frac{1}{2} * 9.81 * 1 * 2 = 9.81 \text{ KN}$

$y_p = \frac{2}{3} * 1 = 0.667 m$



H.W. use $F_1 = \gamma h_{c_1} A_1$

$h_{c_1} = \frac{1}{2}$

$A_1 = 1 * 2$

$F_1 = 9.81 * \frac{1}{2} * 2 = 9.81 \text{ KN}$

$y_{p_1} = y_{c_1} + \frac{I_{c_1}}{y_{c_1} A_1} = 0.5 + \frac{\frac{2 * 1^3}{12}}{0.5(1 * 2)}$

$= 0.5 + \frac{2}{12} = 0.5 + 0.1667$

$= 0.667 m$

For F2: $F_2 = \gamma_w h_{c_2} A_2$

$F_2 = 9.81 * h_{c_2} * (2 * 2) = 39.24 h_{c_2}$ (KN)

$y_{p_2} = y_{c_2} + \frac{I_{c_2}}{y_{c_2} A_2} \Rightarrow$

$\therefore \frac{I_{c_2}}{y_{c_2} A_2} = \frac{2(2)^3/12}{1.155 h_{c_2} (2 * 2)} = \frac{0.288}{h_{c_2}}$

$\cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 60^\circ$

$\sin 60^\circ = \frac{h_1}{1}$

$\therefore h_1 = 0.886 m$

$\sin 60^\circ = \frac{h_{c_2}}{y_{c_2}}$

$\therefore y_{c_2} = \frac{h_{c_2}}{\sin 60^\circ}$

$\therefore y_{c_2} = 1.155 h_{c_2}$

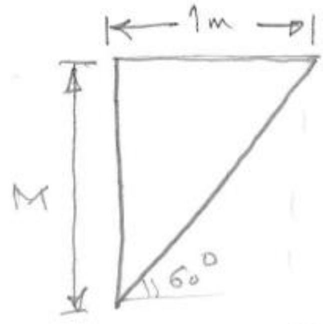
$W = m.g$; $\rho = \frac{m}{\forall} \Rightarrow m = \rho \forall$

$\therefore W = \rho g \forall = \gamma \forall$

$W_{gate} = S\gamma_w * \forall = 4.5 * 9.81 * \forall$

$\forall = \frac{1}{2} M * 1 * 2 = 1.732 \; m^3$

$W_{gate} = 4.5 * 9.81 * 1.732 = 76.46 \; KN$



$\tan 60^\circ = \frac{M}{1}$

$M = \tan 60^\circ$

$M = 1.732 m$

$\Sigma M_{hinge} = 0.$

$F_2 * \left[1 - (y_{p_2} - y_{c_2})\right] = F_1 * \frac{1}{3} + W_{gate} * \frac{1}{3}$

$39.24 h_{c_2} \left[1 - \frac{0.288}{h_{c_2}}\right] = \frac{9.81}{3} + \frac{76.46}{3}$

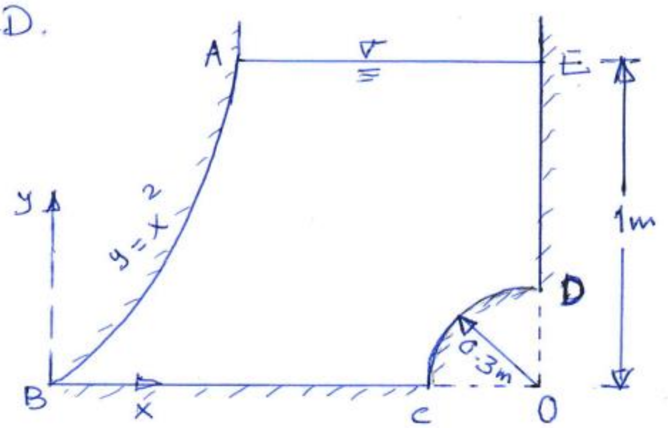
$39.24 h_{c_2} - 11.3 = 28.756$

$\therefore h_{c_2} = 1.021 \; m$

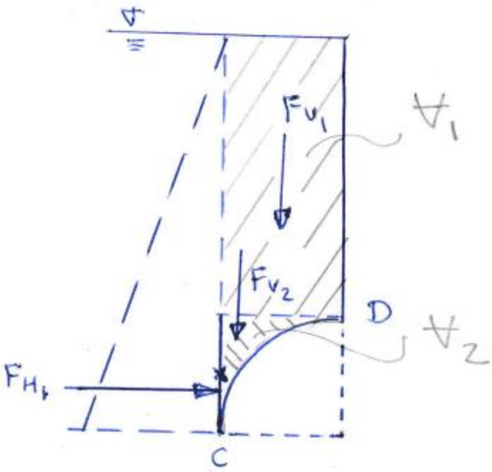
since $h = h_1 + h_{c_2}$

$\therefore h = 0.886 + 1.021 = 1.89 \; m$

Ex.4: A tank ABCDE contains water upto a depth of 1m and is 2m wide. The curve AB is defined by $y = x^2$ and curve CD is a quadrant of a circle of radius 0.3m. Calculate the forces on surfaces AB & CD.



Solution: Forces on surface CD:



$F_{H_1} = \gamma_w h_{c_1} A_{v_1}$

$h_{c_1} = 1 - \frac{0.3}{2} = 0.85\,m$

$A_{v_1} = 0.3(2) = 0.6\,m^2$

$\therefore F_{H_1} = 9.81\,(0.85)(0.6) = 5\,KN \rightarrow$

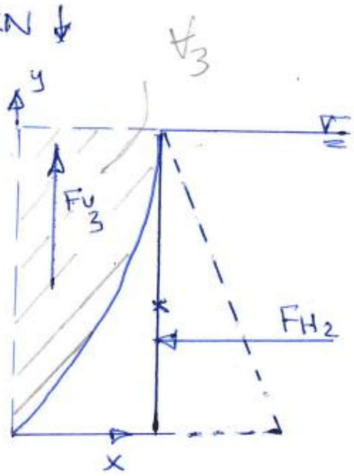
$F_v = \gamma_w \forall$

$F_{v_1} = \gamma_w \forall_1 = 9.81(0.3 * 0.7 * 2)$
$= 4.12\,KN \downarrow$

$F_{v_2} = \gamma_w \forall_2 = 9.81\left[(0.3)^2 - \frac{\pi}{4}(0.3)^2\right] * 2 = 0.38\,KN \downarrow$

$\therefore F_v = F_{v_1} + F_{v_2} = 4.5\,KN \downarrow$

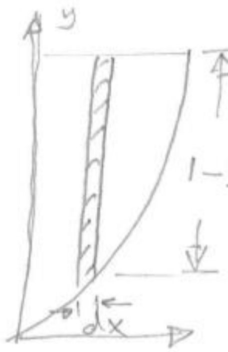
Forces of Surface AB



$F_{H_2} = \gamma_w h_{c_2} A_{v_2} = 9.81 * 0.5 * (1 * 2)$
$= 9.81\,KN \leftarrow$

$F_{v_3} = \gamma_w \forall_3$

$\forall_3 = A_3 * 2$



$$A_3 = \int_a^b (1-y)\,dx$$

at $y=0 \Rightarrow x=0$.

at $y=1 \Rightarrow x=\pm 1 \Rightarrow x=1$ only according to the sketch

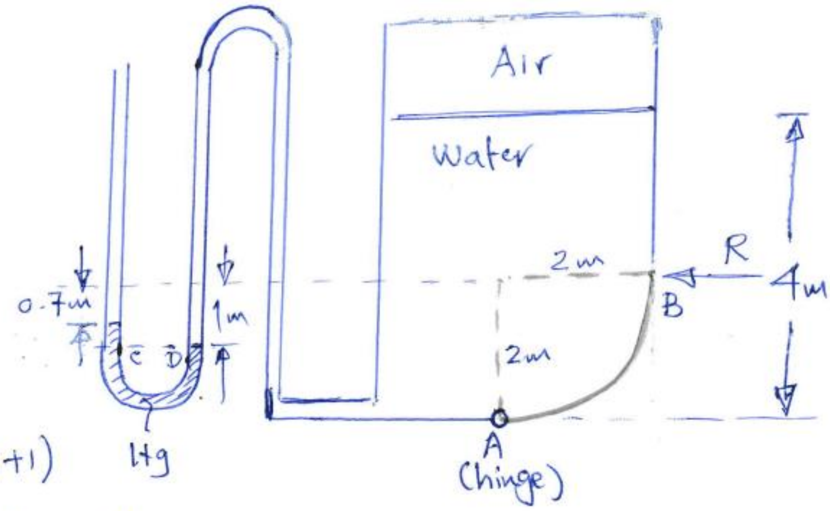
$$A_3 = \int_0^1 (1-x^2)\,dx = x\Big|_0^1 - \frac{1}{3}x^3\Big|_0^1 = 1-\left(\frac{1}{3}\right) = \frac{2}{3}\ m^2$$

$$\therefore \forall_3 = \frac{2}{3} * 2 = \frac{4}{3}\ m^3$$

$$\therefore F_{V_3} = 9.81 * \frac{4}{3} = 13.1\ KN \uparrow$$

H.W.: prove that the resultant of forces acting on surface CD must pass through point O.

Ex.5: Calculate the force R required to hold the gate AB in a closed position. The gate width is 3m. Neglect the weight of the gate.



Solution:

From the manometer;

$P_C = P_D$

$13.6\gamma_w(0.3) = P_{air} + \gamma_w(2+1)$

$\therefore P_{air} = 1.08\gamma_w = h_{w*} * \gamma_w$

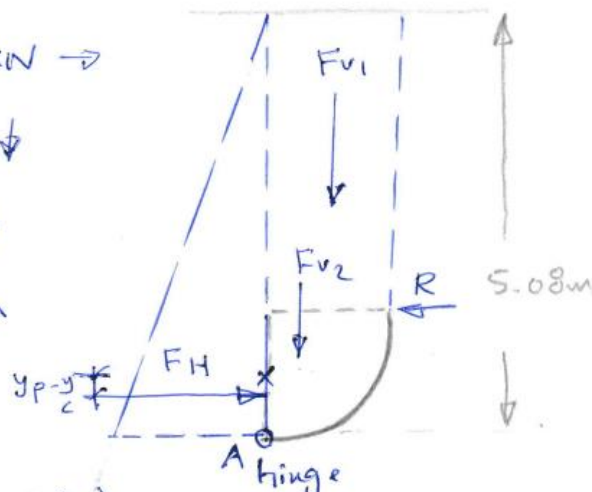
$\therefore h_w = 1.08m$

$F_H = \gamma_w(5.08-1)(2*3) = 240.15\ KN \rightarrow$

$F_{v_1} = \gamma_w(2*3.08*3) = 181.28\ KN \downarrow$

$F_{v_2} = \gamma_w(\frac{\pi}{4}(2)^2*3) = 92.46\ KN \downarrow$

$y_p - y_c = \frac{I_c}{y_c A} = \frac{3(2)^3/12}{4.08(2*3)} = 0.082m$



$\Sigma M_{hinge} = 0.$

$F_H * (1-0.082) + F_{v_1} * 1 + F_{v_2} * \frac{4(2)}{3\pi}$

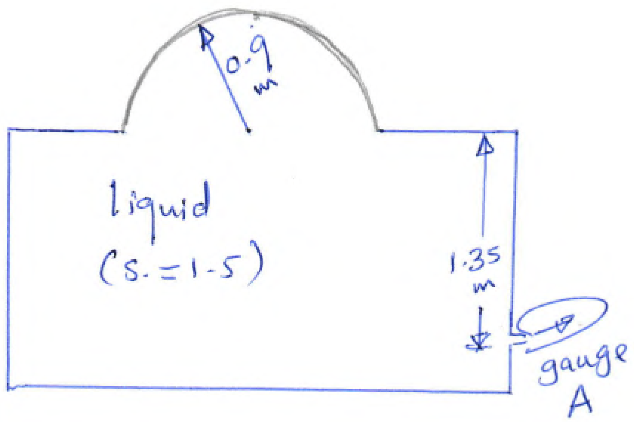
$- R * 2 = 0.$

$240.15(0.918) + 181.28*1 + 92.46 * 0.85 = 2R$

$\therefore R = 240.16\ KN$

Ex.-6: Find the Vertical component of force in the metal spherical dome shown in figure below, when gauge A reads 69 kPa. Assume the dome weight 4500N

Note: The volume of sphere = $\frac{\pi D^3}{6}$



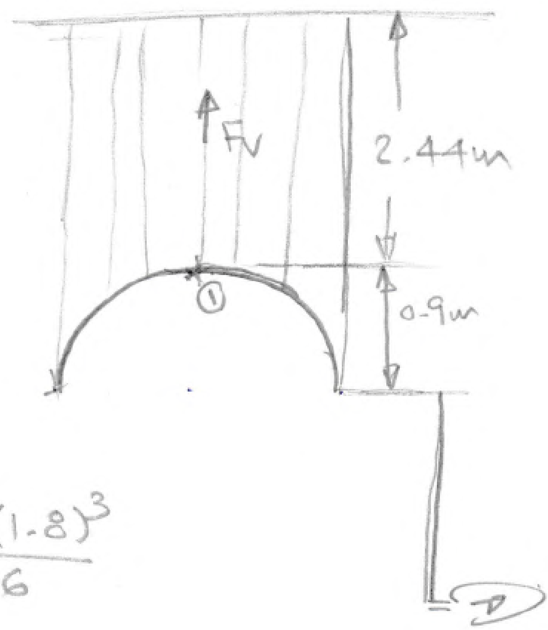
Solution:

$P_{gauge\,A} = P_1 + 1.5\,\gamma_w\,(0.9 + 1.35)$

$69 = P_1 + 1.5(9.81)(2.25)$

$\therefore P_1 = 35.89$ KN

$h_{liquid} = \frac{35.89}{1.5(9.81)} = 2.44\,m$



$F_V = \gamma_{liquid}\,\forall$

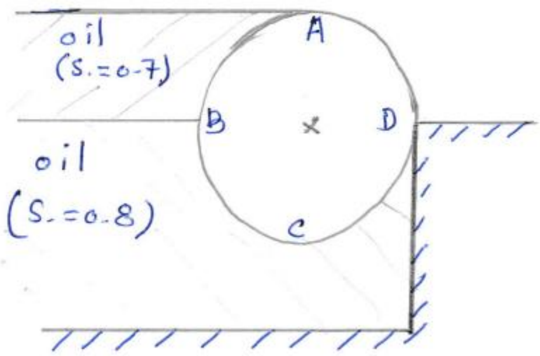
$\forall = \frac{\pi}{4}(1.8)^2 * (0.9 + 2.44) - \frac{1}{2}\frac{\pi (1.8)^3}{6}$

$= 6.97\,m^3$

$F_V = 1.5(9.81)(6.97) = 102.56$ KN

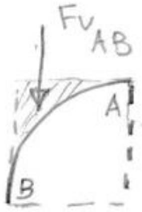
$F_{V\,net} = F_V - W = 102.56 - 4.5 = 98.06$ KN

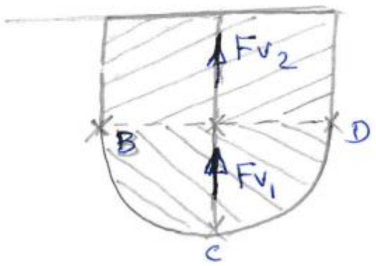
Ex. 7. A cylinder of 1m dia. & 2m length stays in equilibrium, as shown in figure below. Calculate the specific gravity of the material of the cylinder.



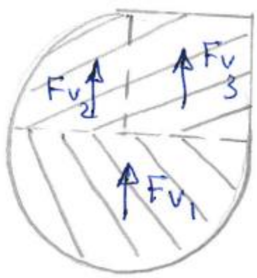
Solution:

For surface AB.





⇒ Net Vertical forces



$$Fv_1 = 0.8\gamma_w * \frac{\pi}{2}(0.5)^2 * 2$$

$$= 0.628\,\gamma_w$$

$$Fv_2 = 0.7\,\gamma_w * \frac{\pi}{4}(0.5)^2 * 2$$

$$= 0.275\,\gamma_w$$

$$Fv_3 = 0.7\,\gamma_w\,(0.5 * 0.5 * 2)$$

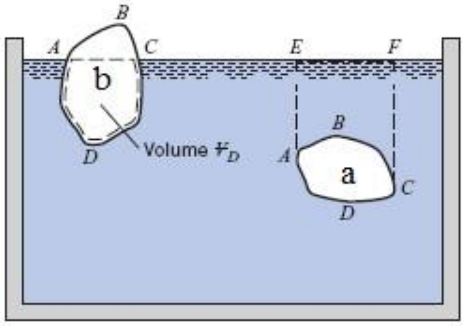
$$= 0.35\,\gamma_w$$

$$\therefore\ Fv = Fv_1 + Fv_2 + Fv_3 = 1.253\,\gamma_w$$

$$\Sigma F_y = 0 \Rightarrow N = Fv = 1.253\,\gamma_w = \gamma\,\forall = S.\,\gamma_w * \forall$$

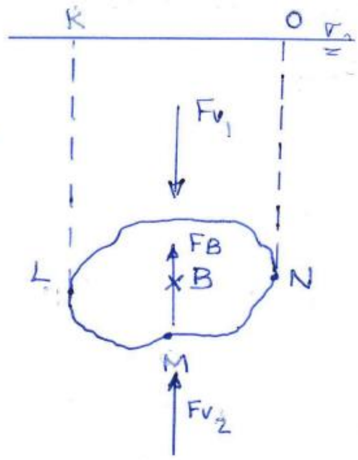
$$1.253\,\gamma_w = S.\,\gamma_w * \frac{\pi}{4}(1)^2 * 2 \Rightarrow S. = 0.8$$

3.5 The Buoyant Force Equation



The buoyant force (F_B) passes through the center of buoyancy (B).

Submerged Body



$F_{v_2} > F_{v_1}$ because pressure increase with depth

$F_{v_2} = \gamma V_2$

$F_{v_1} = \gamma V_1$

where, γ = specific weight of the fluid.

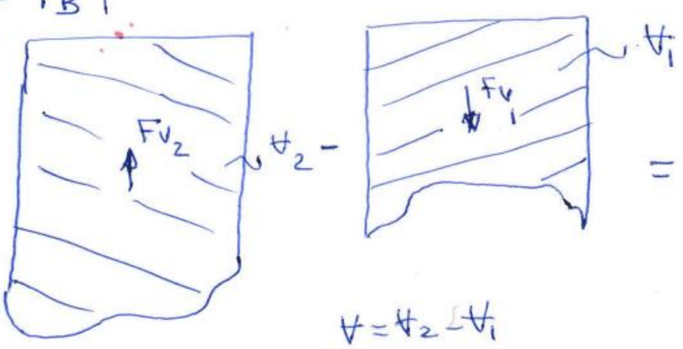
$V_1 = V_{KLNOK}$

$V_2 = V_{KLMNOK}$

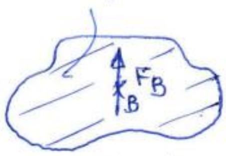
$V_2 - V_1$ = volume of displaced fluid = volume of submerged body $= V$

$\therefore F_{v_2} - F_{v_1} = \gamma V = F_B \uparrow$

$\therefore \boxed{F_B = \gamma V} \uparrow$



$V = V_2 - V_1$



-49-

$F_B = \gamma V$

Stability of Immersed Bodies

When a body is completely immersed in a liquid, its stability depends on the relative positions of the center of gravity of the body and the centroid of the displaced volume of fluid, which is called the *center of buoyancy*.

- If the center of buoyancy is above the center of gravity (Fig. 3.11a), any tipping of the body produces a righting couple, and consequently, the body is stable.
- If the center of gravity is above the center of buoyancy (Fig. 3.11c), any tipping produces an increasing overturning moment, thus causing the body to turn through 180°.
- Finally, if the center of buoyancy and center of gravity are coincident, the body is neutrally stable—that is, it lacks a tendency for righting or for overturning, as shown in Fig. 3.11*b*.

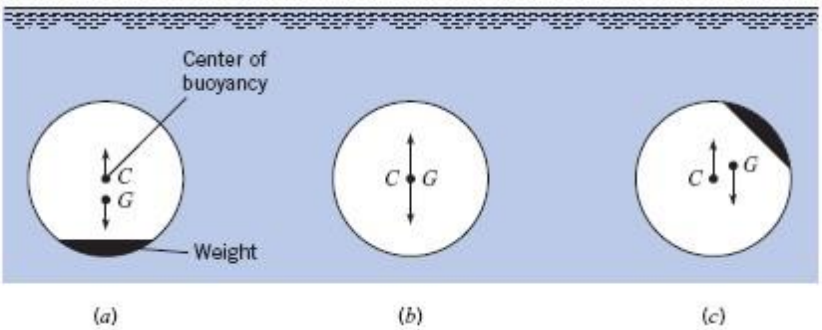


Fig. 3.11

Stability Floating Bodies

The stability for floating bodies than for immersed bodies is very important because the center of buoyancy may take different positions with respect to the center of gravity, depending on the shape of the body and the position in which it is floating. When the center of gravity *G* is above the center of buoyancy *C* (center of displaced volume) for floating body, the body will be stable and equilibrium. The reason for the change in the center of buoyancy for the ship is that part of the original buoyant volume, as shown in Fig.3.12by the wedge shape *AOB*, is transferred to a new buoyant volume *EOD*. Because the buoyant center is at the centroid of the displaced volume, it follows that for this case the buoyant center must move laterally to the right. The point of intersection of the lines of action of the buoyant force before and after heel is called the ***metacenter M***, and the distance ***GM*** is called the ***metacentric height***.

- If ***GM*** is positive—that is, if ***M*** is above ***G***,the body is stable
- If ***GM*** is negative, the body is unstable.

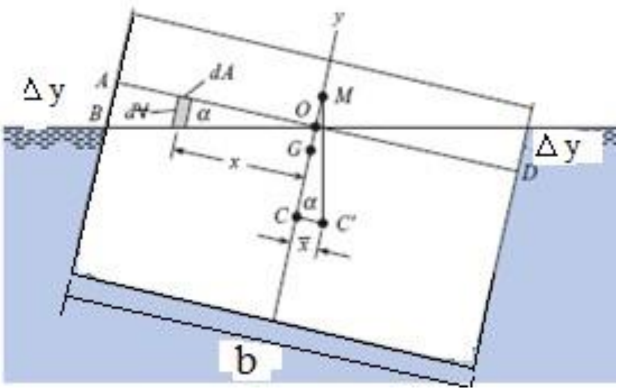


Fig.3.12

Consider the prismatic body shown in Fig. 3.12, which has taken a small angle of heel α. First evaluate the lateral displacement of the center of buoyancy **CC'**, then it will be easy by simple trigonometry to solve for the metacentric height *GM* or to evaluate the righting moment.

The righting couple $= W.\, MG.\sin\alpha$

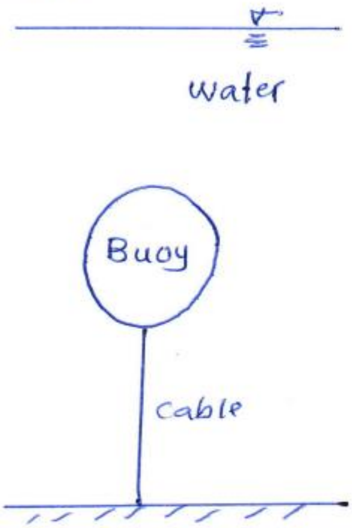
Where : *W* weight of body and α angle of heel.

By similar triangle ***EOD*** and ***C'CM***: $\frac{\Delta y}{b/2} = \frac{C'C}{CM}$ find ***CM***

$$GM = MC - GC$$

Ex.1: A spherical buoy has a dia. of (1.5m), weighs 8.5KN, and is attached as shown in figure below with a cable. Determine the tension force at the cable.

Note: volume of sphere $= \frac{\pi D^3}{6}$



Solution

F.B.D of the bouy

$\Sigma F_y = 0.$

$F_B - W - T = 0.$

$$T = F_B - W = \gamma_w \forall_{sphere} - W$$

$$= 9.81 * \frac{\pi (1.5)^3}{6} - 8.5$$

$$= 17.34 - 8.5 = 8.84 KN$$

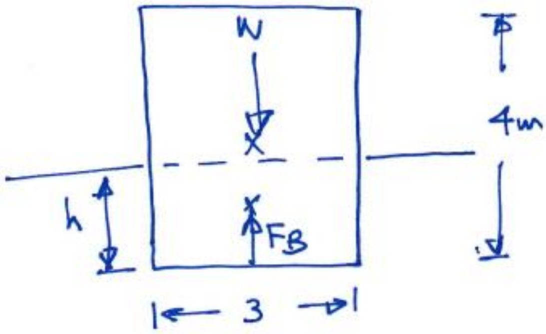
Ex.2: A rectangular box of dimension 7.6m x 3m x 4m deep floats in water. If the box weighs 40ton, determine:

1- the deep it will sink

2- the mass of stone placed on the box to sink it 4m depth.

Solution :

$b = 7.6\,m$

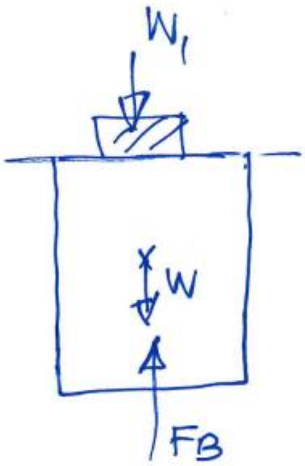


1- $F_B = W$

$$\gamma_{water} \forall_{sink} = m \cdot g$$

$$98\cancel{1}0 * 3(7.6)h = 40 * \cancel{10}^{3} * \cancel{9}.81$$

$$h = \frac{40}{22.8} = 1.754\,m$$



2- $\Sigma F_y = 0.$

$$W_1 + W = F_B$$

$$W_1 + 40 * 9810 = 9810\,(3)(4)(7.6)$$

$$W_1 = 502272\ N$$

$$\therefore m_1 \cdot g = 502272$$

$$\therefore m_1 = \frac{502272}{9.81} = 51200\,kg = 51.2\ ton$$

Ex3: An object weighs 3N in Water and 4N in oil ($s = 0.83$). Determine its volume & specific gravity (S.).

$$W_{liquid} = W_{air} - F_B$$

Sol.:

$$W_{water} = W_{air} - \gamma_w \forall \quad \text{—} (1)$$

$$W_{oil} = W_{air} - \gamma_{oil} \forall \quad \text{—} (2)$$

From Eq (1)

$$\therefore 3 = W_{air} - 9810 \forall \Rightarrow W_{air} = 3 + 9810 \forall \quad \text{—} (3)$$

subs. Eq. (3) into eq. (2)

$$4 = 3 + 9810 \forall - 0.83 (9810)(\forall)$$

$$\cancel{4-3} \quad 1 = 0.17(9810) \forall$$

$$\therefore \forall = 6 * 10^{-4} \ m^3$$

From Eq. (3) $\Rightarrow W_{air} = 3 + 9810 * 6 \times 10^{-4}$

$$W_{air} = 8.886 \ N$$

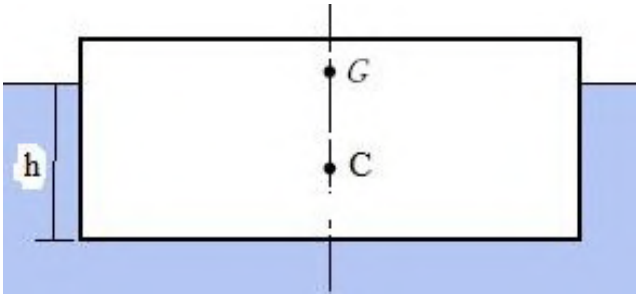
since $W = \gamma_{object} \forall = \rho_{object} * g * \forall$

$$8.886 = \rho_{object} * 9.81 * 6 * 10^{-4}$$

$$\rho_{obj} = 1509.7 \ Kg/m^3$$

$$S. = \underline{1.51}$$

Example 2.13 In Fig. a scow 20 ft wide and 60 ft long has a gross weight of 225 short tons (2000 lb). Its center of gravity is 1.0 ft above the water surface. Find the metacentric height and restoring couple when $\Delta y = 1.0$ ft.



SOLUTION

1. Find the depth (h):

The depth of submergence h in the water is

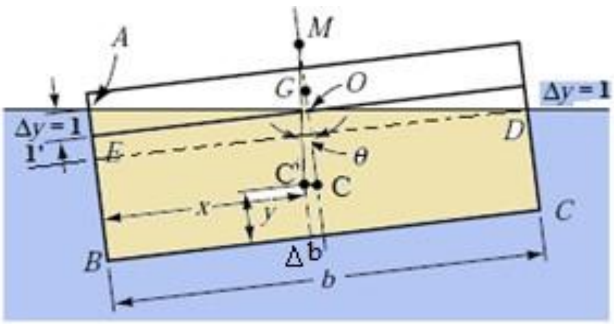
$$h = \frac{225(2000)}{20(60)(62.4)} = 6.0 \text{ ft}$$

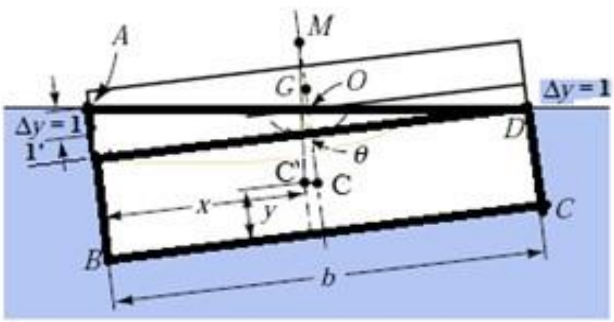
2. Find the location of new center of buoyancy (C'):

The centroid in the tipped position is located with moments about AB and BC

$$x = \frac{5(20)(10) + 2(20)(\frac{1}{2})(\frac{20}{3})}{6(20)} = 9.46 \text{ ft}$$

$$y = \frac{5(20)(\frac{5}{2}) + 2(20)(\frac{1}{2})(5\frac{2}{3})}{6(20)} = 3.03 \text{ ft}$$



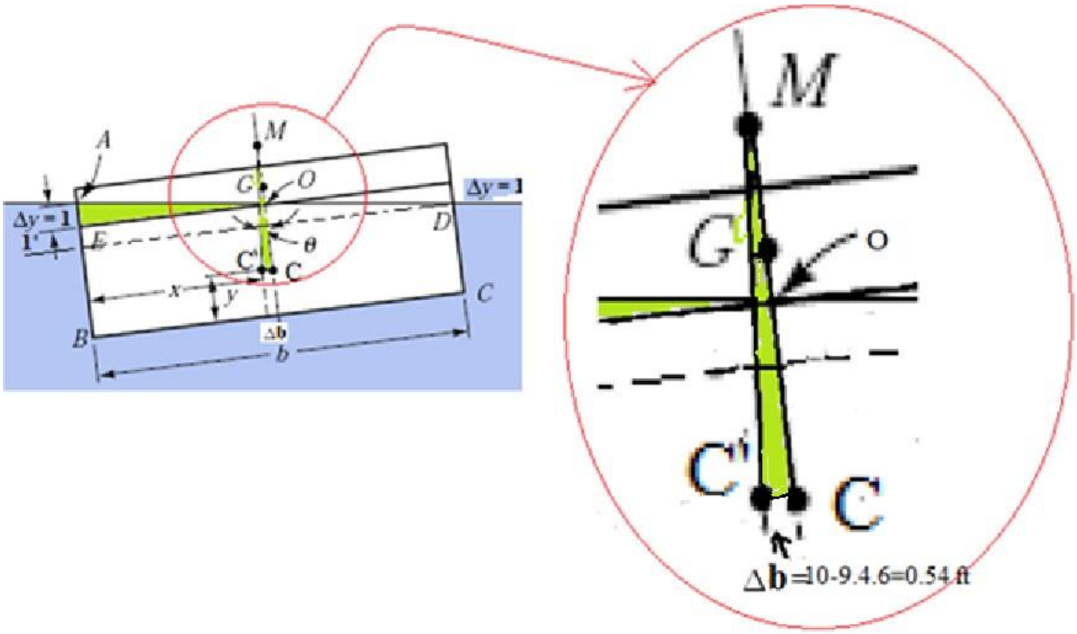


3. Find the distance **CM** :

By similar triangles **AOE** ***and CC'M*** $\frac{\Delta y}{b/2} = \frac{\Delta \mathbf{b}}{\boldsymbol{CM}}$

$\Delta y = 1$, $b/2 = 10$, $\Delta \mathbf{b} = 10 - 9.46 = 0.54$ ft; then

$$\overline{\boldsymbol{CM}} = \frac{0.54(10)}{1} = 5.40 \text{ ft}$$



4.Find **MG**

G is 7.0 ft from the bottom; hence,

$$\overline{\boldsymbol{GC}} = 7.00 - 3.03 = 3.97 \text{ ft}$$

and $\overline{MG} = \overline{\boldsymbol{MC}} - \overline{\boldsymbol{GC}} = 5.40 - 3.97 = 1.43$ ft

The scow is stable since $\overline{MG}$ is positive; the righting moment is

$$W\overline{MG}\sin\theta = 225(2000)(1.43)\frac{1}{\sqrt{101}} = 64{,}000 \text{ lb}\cdot\text{ft}$$

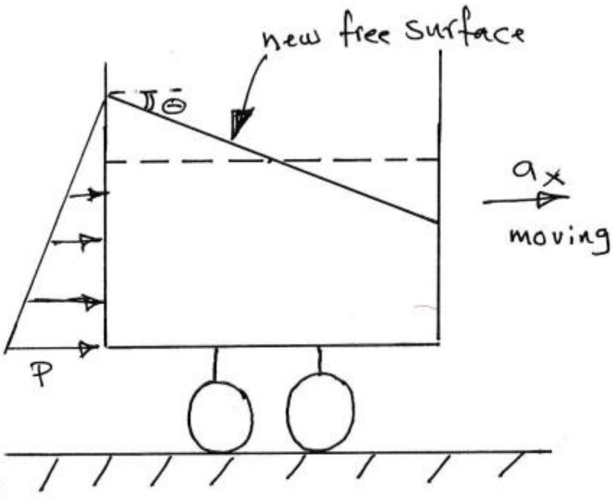
3.6 Equilibrium of accelerated fluid masses

If a body of fluid is moved at a constant velocity, then it obeys the equations derived earlier for static equilibrium.

If a body of fluid is accelerated such that, after some time, it has adjusted so that there are no shearing forces, there is no motion between fluid particles, and it moves as a solid block, then the pressure distribution within the fluid can be described by equations similar to those applying to static fluids.

a- Horizontal Acceleration

$$\boxed{\tan\theta = \frac{a_x}{g}} \qquad \boxed{P = \gamma h}$$



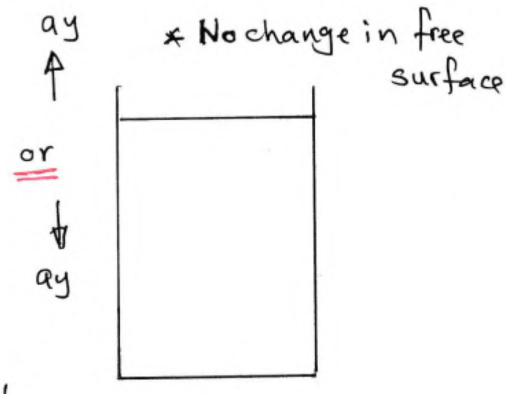
where;

θ = Inclination angle of the new free surface with the horizontal (degree).

a_x = constant horizontal acceleration (m/s^2).

P = pressure value (Pa).

b- Vertical Acceleration



$$\boxed{P = \gamma h \left(1 + \frac{a_y}{g}\right)}$$ moving upwards ↑

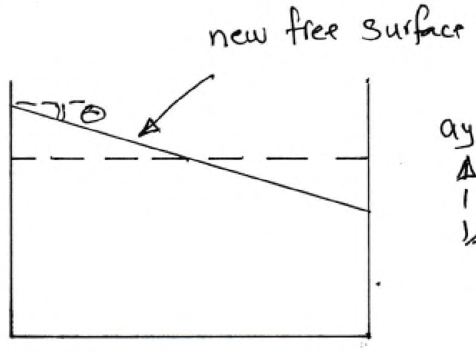
&

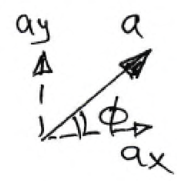
$$\boxed{P = \gamma h \left(1 - \frac{a_y}{g}\right)}$$ moving downwards ↓

c- Inclined Acceleration

$$\boxed{\tan\theta = \frac{a\cdot\cos\phi}{a\sin\phi + g} = \frac{a_x}{a_y + g}}$$

moving upwards ↗





$$\boxed{\tan\theta = \frac{a\cos\phi}{a\sin\phi - g} = \frac{a_x}{a_y - g}}$$

moving downwards ↙

$$\boxed{p = \gamma h\left(1 + \frac{a_y}{g}\right)}$$ moving upwards

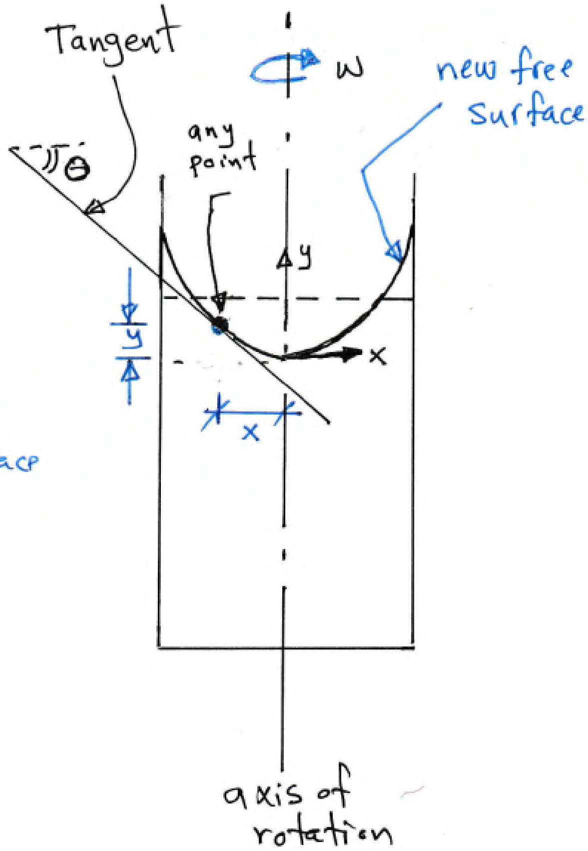
$$\boxed{p = \gamma h\left(1 - \frac{a_y}{g}\right)}$$ moving downwards

where; θ: Inclination angle of the new free surface (degree).

a: constant inclined acceleration (m/s^2).

Liquid in A Container Subjected to A Constant Rotation

A liquid, contained in a vessel, may be rotated at a constant rotational velocity (w) without any relative movement being created between different elements of the liquid in the vessel. The liquid reorients itself once & for all to stay in that position with respect to the axis of rotation.



$$\boxed{\tan\theta = \frac{w^2 \cdot x}{g}}$$

$$\boxed{y = \frac{w^2 \cdot x^2}{2g}}$$

parabola Equation (new free surface eq.)

where;

θ = Inclination angle of tangent of any point located along new free surface (degree).

x, y = x & y -values for any point located along new free surface.

ω : constant angular velocity (rad./s.) $= \frac{2\pi N}{60}$

N : constant angular velocity (r.p.m).

Noting that ; $V = \omega . x$

$a = \omega^2 . x$

where; V = velocity vector (m/s.)

a = acceleration (m/s.²).

Ex.1: An open rectangular tank (5m X 4m X 3m high) containing water upto a height of (2m) is accelerated at (3 m/s²)

a - horizontally along the longer side.

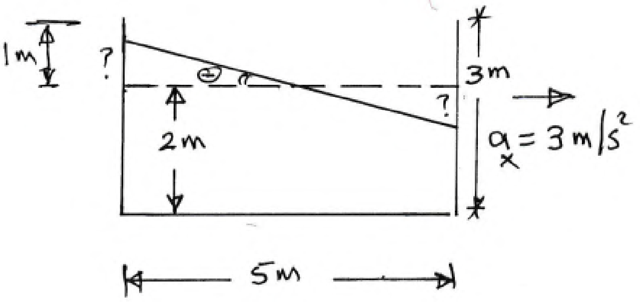
b - Vertically upwards.

c - " downwards &

d - upwards in a direction inclined 30° to the horizontal along the longer side.

Calculate, in each case, the total force on the base of the tank as well as on the vertical faces

b=4m



Sol.: a-

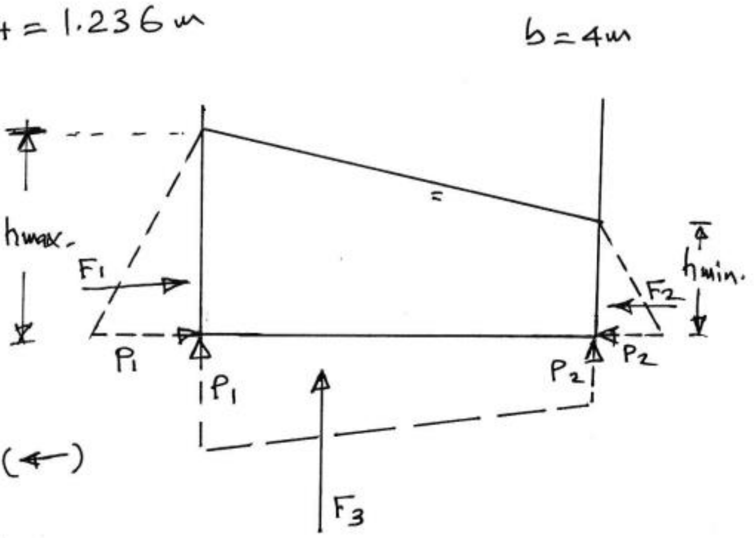
$$\tan\theta = \frac{a_x}{g} = \frac{3}{9.81} = \frac{?}{2.5}$$

$\therefore$? = 0.764m < 1m

The water does not spilt over

$\therefore h_{max.} = 2 + 0.764 = 2.764 m$

$h_{min.} = 2 - 0.764 = 1.236 m$



$P_1 = \gamma_w h_{max.} = \frac{9810}{1000}(2.764)$

$\therefore P_1 = 27 \text{ KPa}$

$F_1 = \frac{1}{2} P_1 \cdot h_{max.} \cdot b = 149.2 \text{ KN} (\leftarrow)$

$P_2 = \gamma_w h_{min.} = 9.81 (1.236)$

$= 12.125 \text{ KPa}$

$\therefore F_2 = \frac{1}{2} P_2 \cdot h_{min.} \cdot b = 30 \text{KN} (\rightarrow)$

Note: All the forces above are reaction forces to the action forces.

$F_3 = \frac{P_1 + P_2}{2} (5)(4) = 391.25 \text{ KN} (\downarrow)$

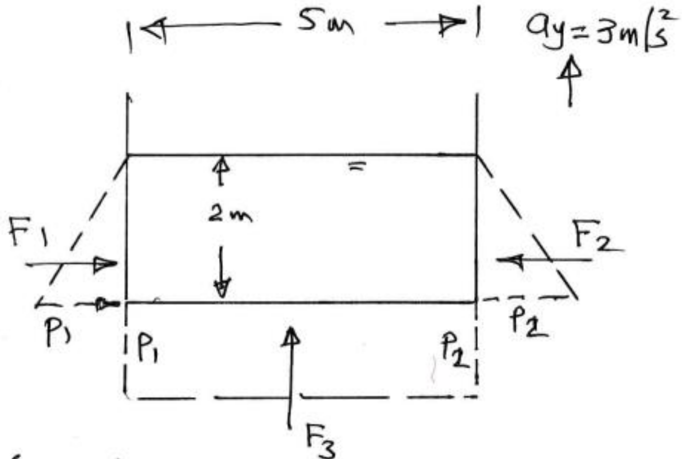
b -

$P_1 = \gamma h \left(1 + \frac{a_y}{g}\right)$

$= 9.81(2)\left(1 + \frac{3}{9.81}\right)$

$= 25.62 \text{ KPa}$

$P_2 = P_1$



$\therefore F_1 = F_2 = \frac{1}{2} P_1 (2)(4)$

$= 102.48 \text{ KN} \quad (F_1 \leftarrow);(F_2 \rightarrow)$

$F_3 = P_1(5)(4) = 512 \text{ KN} (\downarrow)$

c- $P_1 = P_2 = \gamma h\left(1 - \frac{a_y}{g}\right) = 13.62 \text{ KPa}$

$F_1 = F_2 = \frac{1}{2} P_1 (2)(4) = 54.48 \text{ KN}$

$F_3 = P_1 (5)(4) = 272.4 \text{ KN}$

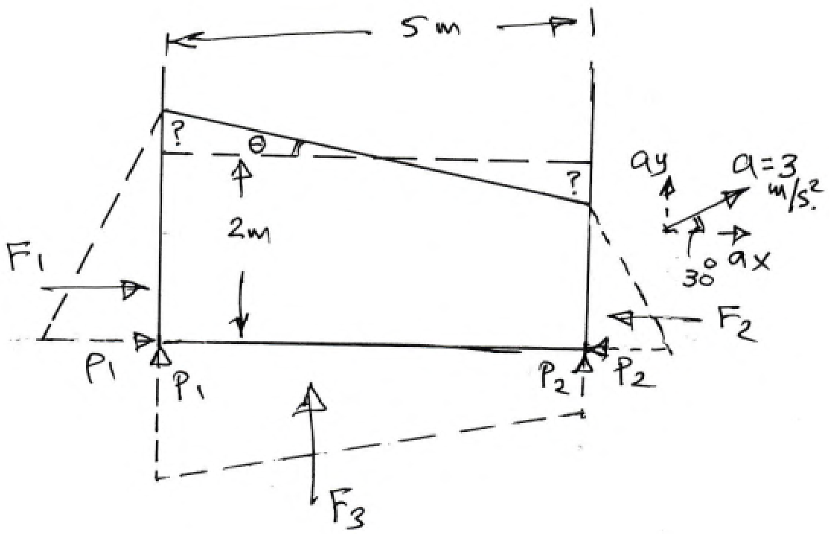
d-

$b = 4m$

$\tan\theta = \frac{a_x}{a_y + g}$

$a_x = 3\cos 30^\circ = 2.6 \text{ m/s}^2$

$a_y = 3\sin 30^\circ = 1.5 \text{ m/s}^2$



$\therefore \frac{2.6}{1.5 + 9.81} = \frac{?}{2.5}$

$\therefore ? = 0.575 m$

$P_1 = \gamma_w h_{max.}\left(1 + \frac{a_y}{g}\right) = 29.123 \text{ KPa}$

$P_2 = \gamma_w h_{min.}\left(1 + \frac{a_y}{g}\right) = 16.116 \text{ KPa}$

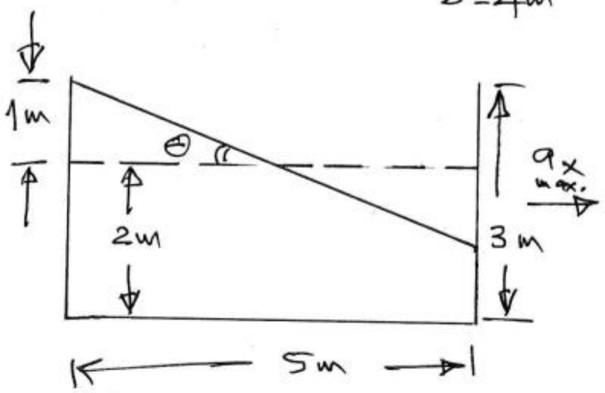
$\therefore F_1 = \frac{1}{2} P_1 \cdot h_{max.} (4) = 150 \text{ KN}$

$F_2 = 45.93 \text{ KN}$

$F_3 = \frac{P_1 + P_2}{2}(5)(4) = 452.4 \text{ KN}$

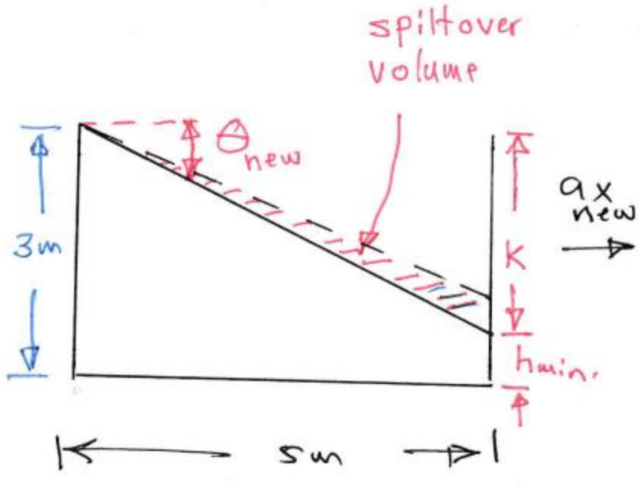
Ex. 2: If the tank in Ex. 1 is accelerated horizontally along the longer side, determine the maximum acceleration that can be given without spilling the water. Also, calculate the percentage of water spilt if this max acceleration is increased by 20 %.

Sol.:



$$\tan\theta = \frac{a_{x\,max.}}{g} = \frac{1}{2.5}$$

$$\therefore a_{x\,max.} = 3.92\ m/s^2$$



$$a_{x\,new} = 1.2\,(a_{x\,max.}) = 4.7\ m/s^2$$

$$\tan\theta_{new} = \frac{a_{x\,new}}{g} = \frac{K}{}$$

$$\therefore \frac{4.7}{9.81} = \frac{K}{5}$$

$$\therefore K = 2.4m$$

$$\therefore h_{min.} = 3 - K = 0.6m$$

$$h_{max.} = 3m$$

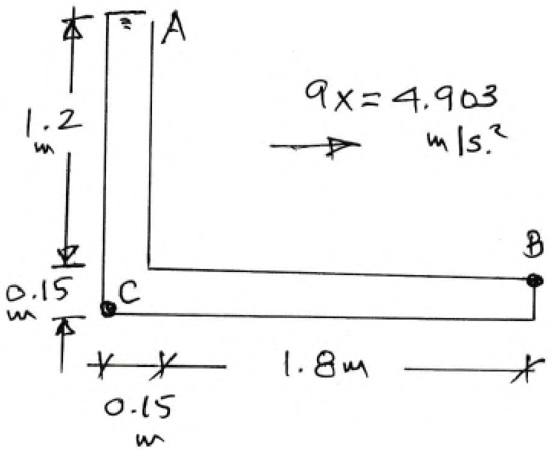
$$\therefore \text{Volume of water that kept in the tank} = \frac{3 + h_{min.}}{2}(5)(4)$$
$$= 36\ m^3$$

$\therefore$ The volume of water that spilt over $= \forall_{original} - \forall_{\text{Kept in the tank}}$

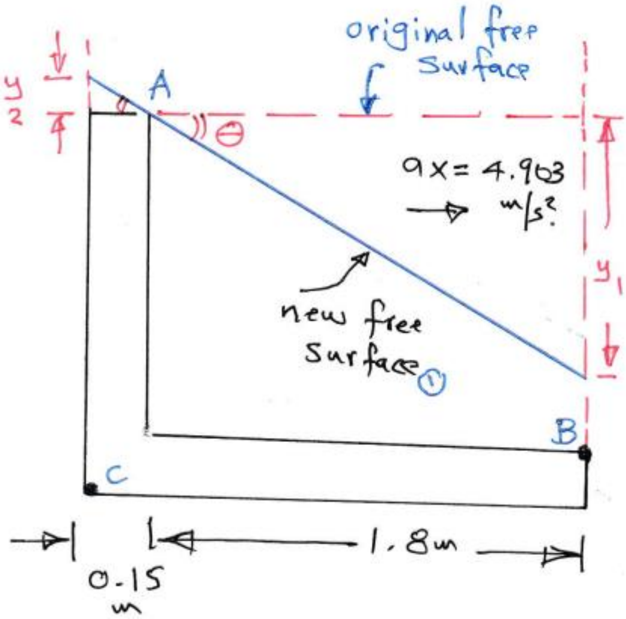
$$= 4\,m^3$$

$$\therefore \% \text{ of water spilt over} = \frac{\forall_{spilt}}{\forall_{original}} * 100 = 10\%$$

Ex.3: The tank in figure is filled with oil (s.=0.8) & acceleration as shown. There is a small opening in the tank at A. Determine the pressure at B & C; and the acceleration (a_x) required to make the pressure at B equals (7 KPa "Vacuum").



Sol.:



$$\tan\theta = \frac{a_x}{g}$$

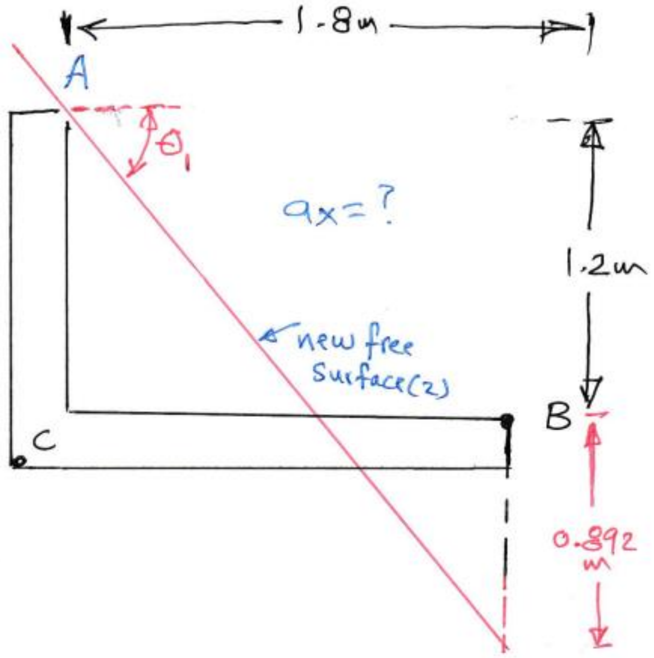
$$\frac{a_x}{g} = \frac{y_1}{1.8} = \frac{y_2}{0.15}$$

$$\therefore y_1 = 0.9m$$

$$y_2 = 0.075m$$

$$P_B = \gamma_{oil}\,(1.2 - y_1) = 2.35\ KPa$$

$$P_C = \gamma_{oil}\,(1.2 + 0.15 + y_2) = 11.18\ KPa.$$



when $P_B = -7$ KPa

convert P_B to oil depth

$$-7 * 10^3 = \gamma_{oil}\, h_{oil}$$

$$\therefore h_{oil} = -0.892m$$

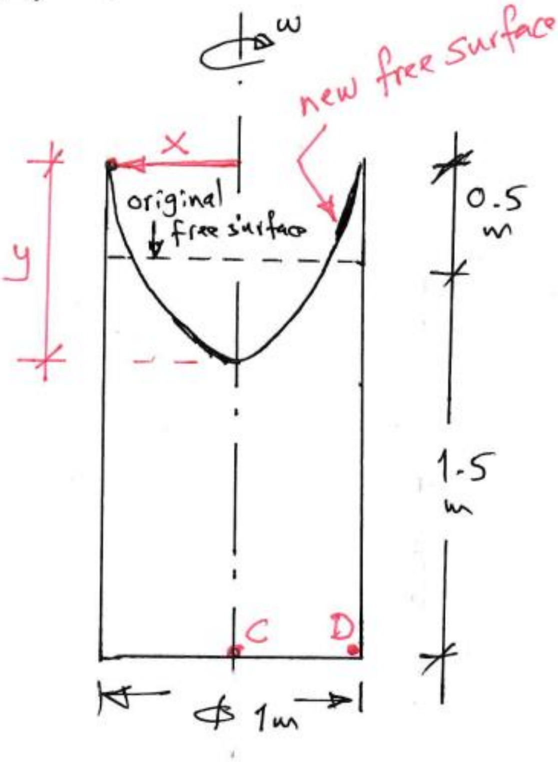
$$\tan\theta_2 = \frac{a_x}{g} = \frac{1.2 + 0.892}{1.8}$$

$$\therefore a_x = 11.4\ m/s^2$$

Ex. 4 : An open cylindrical tank (2m) high & (1m) in diameter contains (1.5m) of water. If the cylinder rotates about its geometric axis,

(a) hat constant angular velocity in (r.p.m) can be attained without spilling any water?

(b) what is the pressure at the tank bottom (at C & D points) when $\omega = 6$ rad/s. ?



Sol.: (a)

Volume of paraboloid of revolution $= \frac{1}{2}$ (volume of circumscribed cylinder)

If no liquid is spilled, this volume equals the volume above the original free surface

Volume of paraboloid of revolution $= \frac{1}{2}(\pi x^2 y)$

Volume above the original free surface $= \pi (0.5)^2 (0.5)$

since there is no spilling of water ⇒

$$\therefore \frac{1}{2}\left(\pi (0.5)^2 y\right) = \pi (0.5)^2 (0.5)$$

$$\therefore y = 1 \text{ m}$$

$$\text{since ; } y = \frac{w^2 \cdot x^2}{2g}$$

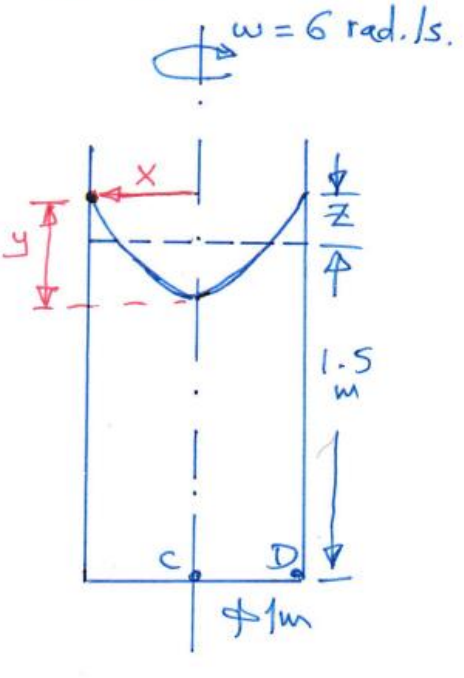
$$\therefore 1 = \frac{w^2 (0.5)^2}{2g}$$

$$\therefore w = 8.86 \text{ rad./s.}$$

$$\text{since } w = \frac{2\pi N}{60}$$

$$\therefore N = 84.6 \text{ r.p.m}$$

(b) Since w < the above w ⇒ this means that the water does not reach the tank top edge & does not spilt over.



volume of paraboloid of revolution = volume above the original free surface

$$\frac{1}{2}\left(\pi (0.5)^2 y\right) = \pi (0.5)^2 * Z \quad \text{—} (1)$$

$$\text{since; } y = \frac{w^2 \cdot x^2}{2g}$$

$$\therefore y = \frac{(6)^2 (0.5)^2}{2g} = 0.46$$

From eq. (1) $\Rightarrow Z = 0.23\,m$

$$\therefore\ P_c = \gamma_w\,(1.5 - (y - Z)) = 12.46\ KPa$$

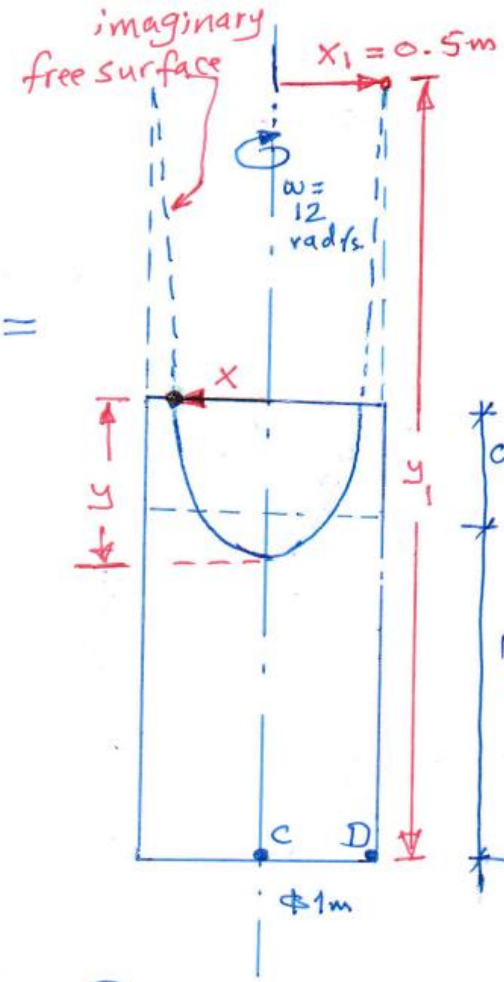
$$P_D = \gamma_w\,(1.5 + Z) = 16.97\ KPa$$

Ex.5: Consider the tank in Ex.4 closed with air space subjected to a pressure of (1.07 bar). When the angular velocity is (12 rad./s.), what are the pressures in (bar) at points C & D?

Sol.: From Ex.4, the ω-value that makes the water reach the tank top edge is 8.86 rad./s.

Since $\omega = 12\ rad./s. > 8.86\ rad./s.$

for closed tank $\Rightarrow$ new free surface has an imaginary part above the tank top edge



Since there is no water spilt over;

volume of paraboloid of revolution = Volume above the original free surface

$$\frac{1}{2}(\pi x^2 y) = \pi (0.5)^2 (0.5)$$

$$y = \frac{1}{4x^2} \quad — (1)$$

Since; $y = \dfrac{\omega^2 \cdot x^2}{2g}$

$$\therefore y = \frac{(12)^2 x^2}{2g} = 7.34\, x^2 \quad — (2)$$

From eqs. (1) & (2):

$$\frac{1}{4x^2} = 7.34\, x^2 \Rightarrow x = 0.43\text{m}$$

From eq.(2) $\Rightarrow y = 1.357$m

$$\therefore \quad h_c = 2 - y = 0.643\text{m}$$

$$\therefore P_c = 1.07 + \frac{\gamma_w}{10^5} h_c = 1.13 \text{ bar}$$

For point D: $P_D = 1.07 + \dfrac{\gamma_w}{10^5} h_D$

$h_D = y_1$

since ; $y = \frac{\omega^2 . x^2}{2g}$

$\therefore \; y_1 = \frac{(12)^2 (0.5)^2}{2g} = 1.83 \text{ m}$

$\therefore \; P_D = 1.31 \text{ bar}$

Chapter Four

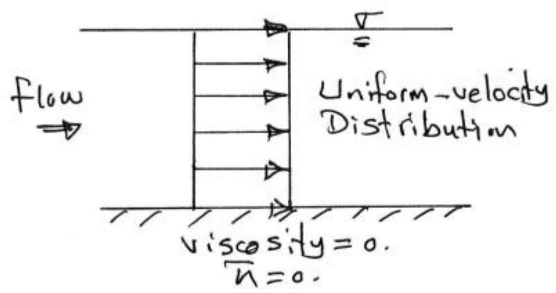
Fluid Dynamics

The force balance between pressure and weight in a static fluid was presented in unit 3, which led to an equation for pressure variation with depth. This unit deal with behaviour of fluid like velocity, acceleration and flow pattern (flow classification). The fluid dynamic deal with fluid having accelerated movement and there is relative acceleration between fluid particles.

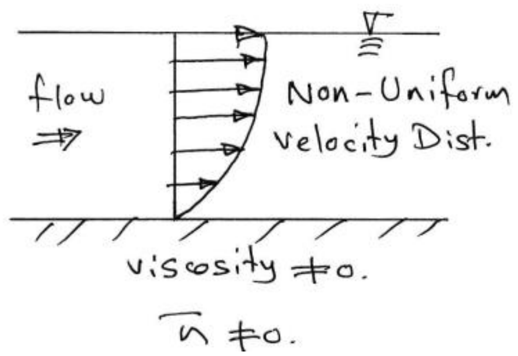
Types of Fluid

There are two types of fluid:
- → Perfect ("Ideal") ("Non-Viscous") Fluid
- Real (viscous) fluid

- Perfect Fluid ⇒ Viscosity = 0.
  * In perfect fluid there is a slipped Boundary condition



- Real fluid ⇒ Viscosity ≠ 0.
  * In Real fluid, there is no slip Boundary condition due to viscous



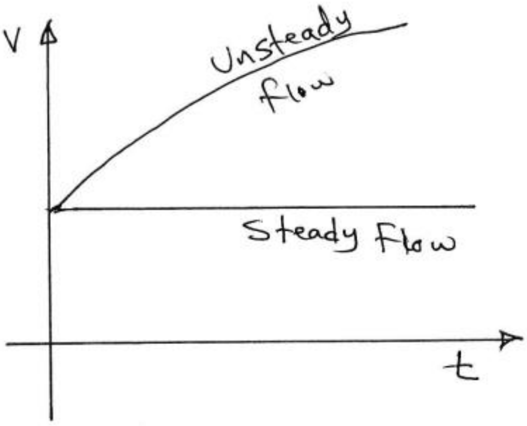
Types of Flow

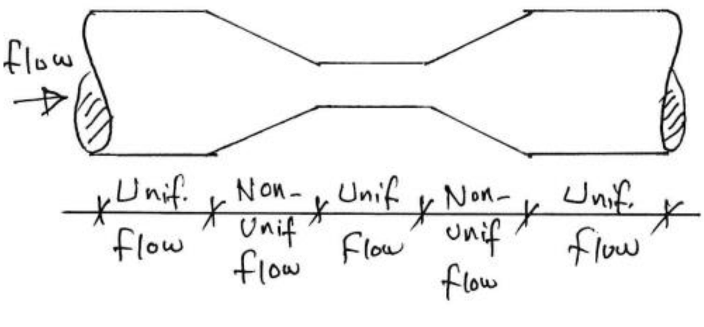
1- Steady Flow: exists if the velocity at a point, for example, remains constant with respect to time ($\frac{\partial V}{\partial t} = 0.$).

2- Unsteady Flow: exists if the velocity at a point, for example, changes either in magnitude or in direction with respect to time ($\frac{\partial V}{\partial t} \neq 0.$).

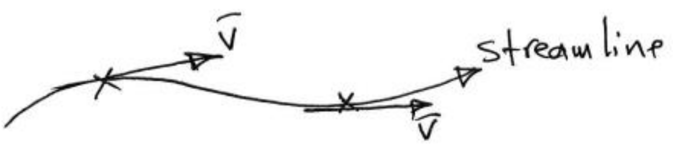
3- Uniform Flow: exists if the velocity, for example, remains constant with respect to distance ($\frac{\partial V}{\partial s} = 0.$).

4- Non-Uniform Flow: exists if the velocity, for example, changes either in magnitude or in direction with respect to distance ($\frac{\partial V}{\partial s} \neq 0.$).

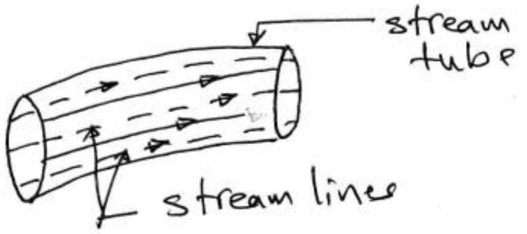




Stream Line: Is an imaginary line within the flow for which the tangent at any point is the time average of the direction of motion at that point.



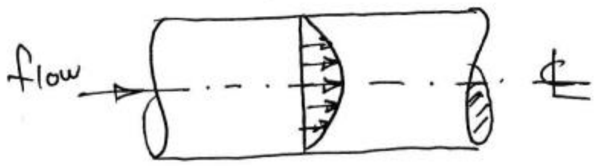
Stream Tube: Is an element of fluid bounded by a special group of stream lines which enclose or confine the flow.



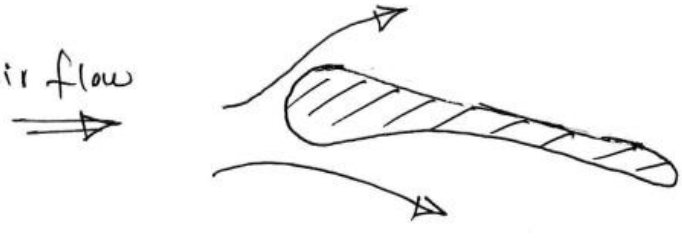
One, Two & Three Dimensional Flow

- 1D-Flow: Such as flow in pipe

1D & Axisymmetry flow



- 2D-Flow: Such as flow around the wing of aircraft.



Velocity & Acceleration

Motion of fluid is specified by velocity components expressed as functions of space & time;

$u = F(x, y, z, t)$ – velocity component in X-dir.

$v = F(x, y, z, t)$ – " " " Y-dir.

$w = F(x, y, z, t)$ – " " " Z-dir.

Acceleration: Rate of change of velocity.

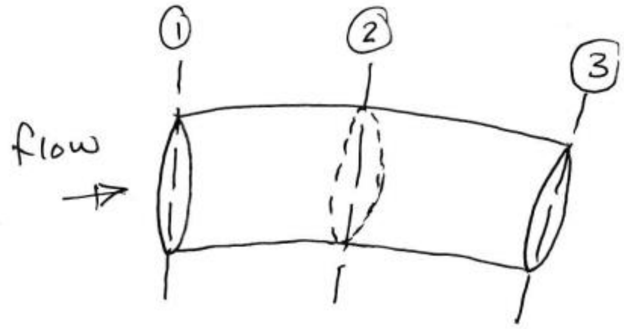
for example;
$$a_x = \frac{Du}{Dt} = \underbrace{\frac{\partial u}{\partial t}}_{\text{local acc.}} + \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\text{convective acc.}}$$

The Continuity Eq.

By conservation of mass:

Inflow – Outflow = Rate of change of accumulating materials inside the control volume

For 1D, steady flow & incompressible fluid,



$$\boxed{\begin{aligned} Q &= A \cdot V \\ Q_1 &= Q_2 = Q_3 \end{aligned}}$$

where: Q = flowrate (discharge) in (L^3/T) L = length unit, T = time unit

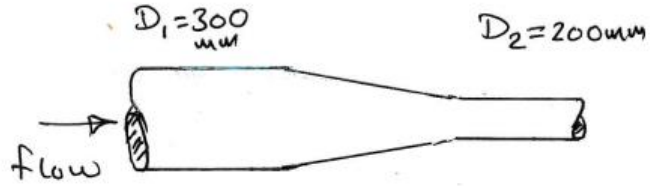
A = cross-sectional area in (L^2)

V = average velocity in (L/T)

$Q_1, Q_2, \& Q_3$: flow rate at sections ①, ②, & ③, respectively.

Ex.1: (3 kN) of water per second flow through pipeline reducer. Calculate the flowrate in (m^3/s.) & the mean velocities in the (300 mm) & (200 mm) pipe diameters.

Solution:



since, $W = \gamma \forall$

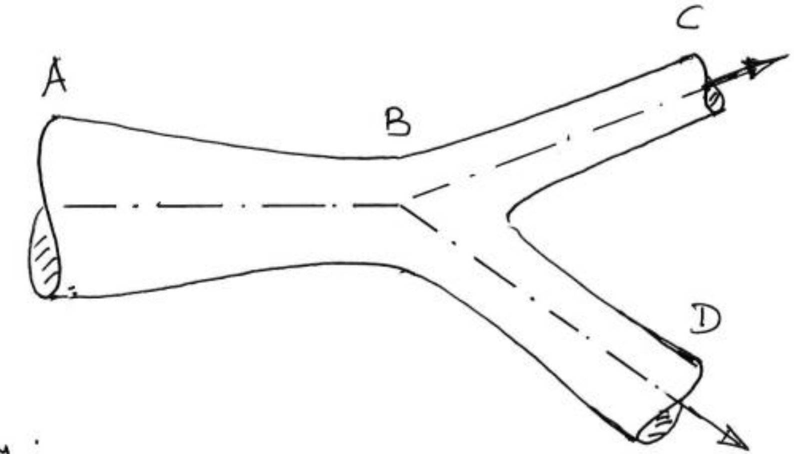
$$\therefore \forall = \frac{W}{\gamma} = \frac{3000}{9810} = 0.306\ m^3$$

$$Q = \frac{\forall}{t} \quad ; \text{ for } t = 1\,s. \Rightarrow Q = \frac{0.306}{1} = 0.306\ m^3/s.$$

$$\because Q = A.V \Rightarrow V_1 = \frac{Q}{A_1} = \frac{0.306}{\frac{\pi}{4}(0.3)^2} = 4.33\ m/s$$

Similarily; $V_2 = \dfrac{Q}{A_2} = 9.74\ m/s$

Ex.2: As shown in figure below, if $D_A = 450\,mm$, $D_B = 300\,mm$, $D_c = 150\,mm$, $D_D = 225\,mm$, $V_A = 1.8\,m/s.$, & $V_D = 3.6\,m/s.$, determine V_B & V_c.



Solution: By continuity;

$$Q_A = Q_B = Q_c + Q_D$$

$$\therefore A_A \cdot V_A = A_B \cdot V_B = A_c \cdot V_c + A_D \cdot V_D \quad \text{— (1)}$$

from eq. (1): $A_A \cdot V_A = A_B \cdot V_B \Rightarrow \frac{\pi}{4}(0.45)^2(1.8) = \frac{\pi}{4}(0.3)^2 V_B$

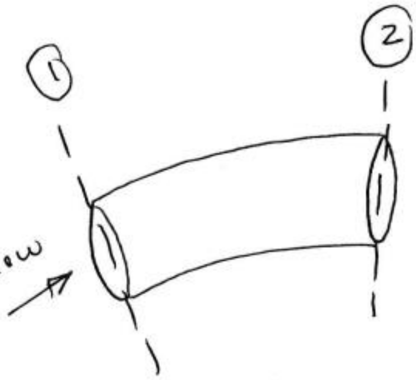
$$\therefore V_B = 4.05\ m/s.$$

from eq. (1): $A_A \cdot V_A = A_c \cdot V_c + A_D \cdot V_D$

$$\frac{\pi}{4}(0.45)^2(1.8) = \frac{\pi}{4}(0.15)^2 V_c + \frac{\pi}{4}(0.225)^2(3.6)$$

$$\therefore V_c = 8.09\ m/s.$$

Equations of Fluid Motion



- Based on Newton's second law ($\Sigma \vec{F}$ = mass * Acceleration) & for 1D, steady flow, & incompressible fluid;

$$\boxed{\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_{L_{1-2}}}$$ Energy Equation

- & for Ideal fluid (viscosity = 0. $\Rightarrow \mu = 0. \Rightarrow h_{L_{1-2}} = 0.$)

$$\boxed{\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g} = \text{constant}}$$ Bernoulli's Equation

where;

$\frac{P}{\gamma}$ = pressure head (L).

Z = elevation head (L).

$\frac{V^2}{2g}$ = velocity head (L)

$h_{L_{1-2}}$ = head loss between section 1 & 2 (L).

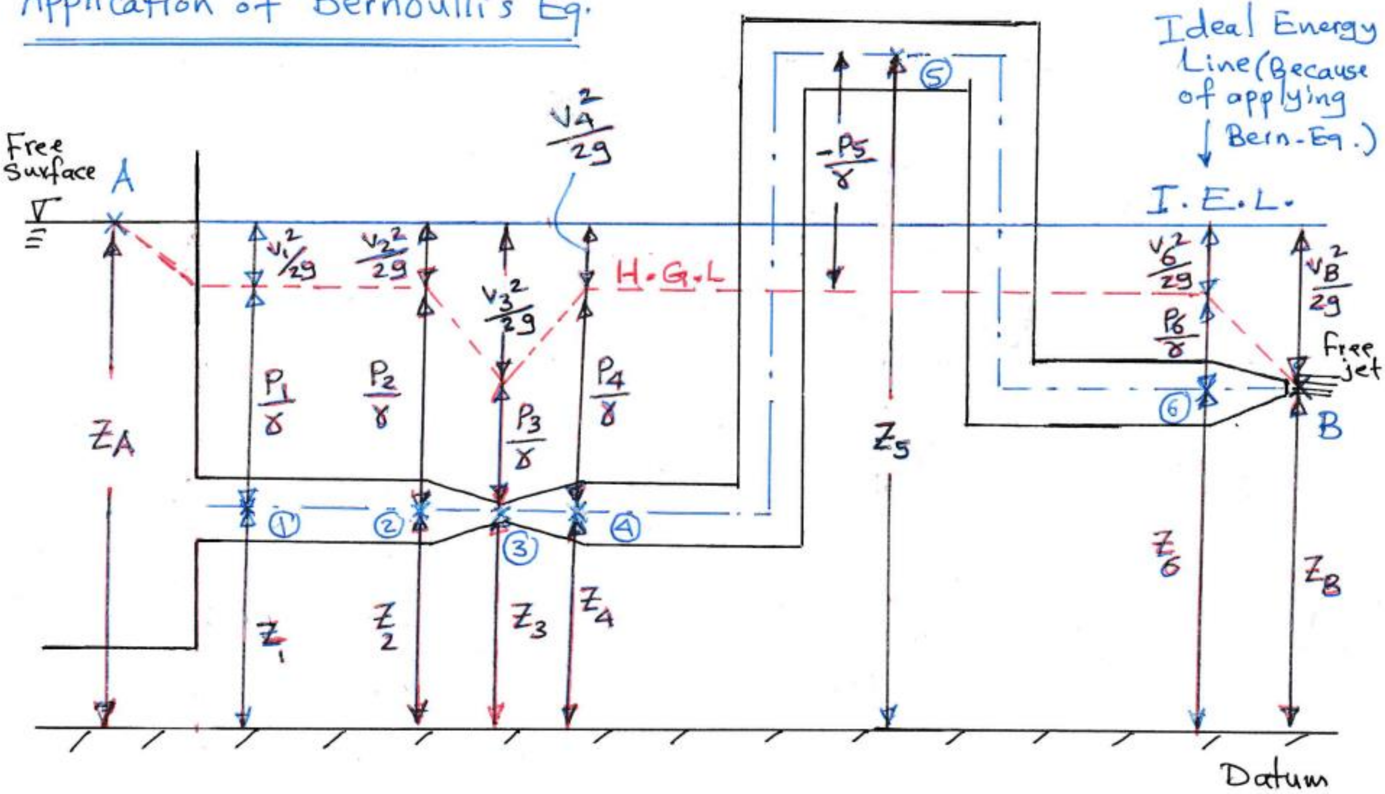
1 & 2 : sections 1 & 2, respectively

Energy Line (E.L.) & Hydraulic Grade Line (H.G.L.)

$$E.L. = \text{Total Energy} = \frac{P}{\gamma} + Z + \frac{V^2}{2g}$$

$$H.G.L = \text{Potential Energy} = \frac{P}{\gamma} + Z$$

Application of Bernoulli's Eq.



System of Pipes with Pump or Turbine

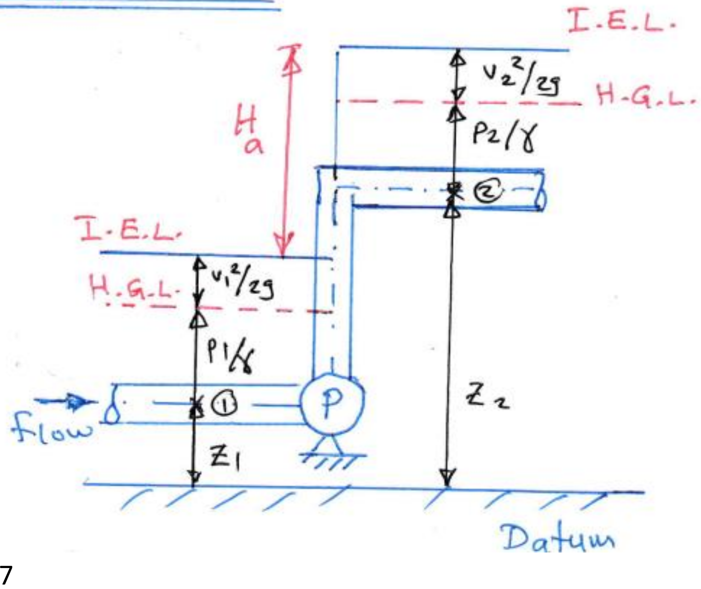
with pump:

By applying Bern. Eq. between points ① & ②:

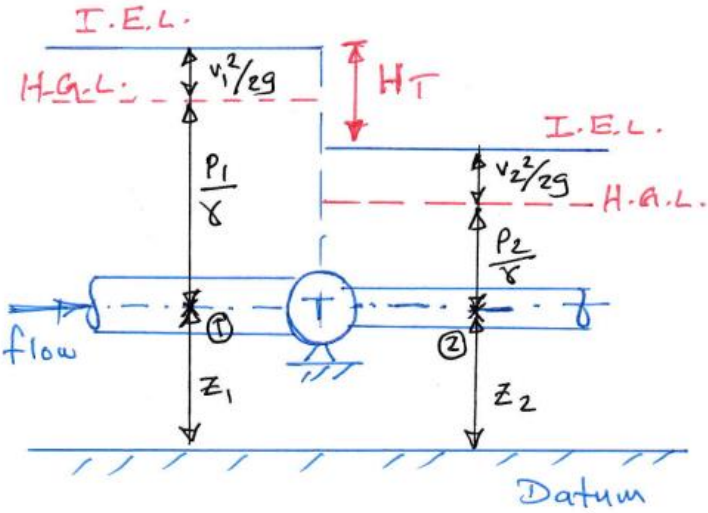
$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} + H_a = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

where;

Ha = pump head (m)



with Turbine :



By applying Bern. Eq. between points ① & ②:

$$\frac{P_1}{\gamma}+Z_1+\frac{V_1^2}{2g}-H_T=\frac{P_2}{\gamma}+Z_2+\frac{V_2^2}{2g}$$

where;

H_T = turbine head (m).

Power :

The power is the Energy per unit time.

$$\boxed{\text{power} = \gamma Q H}$$

where; power measured Watt or horsepower (hp) or J./s.

Note. 1 hp = 746 Watt)

Q = flowrate (m³/s.)

H = head (m).

γ : specific weight of the liquid (N/m³)

So; power of pressure $= \gamma Q \frac{P}{\gamma} = QP$

" " elevation $= \gamma Q Z$

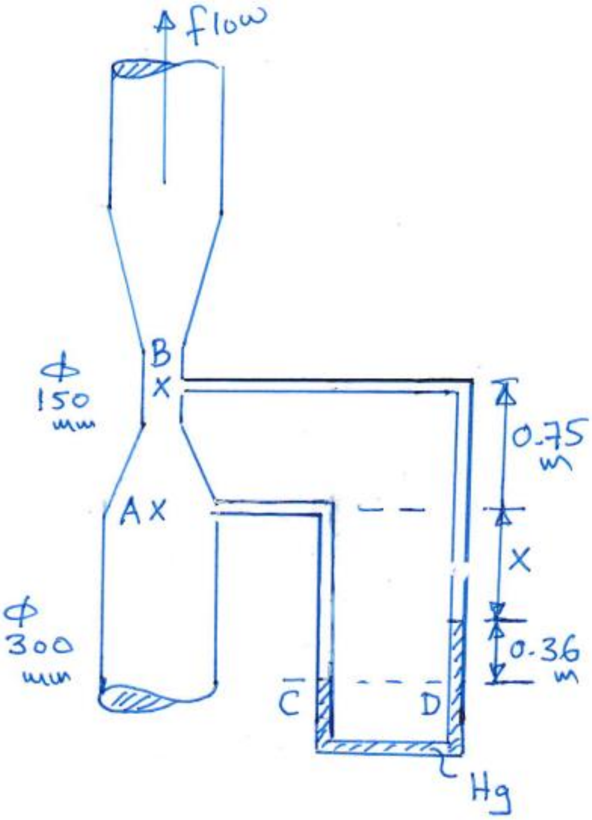
" " velocity $= \gamma Q \frac{V^2}{2g} = \frac{\rho Q V^2}{2}$

" " Pump $= \gamma Q H_a$

" " turbine $= \gamma Q H_T$

dissipation power due to friction $= \gamma Q h_L$

Ex.1: For the Venturi meter shown in figur below, the deflection of the mercury in the differential gauge is 0.36 m. Determine the flow of water through the meter if no energy is lost between A & B.



Sol.: Since there is no energy loss between A & B, by applying Bern. Eq. between points A & B:

$$\frac{P_A}{\gamma_w} + Z_A + \frac{V_A^2}{2g} = \frac{P_B}{\gamma} + Z_B + \frac{V_B^2}{2g}$$

Take datum at A $\Rightarrow Z_A = 0.$
$Z_B = 0.75$

So, Bern. Eq. becomes

$$\frac{P_A}{\gamma_w} + \frac{V_A^2}{2g} = \frac{P_B}{\gamma_w} + 0.75 + \frac{V_B^2}{2g} \quad \text{— (1)}$$

From the differential gauge; $P_C = P_D$

$$P_A + \cancel{\gamma_w X} + \gamma_w (0.36) = P_B + \gamma_w (0.75) + \cancel{\gamma_w \cdot X} + 13.6\gamma_w (0.36) \quad \text{— (2)}$$

Dividing Eq. (2) by γ_w:

$$\therefore \frac{P_A}{\gamma_w} = \frac{P_B}{\gamma_w} + 5.286 \quad \text{— (3)}$$

From continuity $\Rightarrow Q_A = Q_B$

$$\therefore A_A \cdot V_A = A_B V_B$$

$$\therefore V_A = \frac{A_B}{A_A} V_B$$

$$\therefore V_A = 0.25 V_B \quad \text{--- (4)}$$

Subs. eqs. (3) & (4) into eq. (1):

$$\cancel{\frac{P_B}{\gamma_w}} + 5.286 + \frac{(0.25V_B)^2}{2g} = \cancel{\frac{P_B}{\gamma_w}} + 0.75 + \frac{V_B^2}{2g}$$

$$\therefore V_B = 9.74 \text{ m/s.}$$

$$\& \; Q_B = 0.17 \text{ m}^3/\text{s.}$$

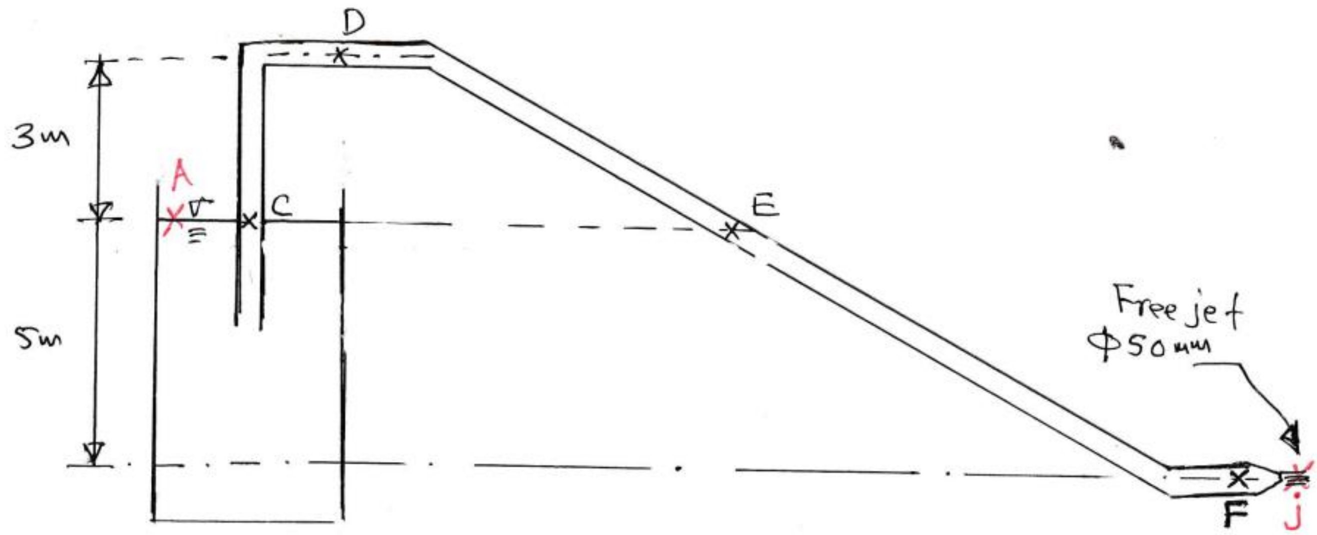
Ex.2: For the siphone shown in figure below, if its diameter is 100m, determine:

i- the outlet flow.

ii- the pressures at points C, D, E, & F.

iii- plot the E.L. & H.G.L.

Note: Assume no energy loss.



Sol.: i- Since there is no energy loss,
by applying Bern. Eq. between points
(A) & (j):

$$\frac{P_A}{\gamma} + Z_A + \frac{V_A^2}{2g} = \frac{P_j}{\gamma} + Z_j + \frac{V_j^2}{2g}$$

Take datum at point j:

Bern. Eq. becomes;

$$0 + 5 + 0 = 0 + 0 + \frac{V_j^2}{2g}$$

$\therefore V_j = 9.9$ m/s.

$Q = 0.019\ m^3/s$.

ii- By continuity $\Rightarrow Q_c = Q_D = Q_E = Q_F = Q = 0.019\ m^3/s$

Since, $A_C = A_D = A_E = A_F$

$$\therefore V_c = V_D = V_E = V_F = \frac{0.019}{\frac{\pi}{4}(0.1)^2} = 2.42\ m/s$$

By applying Bern. Eq. between (A) & (C): Datum at A

$$\frac{P_A}{\gamma} + Z_A + \frac{V_A^2}{2g} = \frac{P_c}{\gamma} + Z_c + \frac{V_c^2}{2g}$$

Datum at A $\Rightarrow$ $0 + 0 + 0 = \frac{P_c}{\gamma} + 0 + \frac{V_c^2}{2g}$

$$\therefore \frac{P_c}{\gamma} = \frac{V_c^2}{2g} \Rightarrow P_c = -2928.2\ Pa.$$

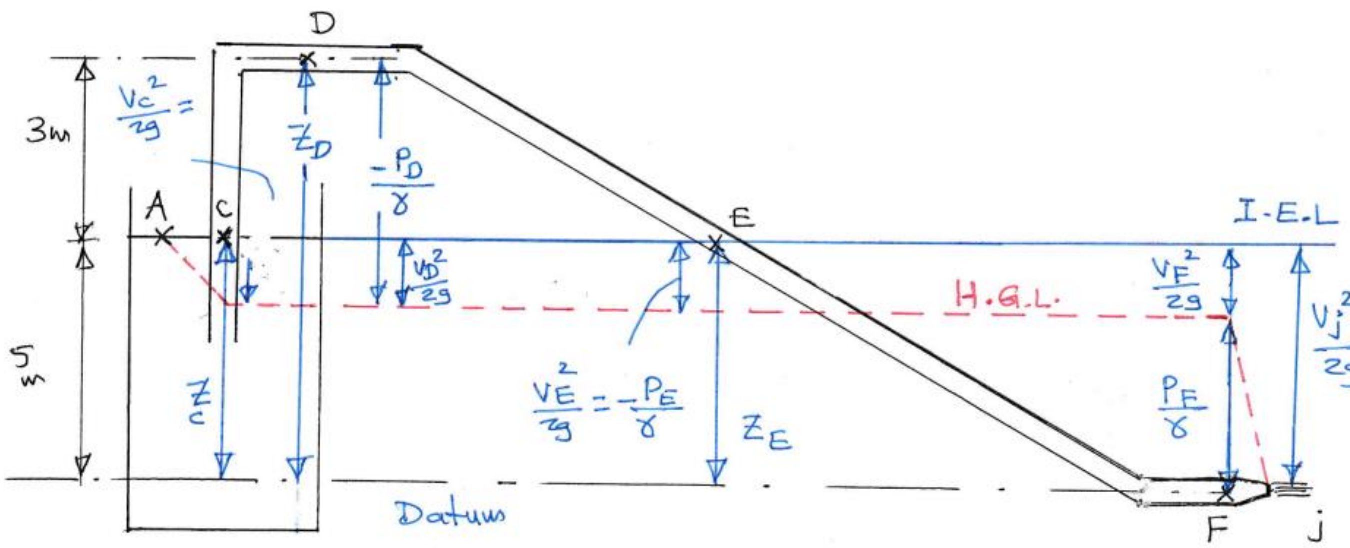
H.W.: Find P_D, P_E, & P_F

Ans.: $P_D = -32358$ Pa.

$P_E = P_c$

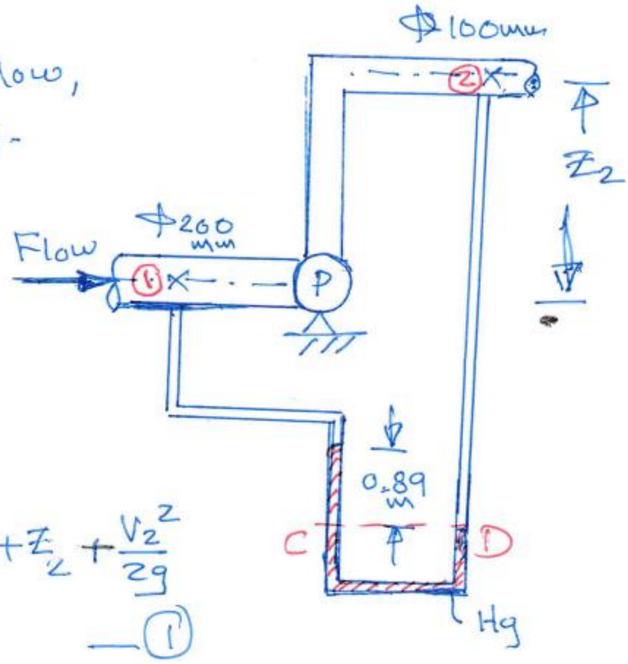
$P_F = 46122$ Pa.

iii-



EX-3: For the figure shown below, Calculate the flowrate of water delivered by a pump which added 12 hp. Assume frictionless flow.

Sol.: For Frictionless flow, applying Bern. Eq. between points ① & ②: (Take datum at point ①):



$$\frac{P_1}{\gamma_w}+0+\frac{V_1^2}{2g}+H_a=\frac{P_2}{\gamma_w}+Z_2+\frac{V_2^2}{2g} \quad \text{—(1)}$$

Since; $\text{Power} = \gamma_w Q\, H_a$

$\therefore\ 12 * 746 = 9810\ Q * H_a$

$\therefore\ H_a = \dfrac{0.912}{Q}$ — (2)

since $Q_1 = Q_2 = Q \Rightarrow V_1 = \dfrac{Q}{A_1} = 31.83Q$ — (3)

$V_2 = \dfrac{Q}{A_2} = 127.32Q$ — (4)

From the manometer; $P_C = P_D$

$\therefore \left[P_1 + \gamma_{Hg}(0.89) = P_2 + \gamma_w Z_2 + \gamma_w(0.89)\right] \div \gamma_w$

$\therefore\ \dfrac{P_2}{\gamma_w} + Z_2 = \dfrac{P_1}{\gamma_w} + 11.214$ — (5)

Subs. eqs. (2), (3), (4), & (5) into eq. (1):

$$\cancel{\frac{P_1}{\gamma_w}} + \frac{(31.83Q)^2}{2g} + \frac{0.912}{Q} = \cancel{\frac{P_1}{\gamma_w}} + 11.214 + \frac{(127.32Q)^2}{2g}$$

$\therefore\ 774.577Q^3 + 11.214Q - 0.912 = 0.$

By trial & error $\Rightarrow Q = 0.0814\ m^3/s$