Reinforced Concrete

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Chapter I
Introduction

Reinforced Concrete Design I

The aim of this subject is to develop the ability of civil engineering students in analysis and design different type of reinforced concrete structures expose to different kind of loads (static and dynamic) using the design equation depending on the essential principals through understanding the procedures of analysis and design easily applied for different type of structures

At the end of this course, the student should be able to:

- State the basis of the analysis of the structure,
- State the objectives of the design of reinforced concrete structures,
- State the method of design of concrete structure,
- Express the design loads in terms of characteristic loads in ultimate strength and working stress methods,
- Define the characteristic load,
- Name the different loads, forces and effects to be considered in the design,
- State the basis of determining the combination of different loads acting on the structure
- Design the beam section for flexural and shear.
- Design the one-way slab.

Course I – syllabus

1- Chapter I: Introduction

2- Chapter II: Flexural Analysis Strength of Concrete Sections

3- Chapter III: Design of concrete Sections

4- CHAPTER IV: Design for Shear

5- Chapter V: Deflection and Control of Cracking

6- Chapter VI: Development Length

7- Chapter VII: One-way Slabs

References :

1- Structural Concrete Theory and Design , By Nadim Hasson, Akthem Aktham Al manseer , 6th Edition 2015

2- Reinforced concrete design , 7th Edition 2007 By Chu Kai Wang, Charles G salmon and Joe A Pincheire

3- Design of Reinforced concrete Structures , 2nd Edition 2008 By Mohammed Tharwat Ghonein, Vol. 3

4- Design of concrete Structure , 14th Edition 2010 By Arthur H. Nilson , Daved Derwin and Charles W. Dolan

5- Reinforced concrete design , 6th Edition 2009 By Edward G. Nawy

6- ACI Code 318- 2019

1. Concrete and Reinforced Concrete

Concrete is a mixture of sand, gravel, crushed rock, or other aggregates held together in a rocklike mass with a paste of cement and water. Sometimes one or more admixtures are added to change certain characteristics of the concrete such as its workability, durability, and time of hardening.

As with most rocklike substances, concrete has a high compressive strength and a very low tensile strength. Reinforced concrete is a combination of concrete and steel wherein the steel reinforcement provides the tensile strength lacking in the concrete. Steel reinforcing is also capable of resisting compression forces and is used in columns as well as in other situations.

1.1. Properties of Concrete

Some of properties of concrete are:

1.1.1. Compressive Strength

The compressive strength of concrete is determined by testing to failure at 28-day-old, concrete cylinders or cubs at a specified rate of loading.

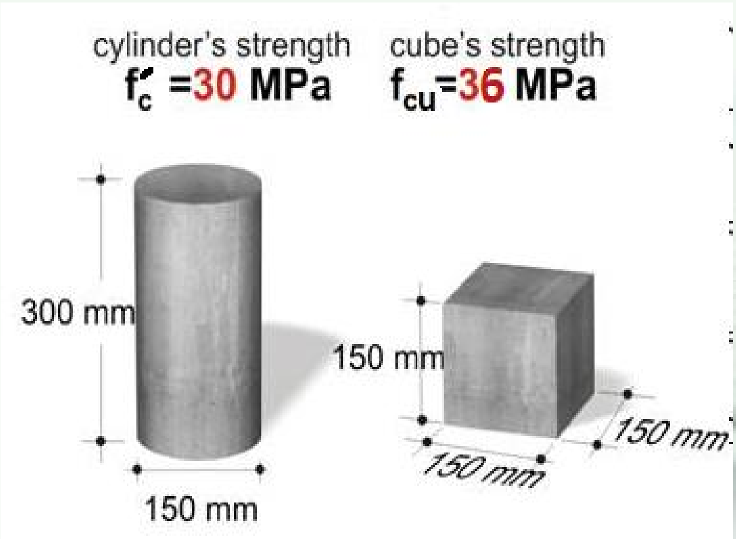
$f'c$: $compressive\ strength\ of\ concrete\ for\ cylinders\ (ACI\ code)$

fcu: $compressive\ strength\ of\ concrete\ for\ cubs\ \ (BS\ code)$

$f'c = about\ 80\%\ fcu$

compressive strength test
For cylinder

compressive strength test
for cube



The stress–strain curves as shown below represent the results obtained from compression tests of sets of 28-day-old standard cylinders of varying strengths. You should carefully study these curves because they bring out several significant points:

(a) The curves are roughly straight while the load is increased from zero to about one-third to one-half the concrete's ultimate strength.

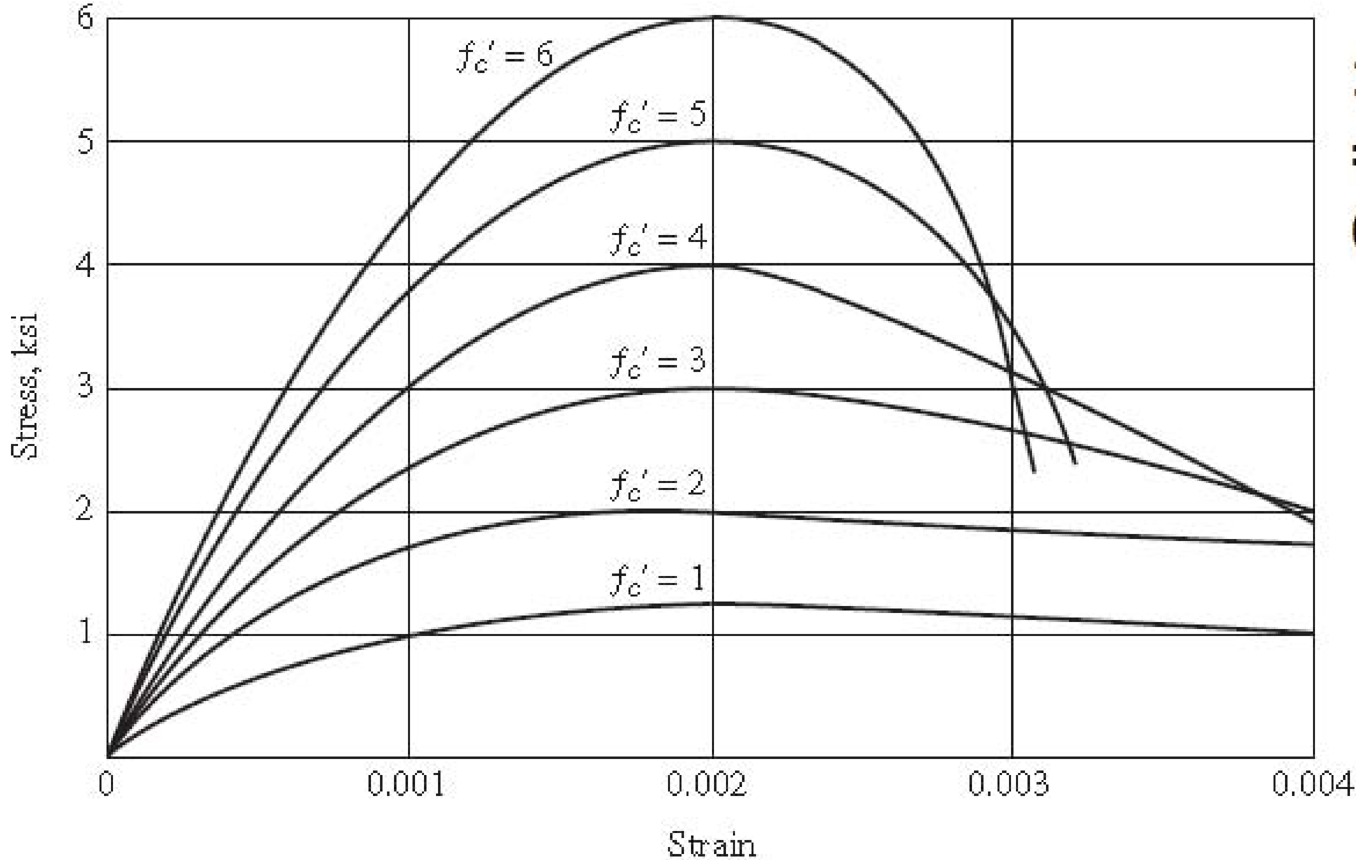
(b) Beyond this range the behavior of concrete is nonlinear. This lack of linearity of concrete stress–strain curves at higher stresses causes some problems in the structural analysis of concrete structures because their behavior is also nonlinear at higher stresses.

(c) particular importance is the fact that regardless of strengths, all the concretes reach their ultimate strengths at strains of about 0.002.

(d) Concrete does not have a definite yield strength; rather, the curves run smoothly on to the point of rupture at strains of from 0.003 to 0.004. It will be assumed for the purpose of future calculations in this text that concrete fails at 0.003 (ACI 318M-14 section 22.2.2.1) or write (ACI 22.2.2.1).

(e) Many tests have clearly shown that stress–strain curves of concrete cylinders are almost identical to those for the compression sides of beams.

(f) It should be further noticed that the weaker grades of concrete are less brittle than the stronger ones—that is, they will take larger strains before breaking.



Typical concrete stress–strain curve, with short-term loading.

1 Ksi

=

6.895 MPa

$$\text{Stress } \sigma = \frac{P}{A}$$

$$\text{Strain } \varepsilon = \frac{\Delta L}{L}$$

1.1.2. Tensile Strength of Concrete

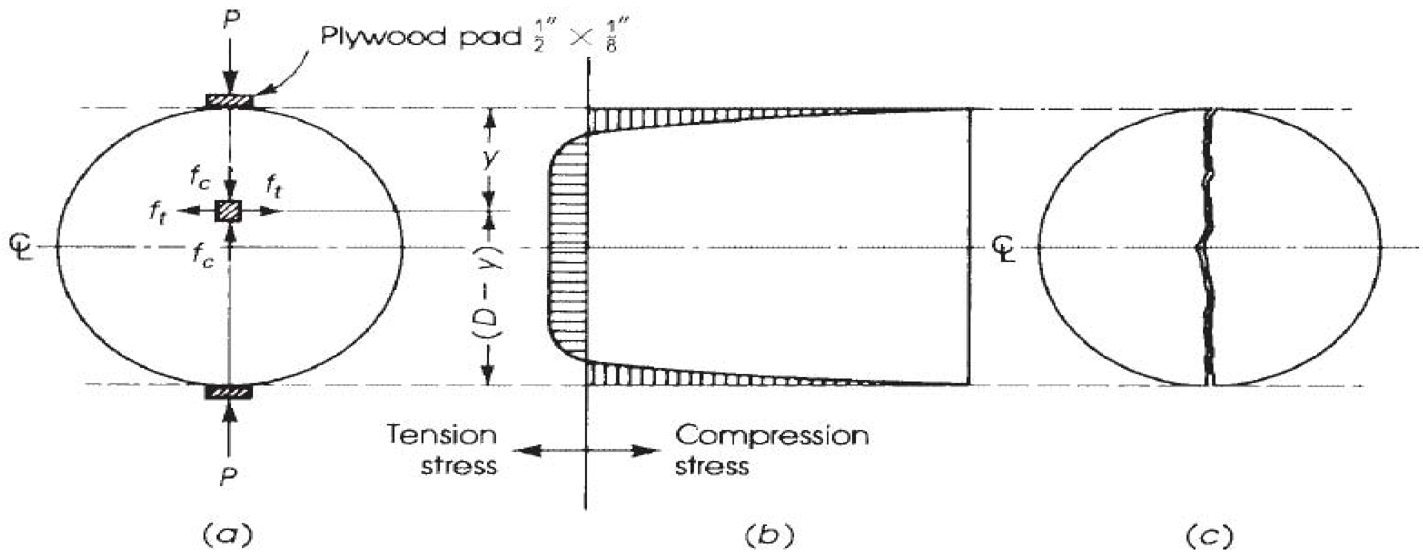
Concrete is a brittle material, and it cannot resist the high tensile stresses that are important when considering cracking, shear, and torsional problems. The low tensile capacity can be attributed to the high stress concentrations in concrete under load, so that a very high stress is reached in some portions of the specimen, causing microscopic cracks, while the other parts of the specimen are subjected to low stress.

Direct tension tests are not reliable for predicting the tensile strength of concrete, due to minor misalignment and stress concentrations in the gripping devices. An indirect tension test is called the splitting test. In this test, the concrete cylinder is placed with its axis horizontal in a compression testing machine. The load is applied uniformly along two opposite lines on the surface of the cylinder through two plywood pads, as shown below. Considering an element on the vertical diameter and at a distance y from the top fibers, the element is subjected to a compressive stress.

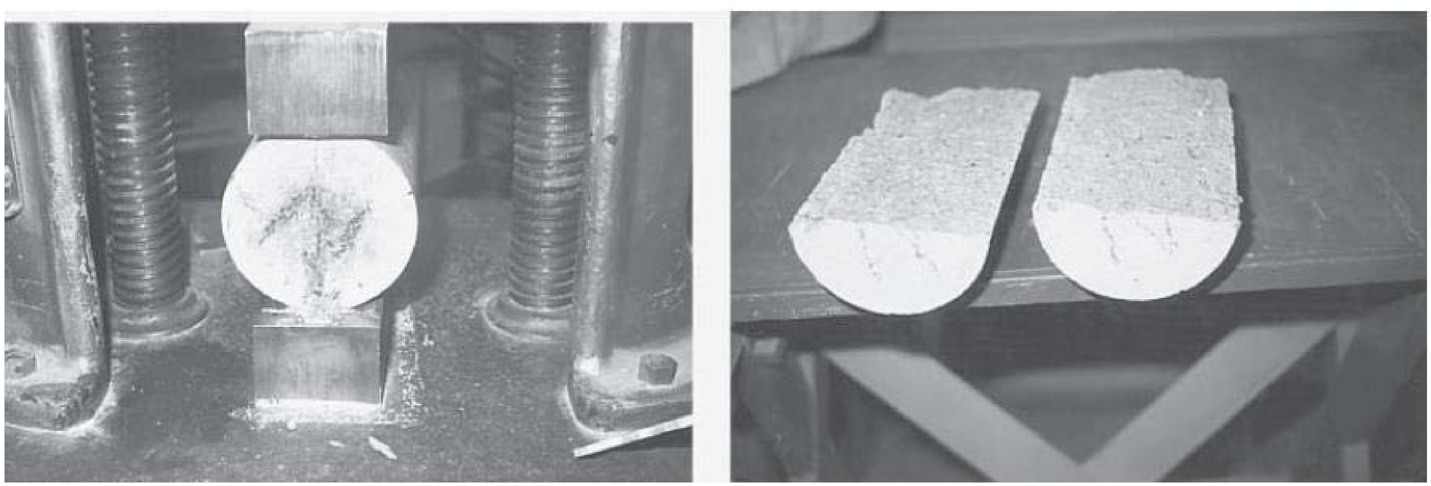
$$f_c = \frac{2P}{\pi L D}\left(\frac{D^2}{y(D-y)} - 1\right)$$

and a tensile stress

$$f'_{sp} = \frac{2P}{\pi L D}$$



Cylinder splitting test [6]: (*a*) configuration of test, (*b*) distribution of horizontal stress, and (*c*) cylinder after testing.



Concrete cylinder splitting test.

1.1.3. Flexural Strength (Modulus of Rupture) of concrete

Experiments on concrete beams have shown that tensile strength in bending is greater than the tensile stress obtained by direct or splitting tests. Flexural strength is expressed in terms of the modulus of rupture of concrete (f_r), which is the maximum tensile stress in concrete in bending.

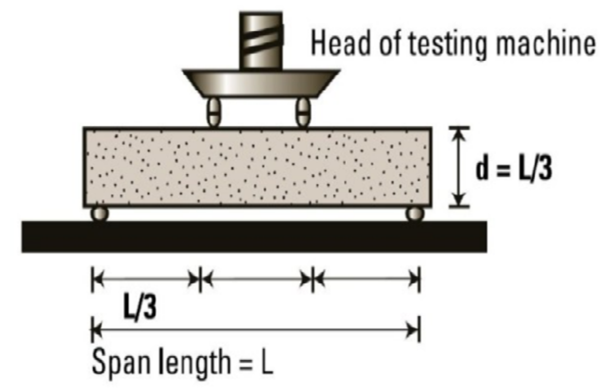
The modulus of rupture can be calculated from the flexural formula used for elastic materials,

$\sigma = f_c = M c / I$, or $f_r = M c / I$

by testing a plain concrete beam. The beam (150 × 150 × 700 mm), is supported on a (600-mm) span and loaded to rupture by two loads on either side of the center. A smaller beam of (100 × 100 × 500 mm) on a (400-mm) span may also be used. The modulus of rupture of concrete ranges between 11 and 23% of the compressive strength.

The ACI Code, Section 19.2.3.1, prescribes the value of the modulus of rupture as

$$f_r = 0.62\lambda\sqrt{f'_c} \quad (N/mm^2)$$



Where the modification factor λ for type of concrete (ACI Table 19.2.4.2 is given as:

$$\lambda = \begin{cases} 1.0 & \text{for normal weight concrete} \\ 0.85 & \text{for sand – light weight concrete} \\ 0.75 & \text{for All light weight concrete} \end{cases}$$

Linear interpolation shall be permitted between 0.85 and 1.0 on the basis of volumetric fractions, for concrete containing normal-weight fine aggregate and a blend of lightweight and normal-weight coarse aggregate.

The modulus of rupture as related to the strength obtained from the split test on cylinders may be taken.

1.1.4. Shear Strength of Concrete

Pure shear is seldom encountered in reinforced concrete members because it is usually accompanied by the action of normal forces. An element subjected to pure shear breaks transversely into two parts. Therefore, the concrete element must be strong enough to resist the applied shear forces.

Shear strength may be considered as 20 to 30% greater than the tensile strength of concrete, or about 12% of its compressive strength. The ACI Code, Section 22.6.6.1, allows a nominal shear stress on plain concrete sections is:

$$= 0.17\lambda\sqrt{f'_c} \quad N/mm^2$$

1.1.5. Modulus of Elasticity of Concrete

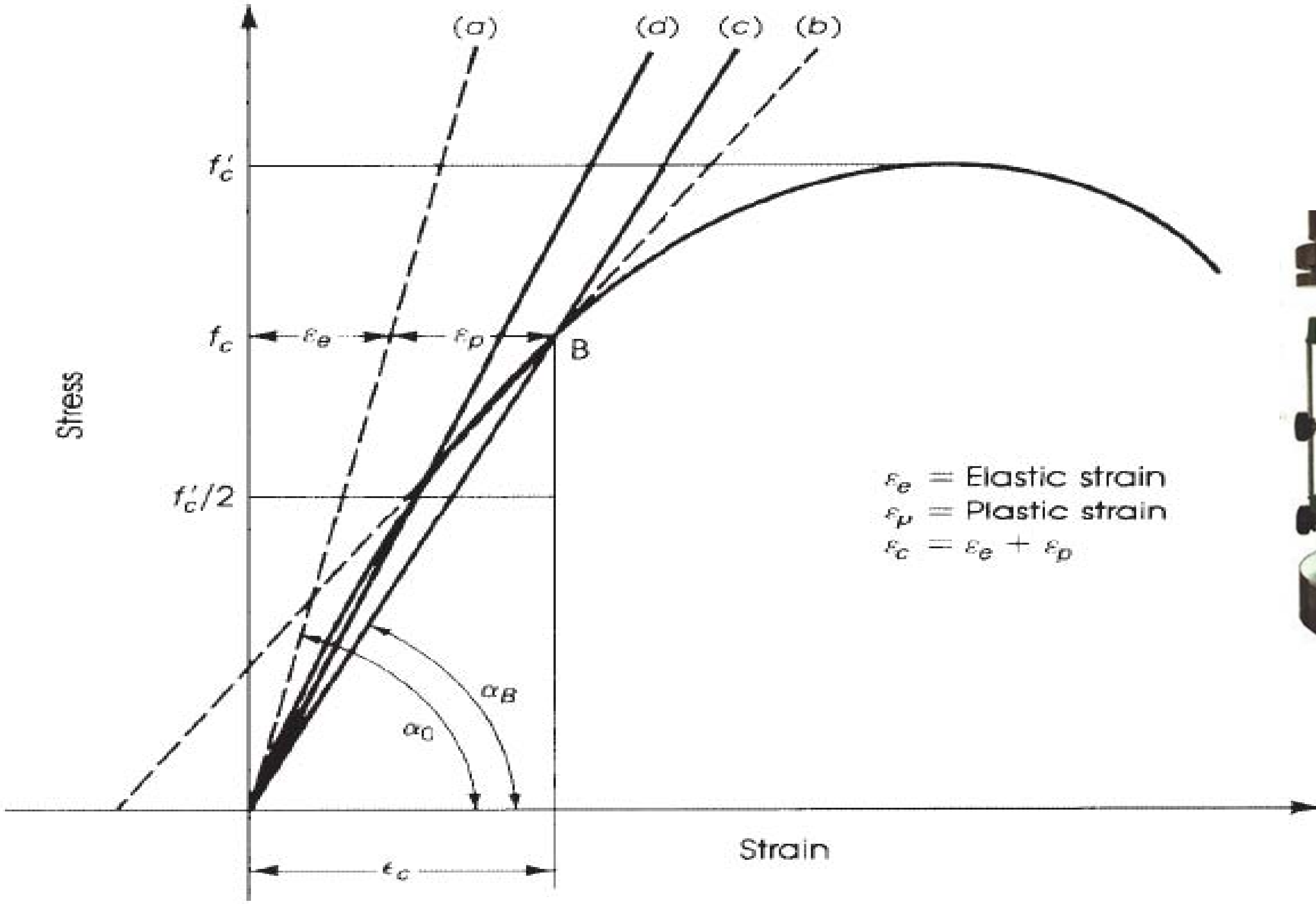
Concrete has no clear-cut modulus of elasticity. Its value varies with different concrete strengths, concrete age, type of loading, and the characteristics and proportions of the cement and aggregates. Furthermore, there are several different definitions of the modulus:

(a) The initial modulus is the slope of the stress–strain diagram at the origin of the curve.

(b) The tangent modulus is the slope of a tangent to the curve at some point along the curve-for instance, at 50% of the ultimate strength of the concrete.

(c) The slope of a line drawn from the origin to a point on the curve somewhere between 25% and 50% of its ultimate compressive strength is referred to as a secant modulus.

(d) Another modulus, called the apparent modulus or the long-term modulus, is determined by using the stresses and strains obtained after the load has been applied for a certain length of time.



Stress–strain curve and modulus of elasticity of concrete. Lines *a–d* represent (*a*) initial tangent modulus, (*b*) tangent modulus at a stress, f_c, (*c*) secant modulus at a stress, f_c, and (*d*) secant modulus at a stress $f'_c/2$.

The ACI Code, Section (19.2.2.1.a), gives a simple formula for calculating the modulus of elasticity of normal and lightweight concrete considering the secant modulus at a level of stress, fc equal to half the specified concrete strength, $f'C$

$$E_c = 0.043\ \omega^{1.5}\sqrt{f'_c}\ \ N/mm^2$$

where ω = unit weight o concrete [between 1400 to 2600 kg/m^3] and f'_c= specified compressive strength of a standard concrete cylinder. For normal-weight concrete. The ACI Code (19.2.2.1.b) allows the use of :

$$E_c = 4700\sqrt{f'_c}\ \ N/mm^2$$

1.1.6. Poisson's Ratio

Poisson's ratio μ is the ratio of the transverse to the longitudinal strains under axial stress within the elastic range. This ratio varies between 0.15 and 0.20 for both normal and lightweight concrete.

1.1.7. Shear Modulus

The modulus of elasticity of concrete in shear ranges from about 0.4 to 0.6 of the corresponding modulus in compression. From the theory of elasticity, the shear modulus is taken as follows

$$G_c = \frac{E_c}{2(1+\mu)}$$

1.1.8. Modular Ratio

The modular ratio n is the ratio of the modulus of elasticity of steel to the modulus of elasticity of concrete:
$n=Es/Ec$

1.1.9. Unit Weight of Concrete

The unit weight, w, of hardened normal concrete ordinarily used in buildings and similar structures depends on the concrete mix, maximum size and grading of aggregates, water–cement ratio, and strength of concrete. The following values of the unit weight of concrete may be used:

1 .Unit weight of plain concrete using maximum aggregate size of 3/4 in. (20 mm) varies between (2320 to 2400 kg/m^3). For concrete of strength less than (28 MPa), a value of (2320 kg/m3) can be used, whereas for higher strength concretes, w can be assumed to be equal to (2400 kg/m^3).

2 .Unit weight of plain concrete of maximum aggregate size of 4 to 6 in. (100 to 150 mm) varies between (2400 to 2560 kg/m3). An average value of 2500 kg/m^3 may be used.

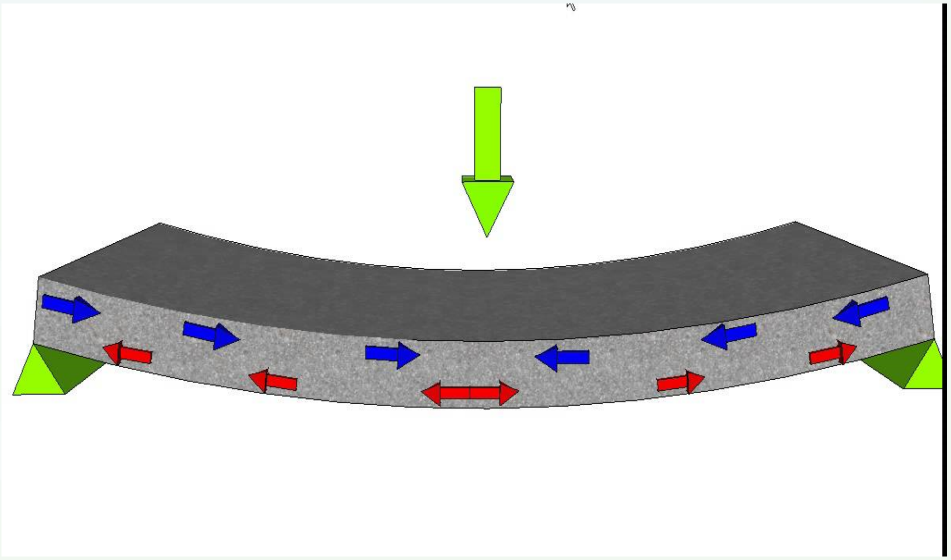
3 .Unit weight of reinforced concrete, using about 0.7 to 1.5% of steel in the concrete section, may be taken as (2400 kg/m^3). For higher percentages of steel, the unit weight, *w*, can be assumed to be (2500 kg/m^3).
4 .Unit weight of lightweight concrete used for fireproofing, masonry, or insulation purposes varies between (320 and 1440 kg/m^3). Concrete of upper values of 1440 kg/m^3 or greater may be used for load-bearing concrete members.
The unit weight of heavy concrete varies between (3200 and 4300 kg/m3). Heavy concrete made with natural barite aggregate of 1.5 in. maximum size (38 mm) weighs about (3600 kg/m^3). Iron of sand and steel-punchings aggregate produce a unit weight of (4320 kg/m^3).

1.1.10. Volume Changes of Concrete

Shrinkage, Creep, and Expansion Due to Rise in Temperature

1.2. Steel Reinforcement

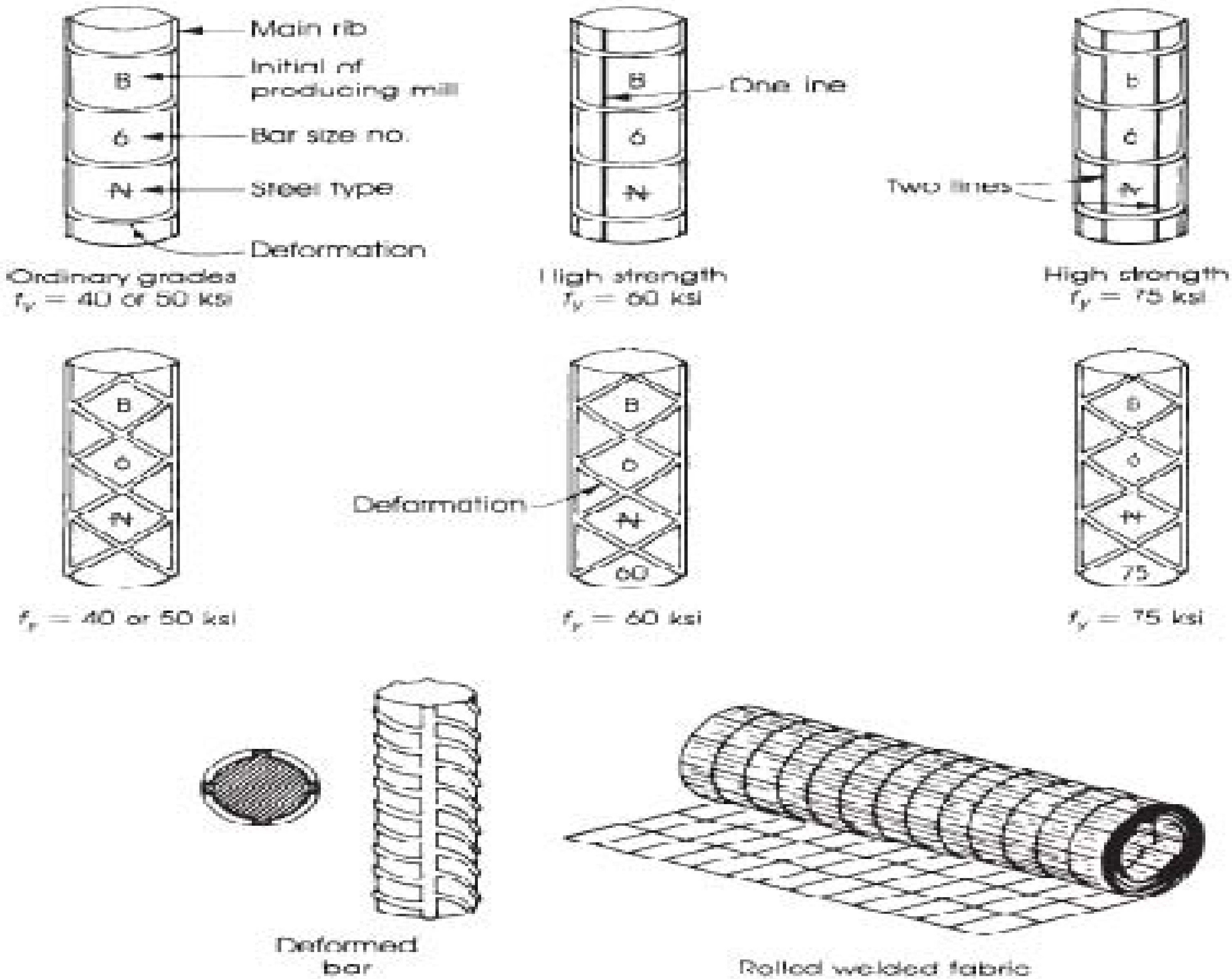
Reinforcement, usually in the form of steel bars, is placed in the concrete member, mainly in the tension zone, to resist the tensile forces resulting from external load on the member. Reinforcement is also used to increase the member’s compression resistance. Steel costs more than concrete, but it has a yield strength about 10 times the compressive strength of concrete.
Longitudinal bars taking either tensile or compression forces in a concrete member are called main reinforcement. Additional reinforcement in slabs, in a direction perpendicular to the main reinforcement, is called secondary, or distribution, reinforcement. In reinforced concrete beams, another type of steel reinforcement is used, transverse to the direction of the main steel and bent in a box or U shape. These are called stirrups. Similar reinforcements are used in columns, where they are called ties.



1.2.1.Types of Steel Reinforcement

Different types of steel reinforcement are used in various reinforced concrete members. These types can be classified as follows:

Round Bars. Round bars are used most widely for reinforced concrete. Round bars are available in a large range of diameters, from 1/4 in. (6 mm) to 1 3/8 in. (36 mm), plus two special types, 1 3/4in. (45 mm) and 2 1/4in. (57 mm). Round bars, depending on their surfaces, are either plain or deformed bars. Plain bars are used mainly for secondary reinforcement or in stirrups and ties. Deformed bars by either the continuous-line system or the number system. In the first system, one longitudinal line is added to the bar, in addition to the main ribs, to indicate the high-strength grade of 60 ksi (420 N/mm2), according to ASTM specification A 617. If only the main ribs are shown on the bar, without any additional lines, the steel is of the ordinary grade according to ASTM A 615 for the structural grade (fy =40 ksi, or 280 N/mm^2). In the number system, the yield strength of the high-strength grades is marked clearly on every bar. For ordinary grades, no strength marks are indicated. The two types are shown in Fig. below.



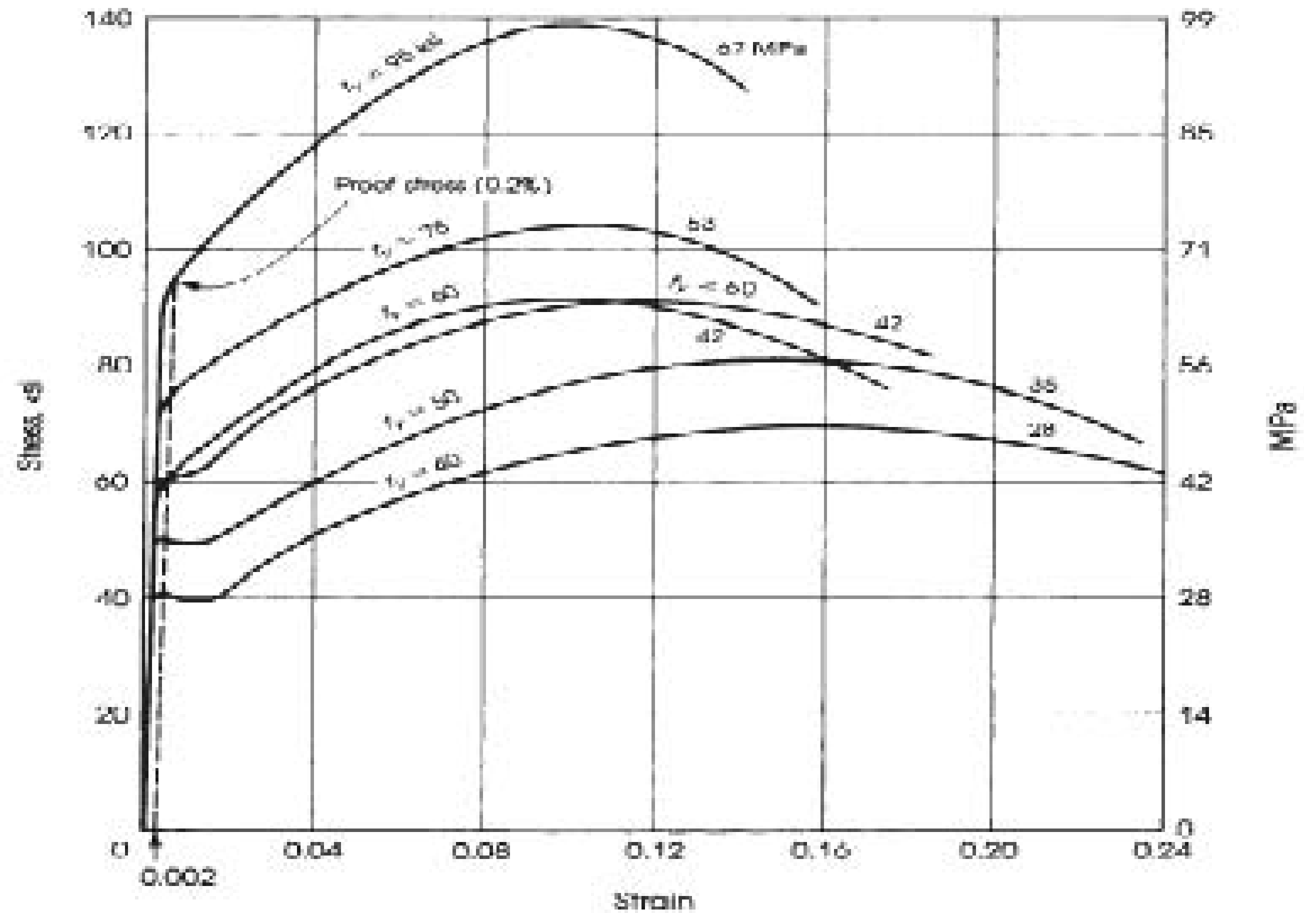
Some types of deformed bars and American standard bar marks.

1.2.2.Stress–Strain Curves of the steel

The most important factor affecting the mechanical properties and stress–strain curve of the steel is its chemical composition. The introduction of carbon and alloying additives in steel increases its strength but reduces its ductility.

Commercial steel rarely contains more than 1.2% carbon; the proportion of carbon used in structural steels varies between 0.2 and 0.3%. Two other properties are of interest in the design of reinforced concrete structures; the first is the modulus of elasticity, Es. It has been shown that the modulus of elasticity is constant for all types of steel. The ACI Code has adopted a value of Es =29 × 10^6 psi (2.0 × 10^5 MPa). The modulus of elasticity is the slope of the stress–strain curve in the elastic range up to the proportional limit; Es =stress/strain. Second is the yield strength, fy.

Typical stress–strain curves for some steel bars are shown in Fig. below. In high-tensile steel, a definite yield point may not show on the stress–strain curve. In this case, ultimate strength is reached gradually under an increase of stress (Fig. below). The yield strength or proof stress is considered the stress that leaves a residual strain of 0.2% on the release of load, or a total strain of 0.5 to 0.6% under load



Typical stress–strain curves for some reinforcing steel bars of different grades. Note that 60-ksi steel may or may not show a definite yield point.

2. DESIGN PHILOSOPHY AND CONCEPTS

The design of a structure may be regarded as the process of selecting the proper materials and proportioning the different elements of the structure according to state-of-the-art engineering science and technology.
In order to fulfill its purpose, the structure must meet the conditions of:

safety, serviceability, economy, and functionality.

The ACI Code emphasizes the unified design method (UDM) which based on the strength of structural members assuming a failure condition, whether due to the crushing of the concrete or to the yield of the reinforcing steel bars. Although there is some additional strength in the bars after yielding (due to strain hardening), this additional strength is not considered in the analysis of reinforced concrete members.
In this approach, the actual loads, or working loads, are multiplied by load factors to obtain the factored design loads. The load factors represent a high percentage of the factor for safety required in the design.

The basic method that is not commonly used (now) is called the **working stress design** or the elastic design method. The design concept is based on the elastic theory assuming a straight-line stress distribution along the depth of the concrete section under service loads. The members are proportioned on the basis of certain allowable stresses in concrete and steel. The allowable stresses are fractions of the crushing strength of concrete and yield strength of steel. This method has been deleted from the ACI Code. The application of this approach is still used in the design of pre-stressed concrete members under service load conditions.

3. CODES OF PRACTICE

The design engineer is usually guided by specifications called the codes of practice. Engineering specifications are set up by various organizations to represent the minimum requirements necessary for the safety of the public, although they are not necessarily for the purpose of restricting engineers.

Most codes specify design loads, allowable stresses, material quality, construction types, and other requirements for building construction.

The most significant standard for structural concrete design in the United States is the Building Code Requirements for Structural Concrete,

1- ACI 318, or the ACI Code.

2- International building Code (IBC),

3-The American Society of Civil Engineers standard ASCE 7,

4- The American Association of State Highway and Transportation Officials (AASHTO).

5- American Society for Testing and Materials (ASTM).

6- American Railway Engineering Association (AREA).

7- Iraqi Standards.

Thank You....................

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LOGO

Chapter I

Introduction

Beam Analysis -Working Stress Method

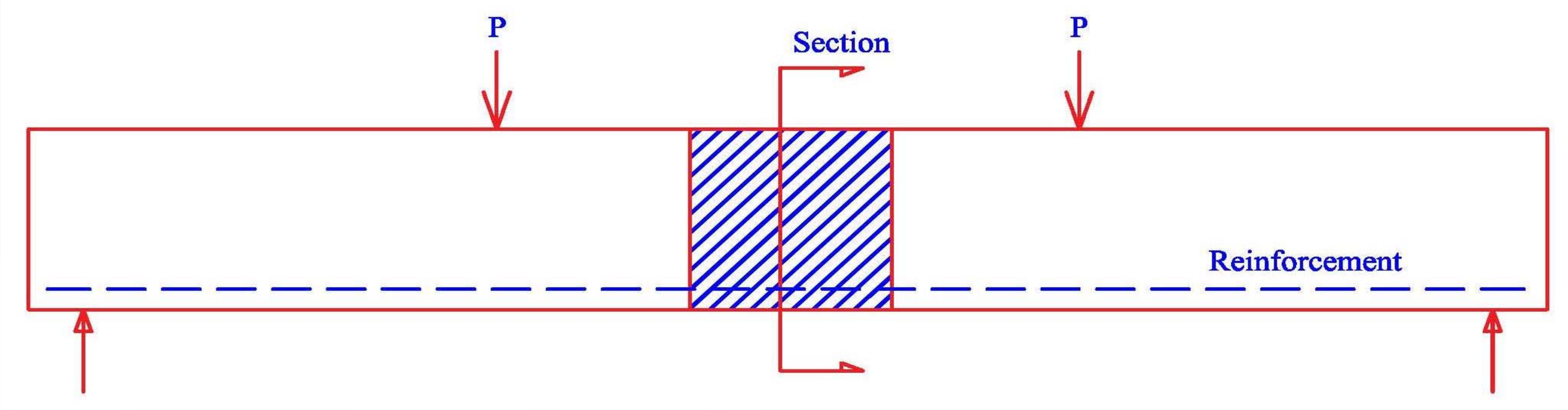
LOGO

FLEXURAL ANALYSIS OF REINFORCED CONCRETE BEAMS

BEHAVIOR OF SIMPLY SUPPORTED REINFORCED CONCRETE BEAM LOADED TO FAILURE

BEAM LOADED TO FAILURE

Concrete being weakest in tension, a concrete beam under an assumed working load will definitely crack at the tension side, and the beam will collapse if tensile reinforcement is not provided. Concrete cracks occur at a loading stage when its maximum tensile stress reaches the modulus of rupture of concrete. Therefore, steel bars are used to increase the moment capacity of the beam; the steel bars resist the tensile force, and the concrete resists the compressive force.



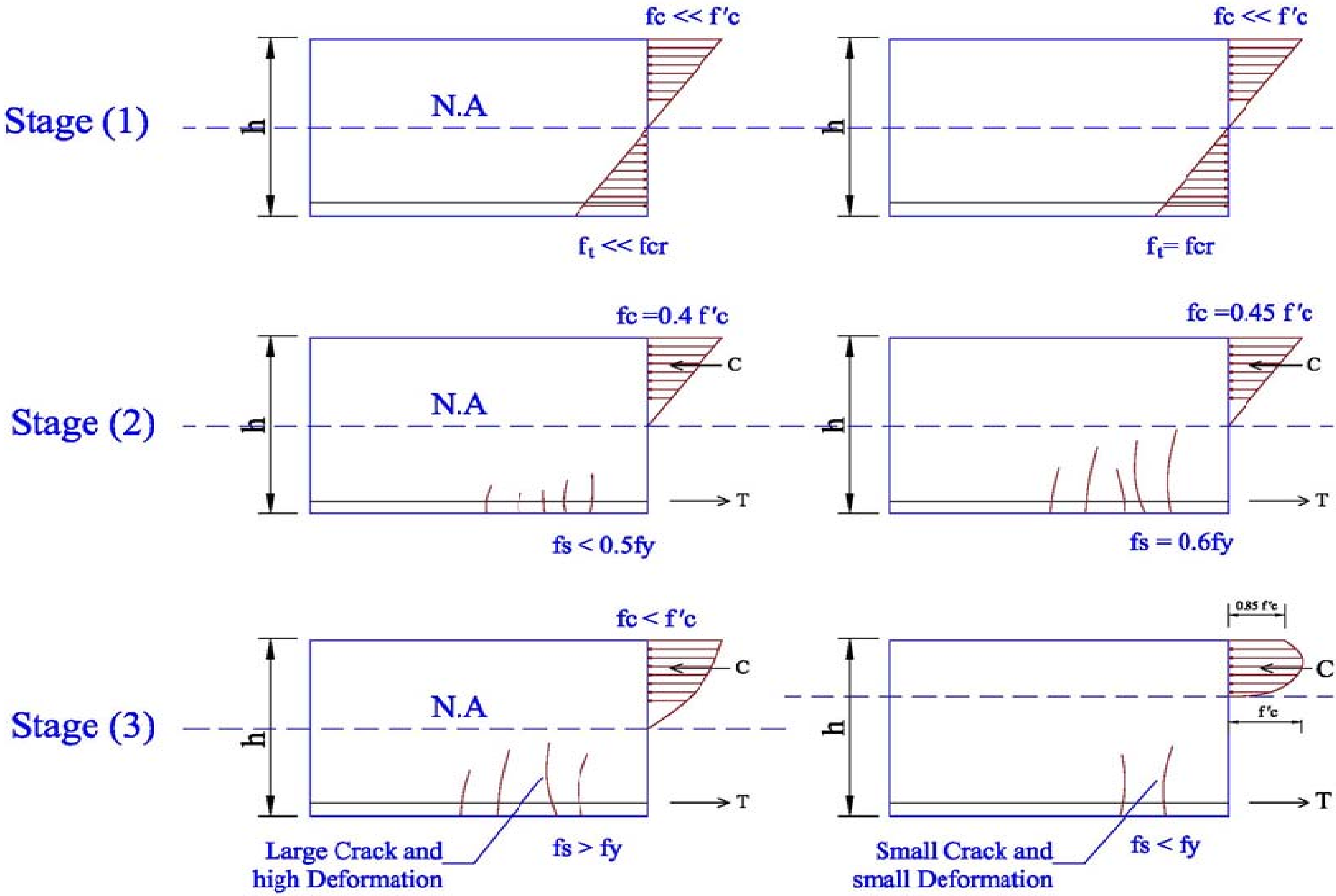
By consider any reinforced concrete beam carry an incrementally accumulative increase load as shown below.

The beam will pass through three stress stages which are:

Stage 1: Elastic Un-cracked Stage: The applied load on beam less than the load which cause cracking.

Stage 2: Elastic Cracked Stage: The applied load makes the bottom fiber stress equal to modulus of rupture of concrete *fr*. Entire concrete section was effective, steel bar at tension side has same strain as surrounding concrete. At this stage before develop any effective cracks the section is under service stresses

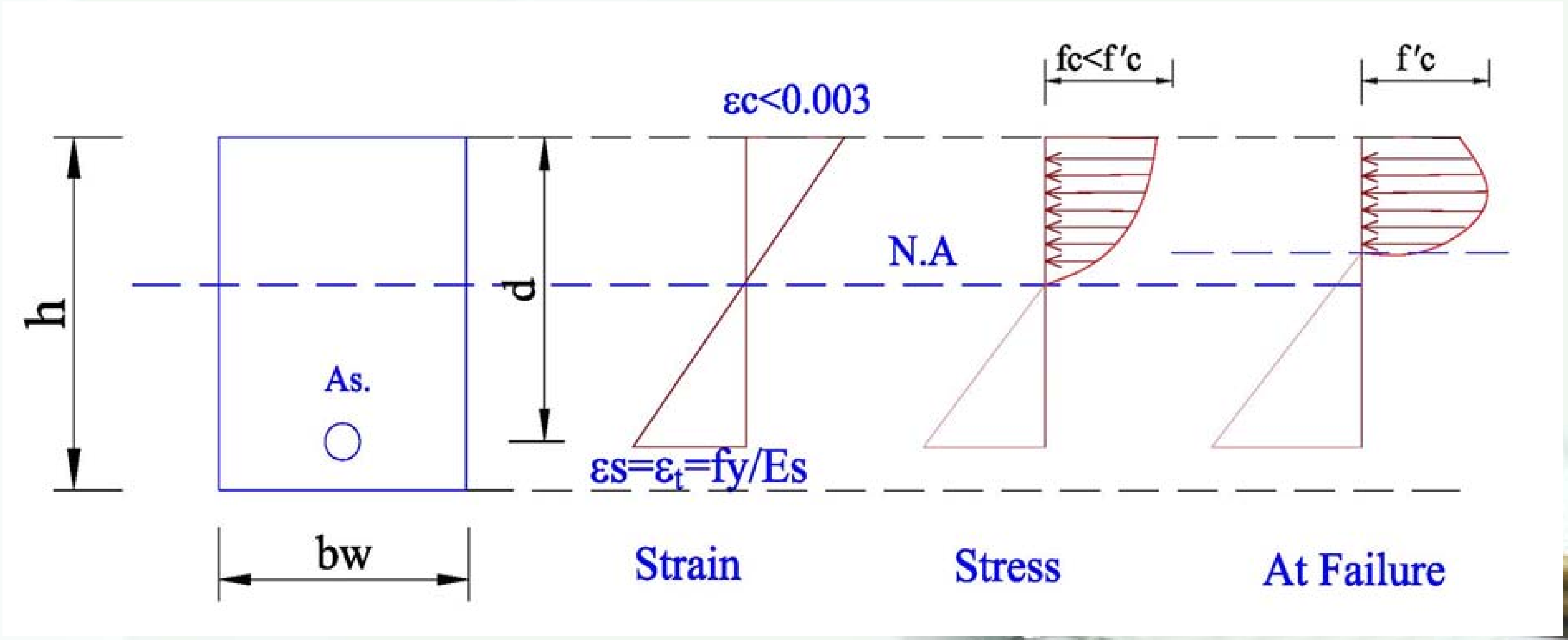
Stage 3: This stage includes two : (a): Inelastic Cracking Stage : The tensile strength of the concrete exceeds the rupture *fr* and cracks develop. The neutral axis shifts upward and cracks extend to neutral axis. Concrete loses tensile strength and steel starts working effectively and resists the entire tensile load. (b): Ultimate Strength Stage: The reinforcement yields. Followed by the failure Stage and the material stresses will be exceed its corresponding capacity.


Stage (1)
Stage (2)
Stage (3)
N.A
h
fc << f′c
fc << f′c
f_t << fcr
f_t= fcr
fc =0.4 f′c
fc =0.45 f′c
C
T
fs < 0.5fy
fs = 0.6fy
fc < f′c
0.85 f′c
f′c
fs > fy
fs < fy
Large Crack and high Deformation
Small Crack and small Deformation

TYPES OF FLEXURAL FAILURE

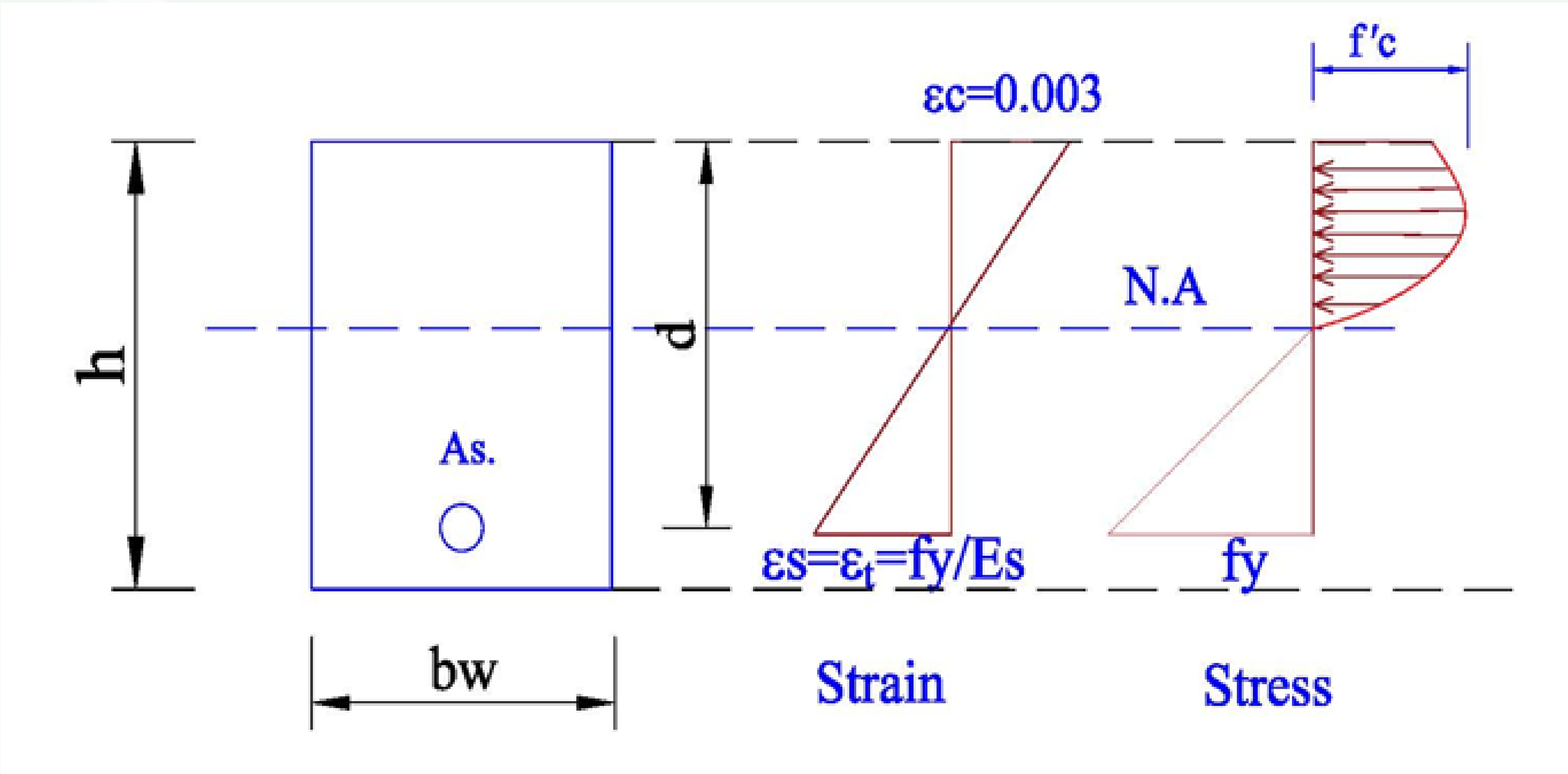
Three types of flexural failure of a structural member can be expected depending on the percentage of steel used in the section.

1 .Steel may reach its yield strength before the concrete reaches its maximum strength. In this case, the failure is due to the yielding of steel reaching a high strain equal to or greater than 0.005. The section contains a relatively small amount of steel and is called a tension-controlled section.

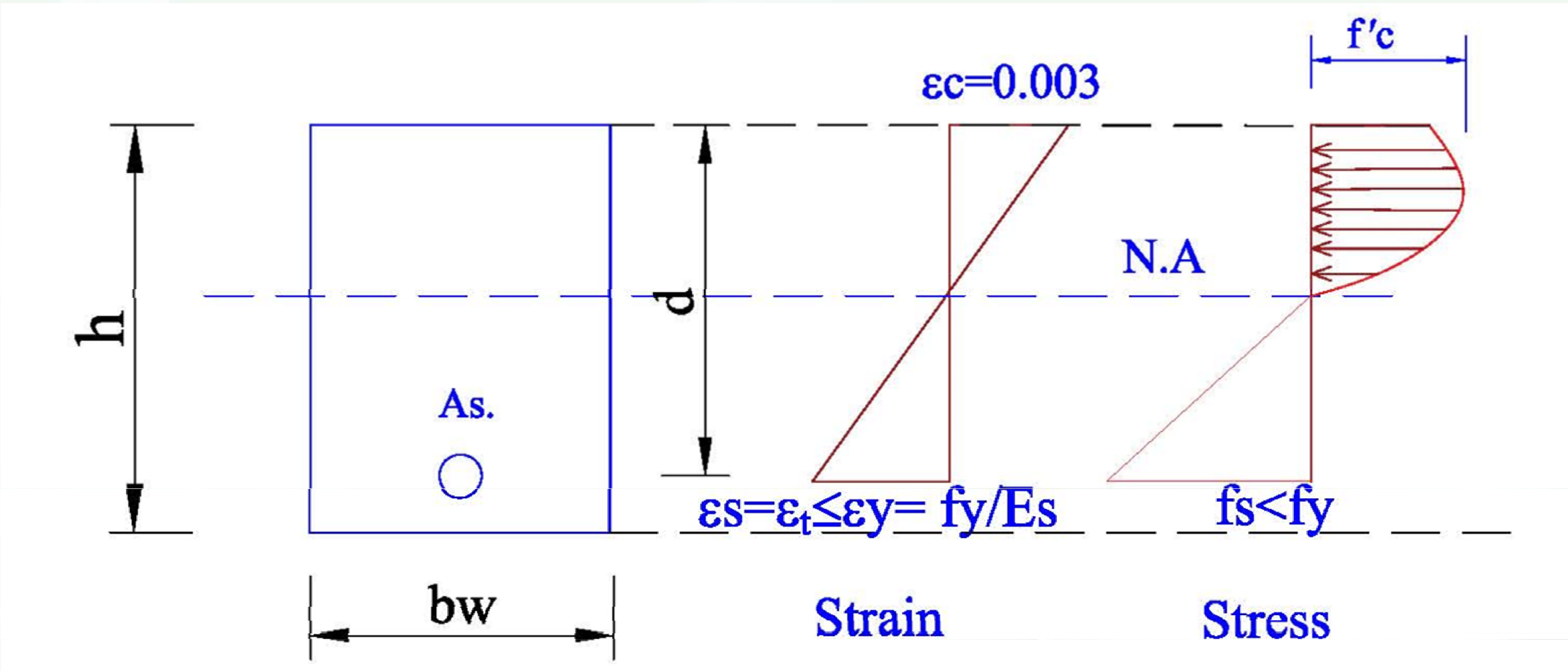


Tension-controlled section.

2 .Steel may reach its yield strength at the same time as concrete reaches its ultimate strength. The section is called a Balanced section



3 .Concrete may fail before the yield of steel, due to the presence of a high percentage of steel in the section. In this case, the concrete strength and its maximum strain of 0.003 are reached, but the steel stress is less than the yield strength, that is, fs is less than fy. The strain in the steel is equal to or less than 0.002. This section is called a compression-controlled section



Analysis and Design Methods of Reinforced Concrete Structure Working Stress Method (WSM)

Stresses are computed in both the concrete and steel using principles of mechanics that include consideration of composite behavior

Actual Stresses < Allowable Stresses

Ultimate design method (UDM)

The Strength of members is computed at ultimate capacity Load Factors are applied to the loads Internal forces are computed from the factored loads

Required Strength < Actual Strength

Working Stress Method (WSM)

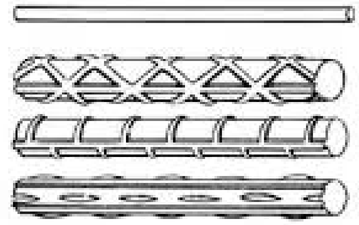
Basic assumptions for design applicable to flexural and compression members are as follows:

(1) Plane section before bending remains plane after bending.

(2) The tensile stress of concrete is neglected unless otherwise mentioned.

(3) The strain-stress relation for concrete as well as for steel reinforcement is linear.

(4) Perfect bond between steel and concrete.



plain and deformed steel bars

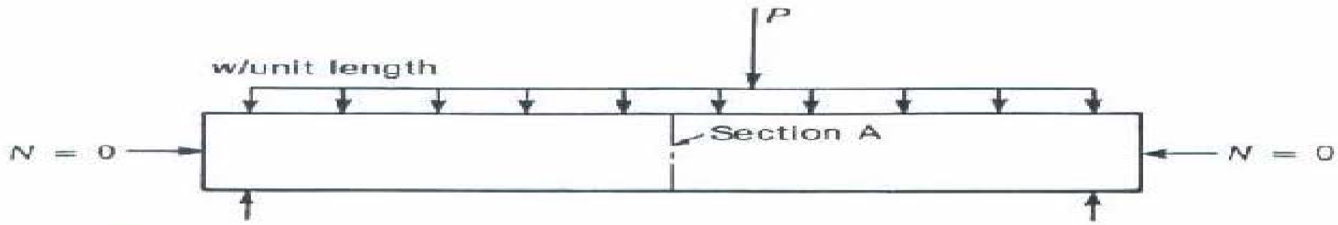
Loading Stages: Un-cracked section and Cracked section and Permissible Stresses

Load factors for all types of loads are taken to be unity for this design method. Permissible stresses are defined as characteristic strength divided by factor of safety.

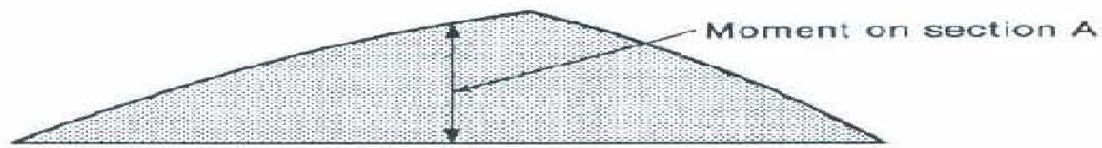
The factor of safety is not unique values either for concrete or for steel; therefore, the permissible stresses at service load must not exceed the following :

- - Flexural Extreme fiber stress in compression : 0.45 $f'c$
- - Tensile stress in reinforcement: 0.5 fy
- - Modular Ratio $n = Es/Ec$
- - Transformer section : Substitute steel area with (n As) of fictitious concrete
- - Location of Neutral axis depends on weather we are analyzing or designing a section

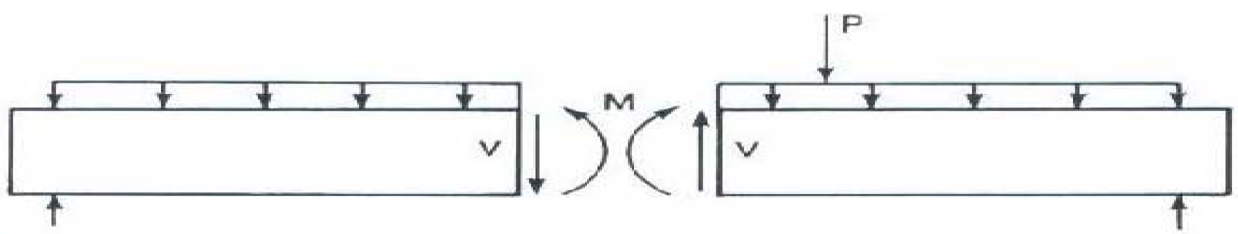
The beam is a structural member used to support the internal moments and shears



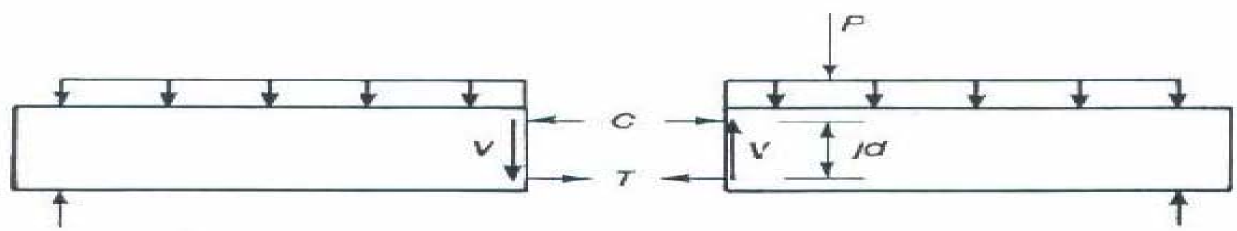
(a) Beam.



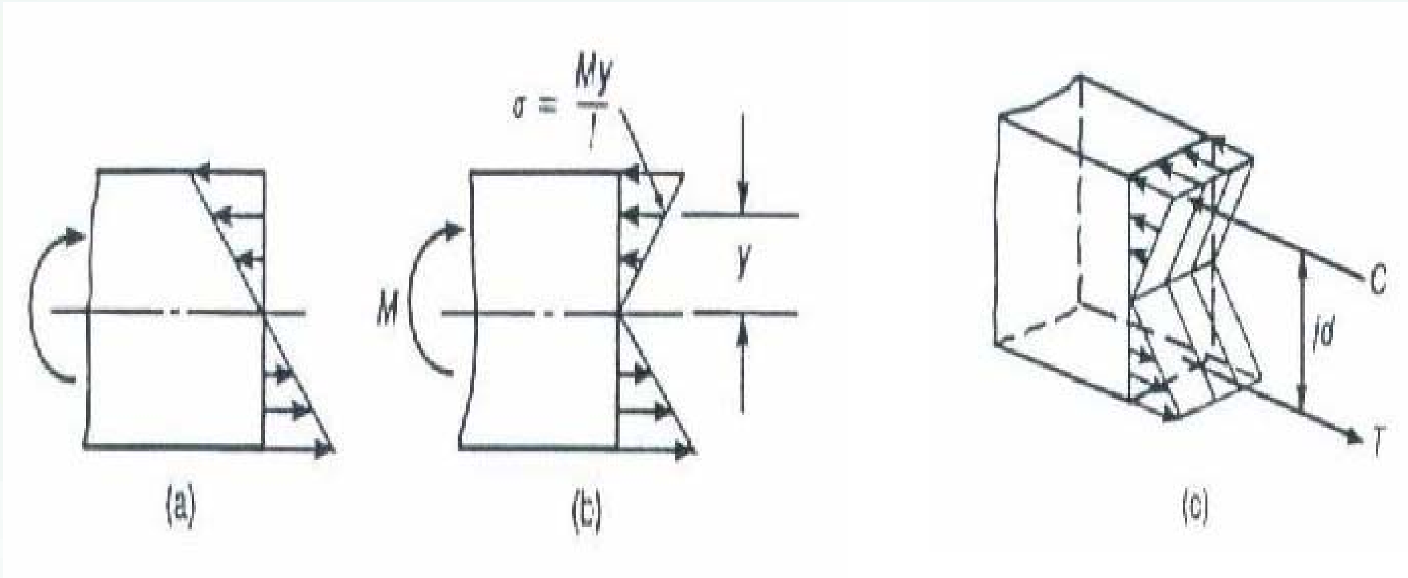
(b) Bending moment diagram.



(c) Free body diagrams showing internal moment and shear force.



(d) Free body diagrams showing internal moment as a compression–tension force couple.



The stress in the block is defined as:

$$\boldsymbol{\sigma} = \frac{\boldsymbol{M} \times \boldsymbol{y}}{\boldsymbol{I}}$$ **(for homogenous section)**

Under the action of transverse loads on a beam strains, normal stresses and internal forces developed on a cross section are as shown below :

1-Stage 1: Before Cracking (Uneconomical).
2- Stage 2: After Cracking (Service Stage).
3- Stage 3: Ultimate (Failure).

1- Un-cracked Section:

Assuming perfect bond between steel and concrete, we have:

$\varepsilon_s = \varepsilon_c$

$\frac{f_s}{Es} = \frac{fc}{E_c}$

$f_s = \frac{E_s}{E_c} fc = nfc$

$\text{Tensile Force} = As\ fs = As.nfc$

$A_{eq} = At = A_c + n\ As$

A_{eq}: *Equivalent Area*

Ac: *Concrete Area*

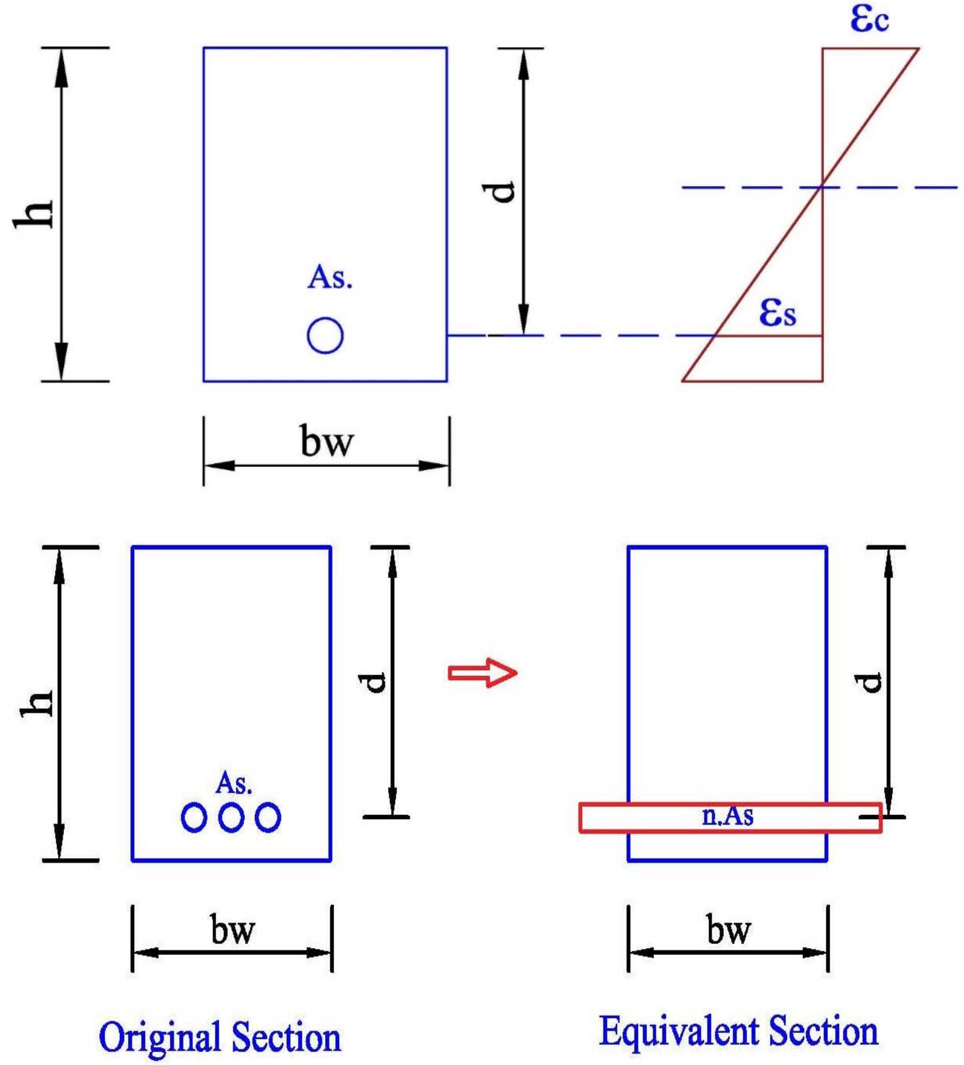
As: *Steel Area*

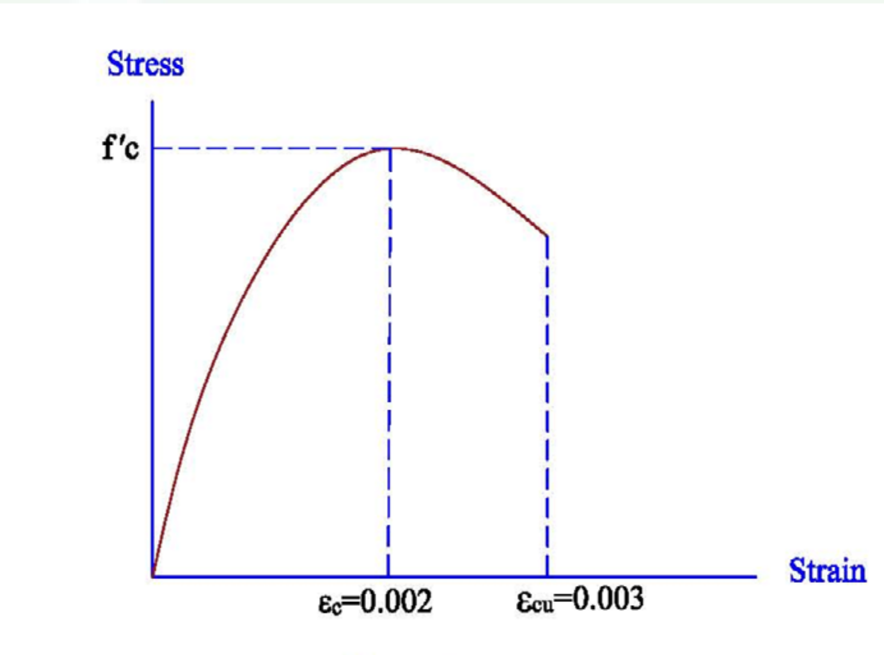
n: *Modular Ratio*

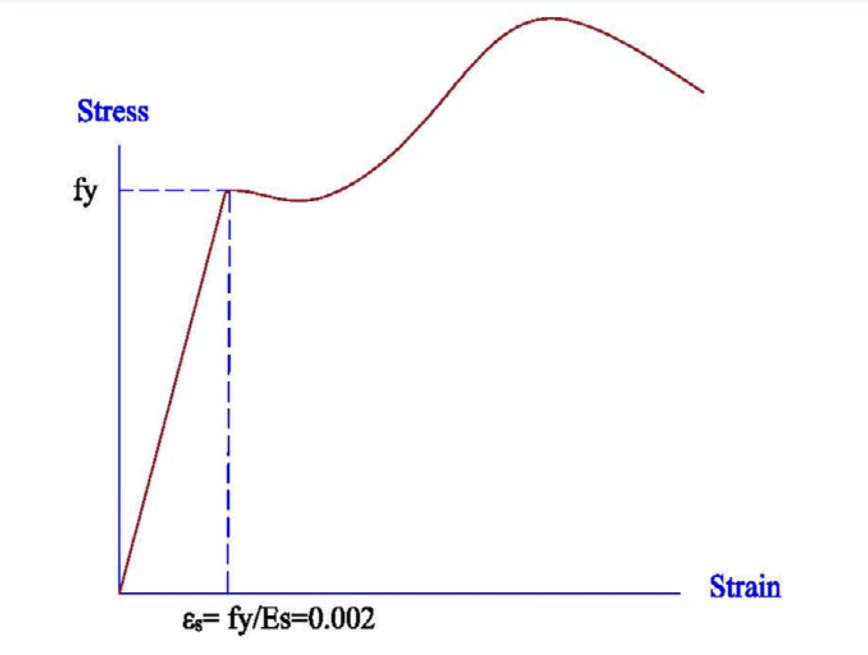
Permissible Stress:

Concrete =**0.45** $\acute{f}_c$

Steel= $\mathbf{0.5} f_y$







Concrete **Steel**

Stress-Strain Relationship of concrete and steel

Homogenous section & under bending:

$$f_c = \frac{M.C}{I}$$

$$f_s = n\ f_c$$

Transformer section:

$$1 - At = (Ac - As) + nA_s = Ac + (n-1)A_s$$

$$2 - \acute{y} = \frac{A_c \times \frac{h}{2} + (n-1)A_s \times d}{A_t}$$

$$3 - I = \frac{b\,h^3}{12} + Ac(\acute{y} - \frac{h}{2})^2 + (n-1)A_s(d - \acute{y})^2$$

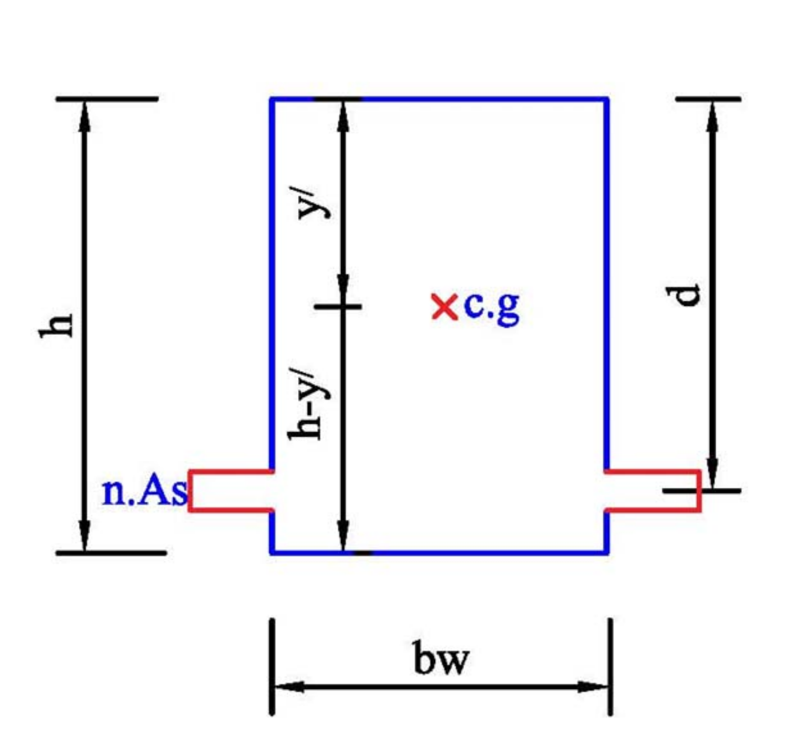
$$where\ At = b\,h + (n-1)A_s$$

Stress:

$$f_c = \frac{M.\acute{y}}{I} \qquad Top\ fiber$$

$$f_c = \frac{M\ (h - \acute{y})}{I} \qquad Bottom\ fiber$$

$$f_s = nf_c = \frac{M.(h - \acute{y} - cover)}{I} \qquad at\ steel\ fiber$$

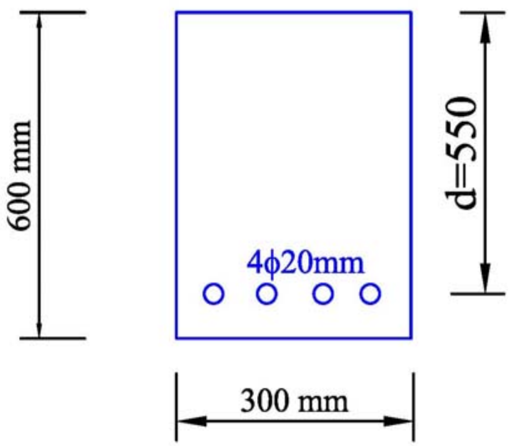


Equivalent Section

Example (1):

Determine the crack moment for the section shown below , and the stresses. Es= 200000 Mpa

$f'c = 28\ Mpa\ , fy = 413\ Mpa\ , b = 300\ , h = 600, concrete\ cover = 50mm$



Solution:

$$f_c = \frac{M.C}{I}$$

$$As = 4 \times 202 \times \pi/4 = 1256\ mm^2$$

$$y' = \frac{bh^2/2 + (n-1)A_s.d}{b.h + (n-1)A_s}$$

$$n = \frac{E_S}{E_c}$$

$$E_c = 4700\sqrt{f'c} = 4700 \times \sqrt{28} = 24870\ \text{MPa}$$

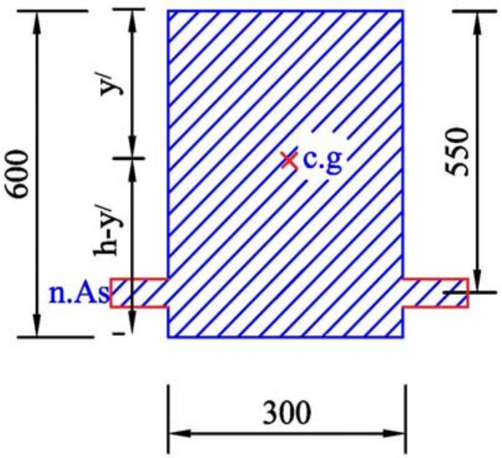
$$n = \frac{Es}{E_c} = \frac{200000}{24870} = \cong 8$$

$$y' = \frac{(300 \times 600 \times \frac{600}{2}) + (8-1) \times 1256 \times (600-50)}{300 \times 600 + (8-1) \times 1256} = 311.63\ mm$$

$$I_{gr} = \frac{bh^3}{12} + bh(y' - \frac{h}{2})^2 + (n-1)As(d-y')^2$$

$$= \frac{300 \times 600^3}{12} + 300 \times 600(311.63 - \frac{600}{2})^2 + (8-1) \times 1256(550 - 311.63)^2$$

$$= 59.232 \times 10^8\ mm^4$$



Equivalent Section

$y_{bottom} = y_b = h - y' = 600 - 311.63 = 288.37\ mm$
$y_{top} = y_t = y' = 311.63\ mm$
$y_{steel} = y_b - \ cover = 288.37 - 50 = \ 238.37\ mm$

From ACI code $f_{cr} = 0.625\sqrt{f'c} = 0.625\sqrt{28} = 3.31MPa$

ft bottom fiber :

$$f_{cr} = \frac{M_{cr} \times y_b}{I_{gr}}$$

$$3.31 = \frac{M_{cr} \times 288.37}{59.232 \times 10^8} \Longrightarrow Mcr = 67.99 \times 10^6\ N.mm$$

or M_{cr}= 67.99 KN.m

$$f_c \text{ top fiber} = f_{ct} = \frac{M_{cr} \times y_t}{I_{gr}}$$

$$= \frac{M_{cr} \times 311.63}{59.232 \times 10^8} = 3.58\ Mpa < f'c = 28\ MPa$$

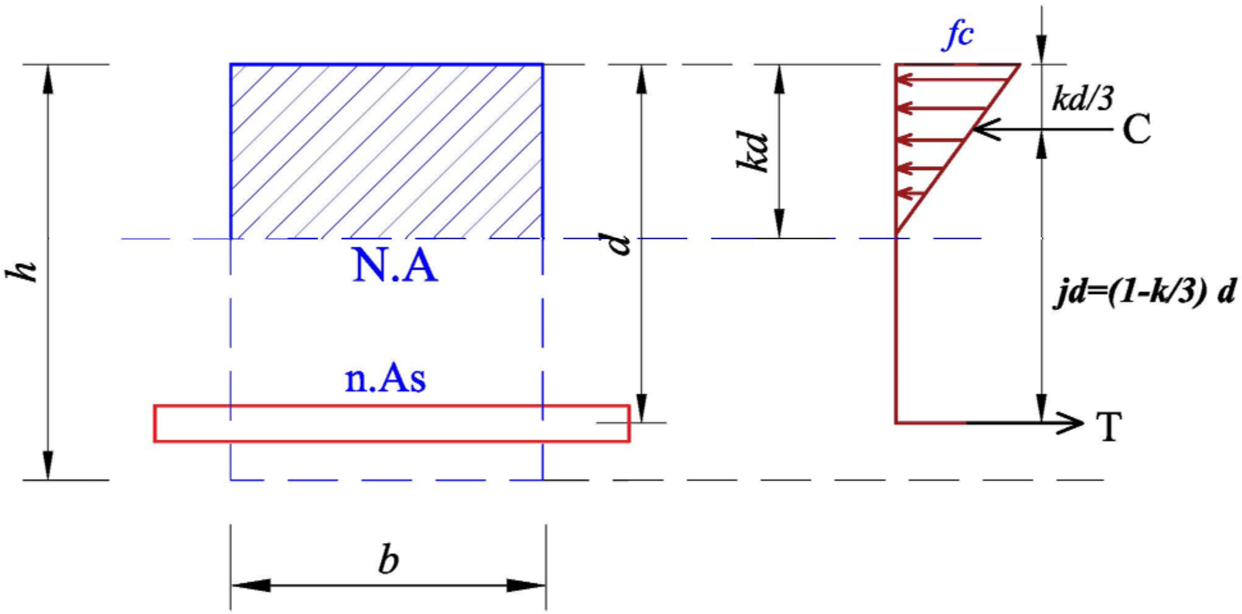
$$f_s = nf_c = n \times \frac{M_{cr} \times (y_b - cover)}{I_{gr}} = 7.99 \times \frac{67.99 \times 10^6(238.37)}{59.232 \times 10^8}$$

$$= 21.86\ MPa << fy = 413\ MPa$$

2-Cracked Section

$$f_t > fr, \qquad fc \leq 0.45f'_c \ and \ fs < 0.5fy$$

Assume the crack goes all the way to the N.A and will use the transformed section



To locate N.A. , tension force = compressive force (by def. NA) (Note, for linear stress distribution and with Tensile and compressive forces are equal to

$$C = b\left(\frac{kd}{2}\right) \times f_c \text{ and } T = As \times fs$$

To determine the location of neutral axis, the moment of the tension is about the axis is set equal to the moment of the compression area, which gives:

$b(kd)\left(\frac{kd}{2}\right) = nAs\,(d - kd)$ second degree equation

$where\ rienforcement\ ratio = \rho = \frac{As}{bd}\ or\ As = \rho bd$

$b(kd)\left(\frac{kd}{2}\right) - n\rho bd^2(\,1 - k) = 0$ multiply by $\left(\frac{1}{bd^2}\right)$

$\left(\frac{k^2}{2}\right) = n\rho(\,1 - k) = 0$

$k^2 + 2n\rho k - 2n\rho + (n\rho)^2 - (n\rho)^2 = 0$

$(k + 2n\rho)^2 = (n\rho)^2 + 2(n\rho) = 0$

Then :

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

Taking moments about C gives:

$M = T\,.\,jd = As\ fs\ jd$

where: jd is the internal lever arm between C and T. From the above equation steel stress is

$$\therefore fs = \frac{M}{As\ jd}$$

Or Conversely, taking moment about T gives

$$M = C\ jd = b\frac{(kd)}{2}f_c\ jd = \frac{f_c}{2}kj\ \mathrm{bd}^2$$

$$\therefore fc = \frac{2M}{kjbd^2}$$

Where :

$$\mathrm{j} = \left(1 - \frac{k}{3}\right)$$

$\boldsymbol{n} = ratio\ of\ modulus\ of\ elasticity\ of\ steel\ to\ that\ of\ concrete = \frac{E_s}{E_c}$

$\boldsymbol{f_c} = compressive\ unit\ stress\ on\ the\ concrete\ at\ the\ surface\ most\ remote$
$from\ the\ neutral\ surface, in\ pound\ per\ square\ inch$

$\boldsymbol{f_s} = tensile\ unit\ stress\ in\ the\ longitudinal\ reinforcement, in\ pound\ per\ square\ inch$

$\boldsymbol{b} = the\ width\ of\ the\ rectangular\ beam, in\ inches.$

$\boldsymbol{d} = the\ effective\ depth\ of\ the\ beam\ in\ inches$

$\boldsymbol{k} = ratio\ of\ distance\ of\ the\ neural\ axis\ of\ the\ cross\ section, from\ extreme$
$fibers\ in\ compression\ to\ the\ effective\ depth\ of\ the\ beam$

$\boldsymbol{kd} = the\ distance\ from\ the\ neutral\ axis\ of\ the\ cross\ section\ to\ the\ extremefibers\ in\ compression$

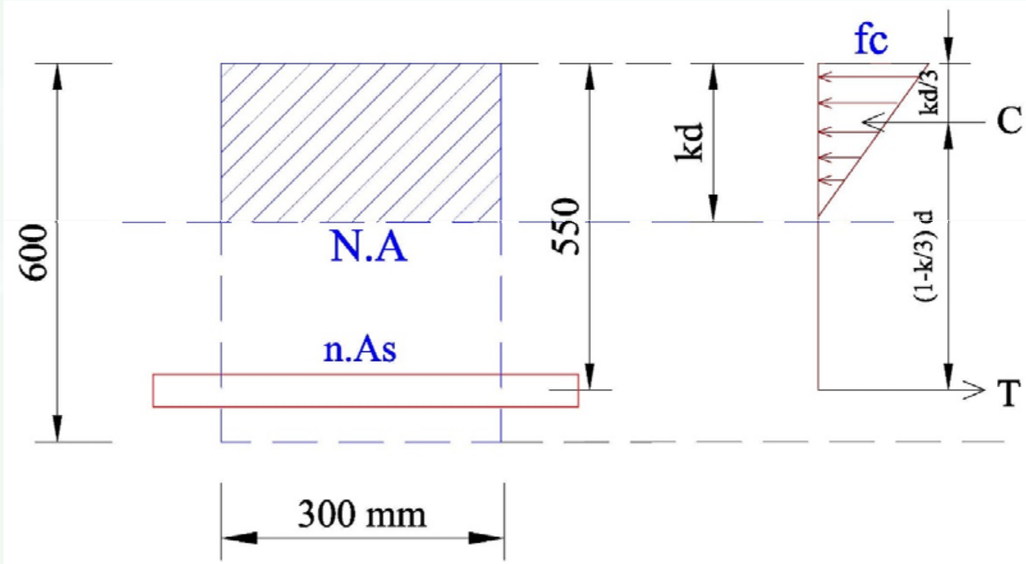
$\boldsymbol{j} = ratio\ of\ the\ distance\ between\ the\ resultant\ of\ the\ compressive\ stresses\ and\ centre\ of\ the$
$tensile\ stresses\ to\ d, the\ effective\ depth\ of\ the\ beam$

$\boldsymbol{jd} = the\ distance\ between\ the\ resultant\ of\ the\ compressive\ stresses\ and\ the\ centre\ of\ the\ tensile$
$stresses. It\ is\ the\ lever\ arm\ of\ the\ resisting\ couple, in\ inches$

$\boldsymbol{\rho} = the\ ratio\ of\ the\ area\ of\ the\ cross\ section\ of\ the\ longitudinal\ steel\ reinforcement$
$to\ the\ effective\ area\ of\ the\ concrete\ beam, \rho\,\frac{A_s}{bd}$

Example (2):

Determine the stresses in concrete and steel of section (300 x 600 mm) as in Exa. (1) subjected to service moment 100 KN.m and $f'_c = 28\ Mpa$, $fy = 413\ Mpa$, cover =50 mm , As= 4ϕ20 mm, Es= 200000 Mpa ,



Solution :

$M_{cr} = 67.99\ KN.m \quad (Example - 1)$

$While\ M\ applied = 100\ KN.m > M_{cr}$

$The\ section\ is\ cracked$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

$$\rho = \frac{A_s}{bd} = \frac{1256}{300 * 550} = 0.007612$$

$n = 7.99$

$$k = \sqrt{2 \times 0.007612 \times 7.99 + (0.007612 \times 7.99)^2} - 0.007612 \times 7.99$$

$= 0.2932$

$C = kd = 0.2932 \times 550 = 161.26mm$

$$I_{cr} = \frac{bc^3}{3} + nA_s(d-c)^2$$

$$= \frac{300 \times 161.26^3}{3} + 7.99 \times 1256\ (550 - 161.26)^2 = 19.359 \times 10^8\ mm^4$$

$Steel\ stress$:

$$f_s = n \times f_{cs} = n \times \frac{M.(d-c)}{Icr}$$

$$= 7.99 \times \frac{100\ \times\ 10^6\ (550 - 161.26)}{19.359 \times 10^8} = 160.6\ MPa\ \ <\ Fs\ allowable = 0.5\ fy$$

$$= 206.5\ MPa$$

Concrete Stress

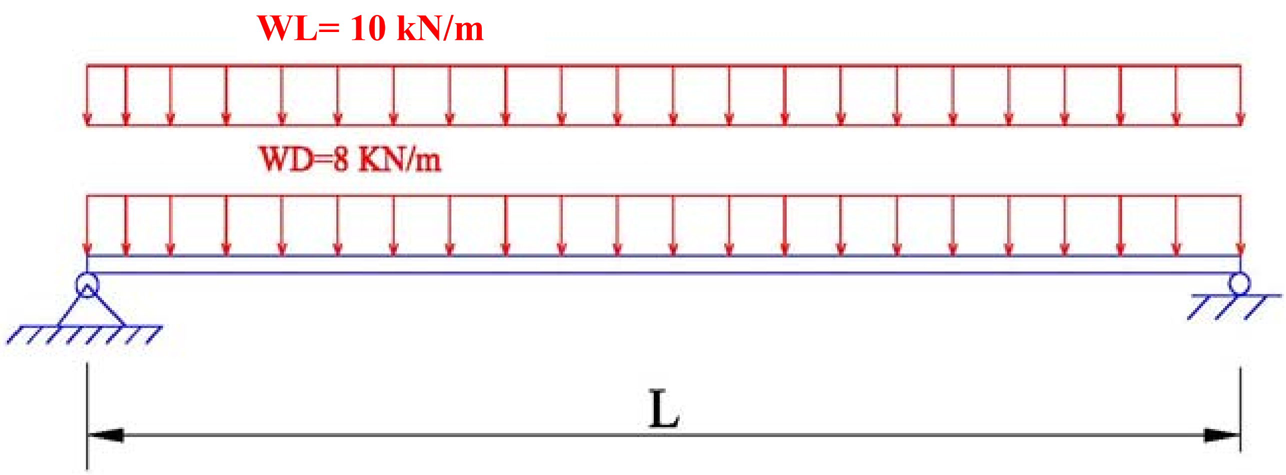
$$f_c = \frac{M.c}{I_{cr}} = \frac{100 \times 10^6 \times\ 161.26}{19.359 \times 10^8} = 8.33\ Mpa < 0.45 \times 28 = 12.6\ Mpa \qquad O.K$$

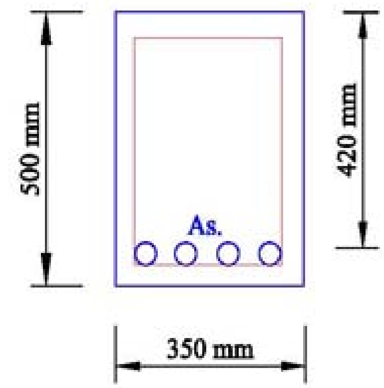
Example (3): For the simply supported beam shown reinforced by 4ϕ 25 mm bars ($fy = 420\ MPa$) , the concrete strength ($f'c = 21\ MPa$) , evaluate the following :

1- If the span beam = 4 m and dead load = 8 KN/m, live load=10 KN/m check the actual flexural stress in concrete and steel.

2- The length of the beam span that make the concrete in tension face start to crack.

3- The actual stress in concrete and steel if the span of beam = 7m





Solution : First

$$Total\ Load\ =W\ =WD+WL=8+10=18\ kN/m^2$$

$$M=\frac{W\ L^2}{8}=\frac{18\times 4^2}{8}=36\ KN/mm^2$$

$$n=\frac{E_s}{E_c}=\frac{200000}{4700\sqrt{21}}=9.22$$

$$A_b=4\times\left(\frac{\pi\times 25^2}{4}\right)=1964\ mm^2$$

Assume $\boldsymbol{f_t=f_r}$

Transformed section area

$$=Ac+(n-1)A_s=500\times 350+(9.22-1)\times 1964=191144.1\ mm^2$$

$$\acute{y}=\frac{A_c\ \times\frac{h}{2}+(n-1)A_s\times d}{A_t}=\frac{350\times 500\times\frac{500}{2}+(9.22-1)\times 1964\times 420}{191144.1}=264.4\ mm$$

$$I=\frac{b\ h^3}{12}+Ac(\acute{y}-\frac{h}{2})^2+(\ n-1)A_{s\,(}d-\acute{y})^2$$

$$=\frac{350\times 500^3}{12}+350\times 500\times\left(264.4-\frac{500}{2}\right)^2+(9.22-1)\times 1964\times(420-264.4)^2$$

$$=4.076\times 10^9\ mm^4$$

$$f_c=\frac{M.C}{I}$$

Compression fiber:

$$f_c=\frac{36\times 10^6\times 264.4}{4.076\times 10^9}=2.33\ MPa$$

Allowable stress in compression $= 0.45f'_c$

$F_c = 0.45 \times 21 = 9.45\ mPa$

$\therefore f_c < f'_c$ O.K

For Tension bottom fiber:

$$f_t = \frac{M.C}{I} = \frac{36 \times 10^6 \times (500 - 264.4)}{4.076 \times 10^9} = 2.08\ MPa$$

$f_r = 0.62\sqrt{f'_c} = 0.62\sqrt{21} = 2.84$ Mpa

$\therefore f_r > f_t$ the assumption is correct and the section is not cracked

$$f_s = nfc = n \times \frac{M.C}{I} = 9.22 \times \frac{36 \times 10^6 \times (420 - 264.4)}{4.076 \times 10^9} = 12.67\ MPa$$

$F_s = 0.5 \times fy = 0.5 \times 420 = 210\ MPa$

$\therefore f_s < F_s$

Second: to make concrete start to crack put the concrete tension stress at the extreme fiber equal to concrete stress at rupture

$(f_r = f_t = 2.84\ mPa)$

$$f_t = \frac{M_{cr}(h - c)}{I}$$

$$2.84 = \frac{M_{cr}(500 - 264.4)}{4.076 \times 10^9}$$

$M_{cr} = 49.12\ kN.m$

$$M_{cr} = \frac{W\ L^2}{8} = \frac{18 \times L^2}{8}$$

$$49.12 = \frac{18 \times L^2}{8} \qquad \therefore L = 4.67\ m$$

Third : $M = \dfrac{W\ L^2}{8} = \dfrac{18 \times 7^2}{8} = 110.25\ kN.m$

$since\ the\ moment\ M = 110.25\ kN.m > M_{cr} = 49.12\ KN.m$ $\therefore$ The concrete section is cracked

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

$$\rho = \frac{A_s}{b\ d} = \frac{1964}{350 \times 420} = 0.0134$$

$$\rho n = 0.0134 \times 9.22 = 0.124$$

$$k = \sqrt{2 \times 0.124 + (0.124)^2} - 0.124 = 0.389$$

$$kd = 0.389 \times 420 = 163.46\ mm$$

$$j = 1 - \frac{k}{3} = 0.87$$

$$jd = 365.54\ mm$$

$$f_c = \frac{2M}{kjbd^2} = \frac{2 \times 110.25 \times 10^6}{0.389 \times 0.87 \times 350 \times (420)^2} = 10.55\ MPa$$

concrete allowable compression stress $F_c = 0.45f'_c = 0.45 \times 21 = 9.45\ mPa$

$\therefore f_c > f'_c$ the concrete behavior is not in elastic range .

$$f_s = \frac{M}{A_S\ jd} = \frac{110.25 \times 10^6}{1694 \times 365.54} = 153.57\ mPa$$

Allowable steel stress = 210 MPa $\therefore f_s > F_s$ the steel stress with in limits (OK)

Thank You……………………

Reinforced Concrete Design

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LOGO

Chapter II

Flexural Analysis Reinforced Beam

Strength Design Approach

The analysis and design of a structural member may be regarded as the process of selecting the proper materials and determining the member dimensions such that the design strength is equal or greater than the required strength. The required strength is determined by multiplying the actual applied loads, the dead load, the assumed live load, and other loads, such as wind, seismic, earth pressure, fluid pressure, snow, and rain loads, by load factors. These loads develop external forces such as bending moments, shear, torsion, or axial forces, depending on how these loads are applied to the structure.

In proportioning reinforced concrete structural members, three main items can be investigated:

1 .The safety of the structure, which is maintained by providing adequate internal design strength.

2 .Deflection of the structural member under service loads. The maximum value of deflection must be limited and is usually specified as a factor of the span, to preserve the appearance of the structure.

3 .Control of cracking conditions under service loads. Visible cracks spoil the appearance of the structure and permit humidity to penetrate the concrete, causing corrosion of steel and consequently weakening the reinforced concrete member. The ACI Code implicitly limits crack widths to 0.016 in. (0.40 mm) for interior members and 0.013 in. (0.33 mm) for exterior members. Control of cracking is achieved by adopting and limiting the spacing of the tension bar.

It is worth mentioning that the strength design approach was first permitted in the United States in 1956 and in Britain in 1957. The latest ACI Code emphasizes the strength concept based on specified strain limits on steel and concrete that develop tension-controlled, compression controlled, or transition conditions.

ASSUMPTIONS

Reinforced concrete sections are heterogeneous (nonhomogeneous), because they are made of two different materials, concrete and steel. Therefore, proportioning structural members by strength design approach is based on the following assumptions:

1. Strain in concrete is the same as in reinforcing bars at the same level, provided that the bond between the steel and concrete is adequate.
2. Strain in concrete is linearly proportional to the distance from the neutral axis.
3. The modulus of elasticity of all grades of steel is taken as Es = (200,000MPa or N/mm^2). The stress in the elastic range is equal to the strain multiplied by Es.
4. Plane cross sections continue to be plane after bending.
5. Tensile strength of concrete is neglected because (a) concrete's tensile strength is about 10% of its compressive strength, (b) cracked concrete is assumed to be not effective, and (c) before cracking, the entire concrete section is effective in resisting the external moment.
6. The method of elastic analysis, assuming an ideal behavior at all levels of stress, is not valid. At high stresses, non-elastic behavior is assumed, which is in close agreement with the actual behavior of concrete and steel.
7. At failure the maximum strain at the extreme compression fibers is assumed equal to 0.003 by the ACI Code provision.
8. For design strength, the shape of the compressive concrete stress distribution may be assumed rectangular, parabolic, or trapezoidal. In this text, a rectangular shape will be assumed (ACI Code, Section 22.2).

TYPES OF FLEXURAL FAILURE AND STRAIN LIMITS

Three types of flexural failure of a structural member can be expected depending on the percentage of steel used as explained before.

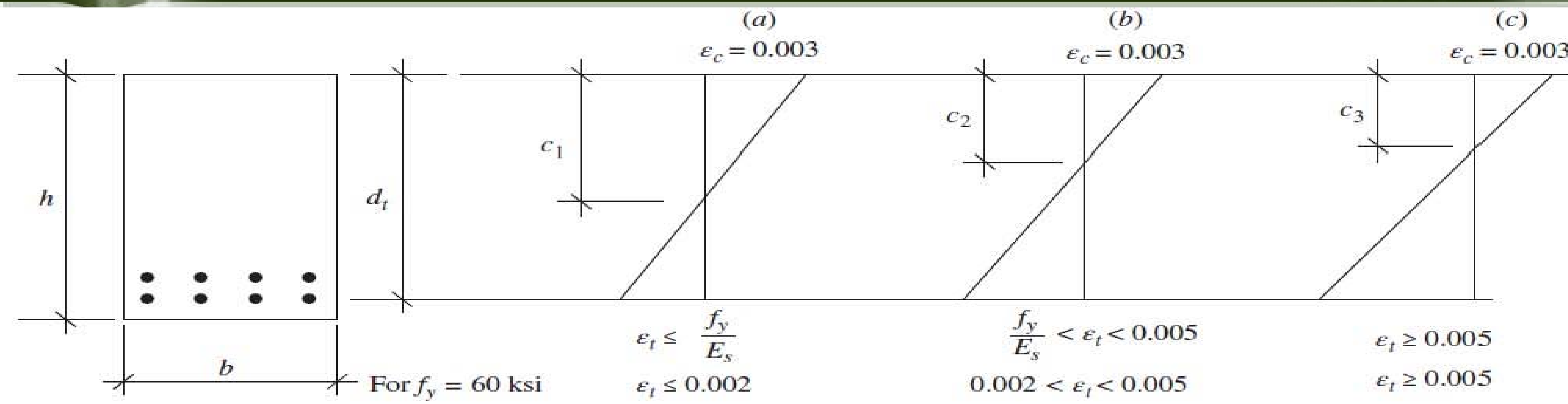
It can be assumed that concrete fails in compression when the concrete strain reaches 0.003.A range of 0.0025 to 0.004 has been obtained from tests and the ACI Code, Section 22.2.2.1, assumes a strain of 0.003.

In beams designed as tension-controlled sections, steel yields before the crushing of concrete. Cracks widen extensively, giving warning before the concrete crushes and the structure collapses. The ACI Code adopts this type of design. In beams designed as balanced or compression-controlled sections, the concrete fails suddenly, and the beam collapses immediately without warning. The ACI Code does not allow this type of design.

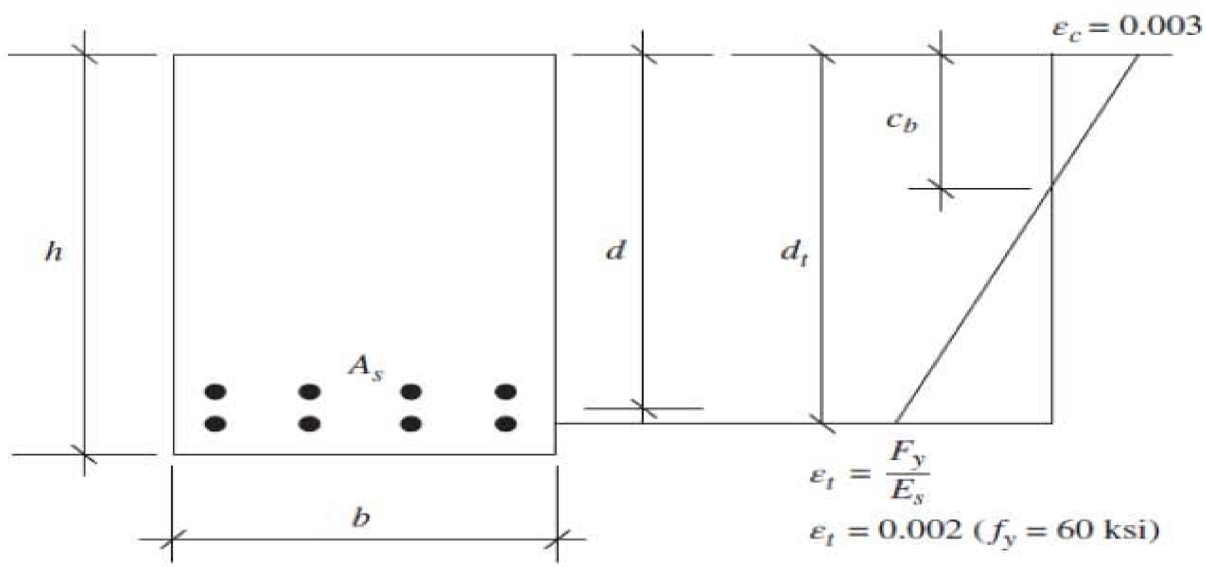
Strain Limits for Tension and Tension-Controlled Sections

The design provisions for both reinforced and pre-stressed concrete members are based on the concept of tension or compression-controlled sections, ACI Code, Section 21.2. Both are defined in terms of net tensile strain (NTS), (ε_t), in the extreme tension steel at nominal strength, exclusive of pre-stress strain. Moreover, two other conditions may develop: (1) the balanced strain condition and (2) the transition region condition. These four conditions are defined as follows:

1. Compression-controlled sections are those sections in which the net tensile strain, NTS, in the extreme tension steel at nominal strength is equal to or less than the compression-controlled strain limit at the time when concrete in compression reaches its assumed strain limit of 0.003,(εc = 0.003). For grade 60 steel, (f_y = 420 MPa), the compression-controlled strain limit may be taken as a net strain of 0.002, Fig. a. This case occurs mainly in columns subjected to axial forces and moments.
2. Tension-controlled sections are those sections in which the NTS, εt, is equal to or greater than 0.005 just as the concrete in the compression reaches its assumed strain limit of 0.003, Fig. c.
3. Sections in which the NTS in the extreme tension steel lies between the compression controlled strain limit (0.002 for f_y = 420 MPa) and the tension-controlled strain limit of 0.005 constitute the transition region, Fig. b.
4. The balanced strain condition develops in the section when the tension steel, with the first yield, reaches a strain corresponding to its yield strength, f_y or $\varepsilon s = f_y/E_s$, just as the maximum strain in concrete at the extreme compression fibers reaches 0.003, Fig. d.



Strain limit distribution, $c_1 > c_2 > c_3$: (*a*) compression-controlled section, (*b*) transition region, and (*c*) tension-controlled section.



d. Balanced strain section (occurs at first yield or at distance d_t).

In addition to the above four conditions, Section 9.3.3.1 of the ACI Code indicates that the net tensile strain, εt, at nominal strength, within the transition region, shall not be less than 0.004 for reinforced concrete flexural members without or with an axial load less than 0.10 f′c Ag, where Ag=gross area of the concrete section.

Note that d_t in Fig. above, is the distance from the extreme concrete compression fiber to the extreme tension steel, while the effective depth, d, equals the distance from the extreme concrete compression fiber to the centroid of the tension reinforcement. These cases are summarized in Table below:

Table 1 Strain Limits of Figure above

Section Condition	Concrete Strain	Steel Strain	Notes (f_y = 60 ksi) 420MPa
Compression controlled	0.003	$\varepsilon_t \leq f_y/E_s$	$\varepsilon_t \leq 0.002$
Tension controlled	0.003	$\varepsilon_t \geq 0.005$	$\varepsilon_t \geq 0.005$
Transition region	0.003	$f_y/E_s < \varepsilon_t < 0.005$	$0.002 < \varepsilon_t < 0.005$
Balanced strain	0.003	$\varepsilon_s = f_y/E_s$	$\varepsilon_s = 0.002$
Transition region (flexure)	0.003	$0.004 \leq \varepsilon_t < 0.005$	$0.004 \leq \varepsilon_t < 0.005$

LOAD FACTORS

For the design of structural members, the factored design load is obtained by multiplying the dead load by a load factor and the specified live load by another load factor. The magnitude of the load factor must be adequate to limit the probability of sudden failure and to permit an economical structural design. The choice of a proper load factor or, in general, a proper factor of safety depends mainly on the importance of the structure (whether a courthouse or a warehouse), the degree of warning needed prior to collapse, the importance of each structural member (whether a beam or column), the expectation of overload and the accuracy of calculations.

Based on historical studies of various structures, experience, and the principles of probability, the ACI Code adopts a load factor of 1.2 for dead loads and 1.6 for live loads. The dead-load factor load. Moreover, the choice of factors reflects the degree of the economical design as well as the degree of safety and serviceability of the structure. It is also based on the fact that the performance of the structure under actual loads must be satisfactorily within specific limits.

If the required strength is denoted by U (ACI Code, Section 5.3.1), and those due to wind and seismic forces are W and E, respectively, according to the ACI and ASCE 7-10 Codes (American society of civil Engineering) , the required strength, U, shall be the most critical of the following factors:

1. In the case of dead, live, and wind loads,

U= 1.4 D
U= 1.2 D+ 1.6L
U= 1.2 D+ 1.0L+1.0 W
U= 0.9 D+1.0 W
U= 1.2 D+ (1.0L+0.5 W)

2. In the Case of Dead Load , Live and seismic load (earthquake) forces , E

U= 1.2 D+ 1.0L+1.0E
U= 0.9 D+ 1.0E

3. For load combination due to roof live load , Lr , rain Load ,R, Snow load ,S, in additional to dead , live load , wind , and earthquake load:

U= 1.2 D + 1.6L+0.5 (Lr or S or R)
U= 1.2 D + 1.6 (Lr or S or R) + (1.0 L or 0.5 W)
U= 1.2 D + 1.0 W + 1.0 L + 0.5 (Lr or S or R)
U= 1.2 D + 1.E + 1.0 L+ 0.2 S

4. Where fluid load F is present, it shall be included as follows:

U= 1.4 (D + F)
U= 1.2 D + 1.2F + (L or 0.5 W)+ 1.6(Lr or S or R)
U= 1.2 D + 1.2F + 1.0 W + L + 0.5 (Lr or S or R)
U= 1.2 D + 1.2F + 1.0 E + L + 0.2 S
U= 0.9 (D+F) + 1.0 E

STRENGTH REDUCTION FACTOR ϕ

The nominal strength of a section, say Mn, for flexural members, calculated in accordance with the requirements of the ACI Code provisions must be multiplied by the strength reduction factor, ϕ, which is always less than 1. The strength reduction factor has several purposes:

1 .To allow for the probability of understrength sections due to variations in dimensions, material properties, and inaccuracies in the design equations.

2 .To reflect the importance of the member in the structure.

3 .To reflect the degree of ductility and required reliability under the applied loads
The ACI Code, Table 21.2.1, specifies the following values to be used

A higher ϕ factor is used for tension-controlled sections than for compression-controlled sections, because the latter sections have less ductility and they are more sensitive to variations in concrete strength. Also, spirally reinforced compression members have a ϕvalue of 0.75 compared to 0.65 for tied compression members; this variation reflects the greater ductility behavior of spirally reinforced concrete members under the applied loads. In the ACI Code provisions, the ϕ factor is based on the behavior of the cross section at nominal strength, (Pn, Mn), defined in terms of the NTS, εt, in the extreme tensile strains, as given below. For tension-controlled members,ϕ= 0.9. For compression-controlled members, ϕ= 0.75 (with spiral reinforcement) and ϕ= 0.65 for other members.

For tension-controlled sections ϕ= 0.9
For Compression -controlled sections
- a- with Spiral Reinforcement ϕ= 0.75
- b- other Reinforced member ϕ= 0.65

For Plain Concrete ϕ= 0.60
For Shear and Torsion ϕ= 0.75
For Bearing on Concrete ϕ= 0.65
For Strut and Tie model ϕ= 0.75

A higher ϕ factor is used for tension-controlled sections than for compression-controlled sections, because the latter sections have less ductility and they are more sensitive to variations in concrete strength. Also, spirally reinforced compression members have a ϕ value of 0.75 compared to 0.65 for tied compression members; this variation reflects the greater ductility behavior of spirally reinforced concrete members under the applied loads. In the ACI Code provisions, the ϕ factor is based on the behavior of the cross section at nominal strength, (Pn, Mn), defined in terms of the NTS, εt, in the extreme tensile strains, as given in Table 1. For tension-controlled members, ϕ= 0.9. For compression-controlled members, ϕ= 0.75 (with spiral reinforcement) and ϕ= 0.65 for other members.

For the transition region, ϕ may be determined by linear interpolation between 0.65 (or 0.75) and 0.9. Figure 3.6a shows the variation of ϕ for grade 60 steel (420 Mpa) . The linear equations are as follows:

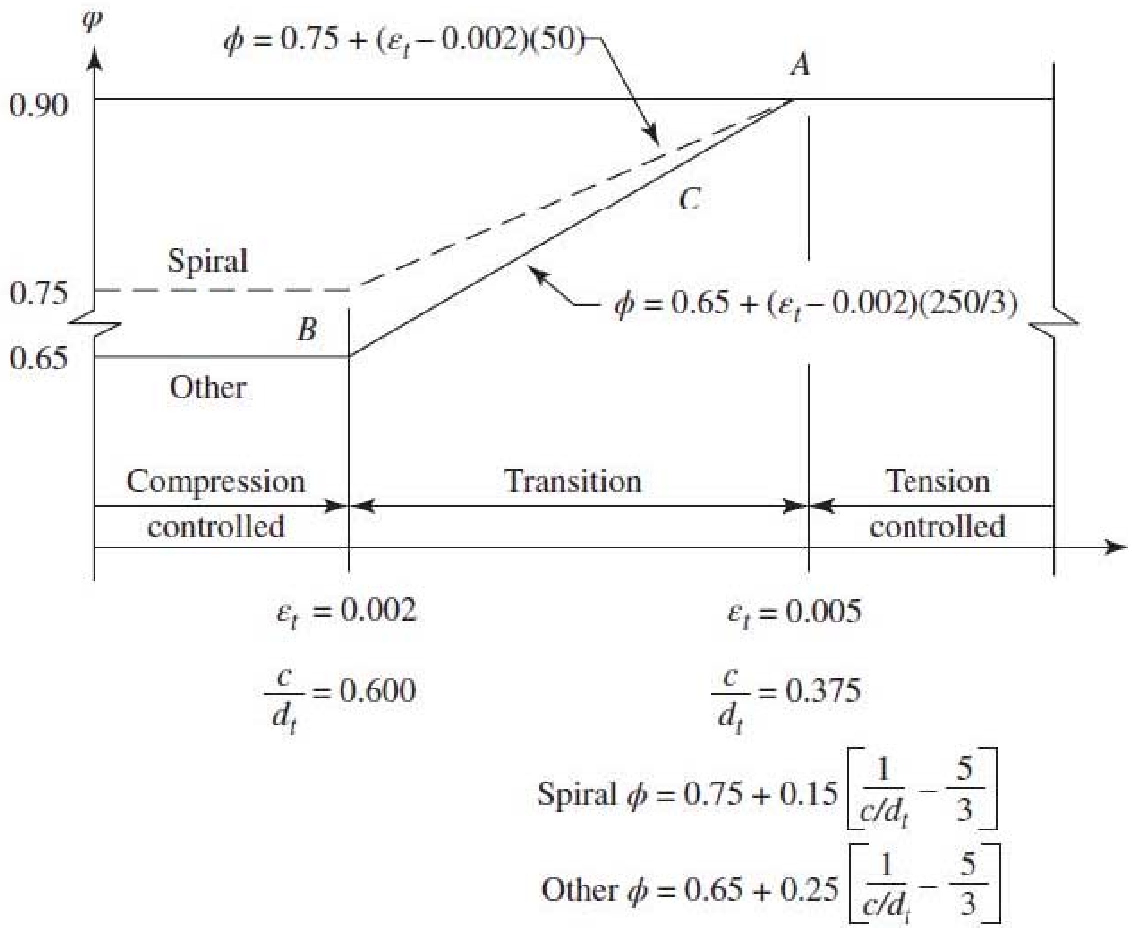
$$\emptyset = 0.75 + (\varepsilon_t - 0.002)\,(50) \qquad \text{For Spiral Members}$$

$$\emptyset = 0.65 + (\varepsilon_t - 0.002)\,\left(\frac{250}{3}\right) \qquad \text{For Other Members}$$

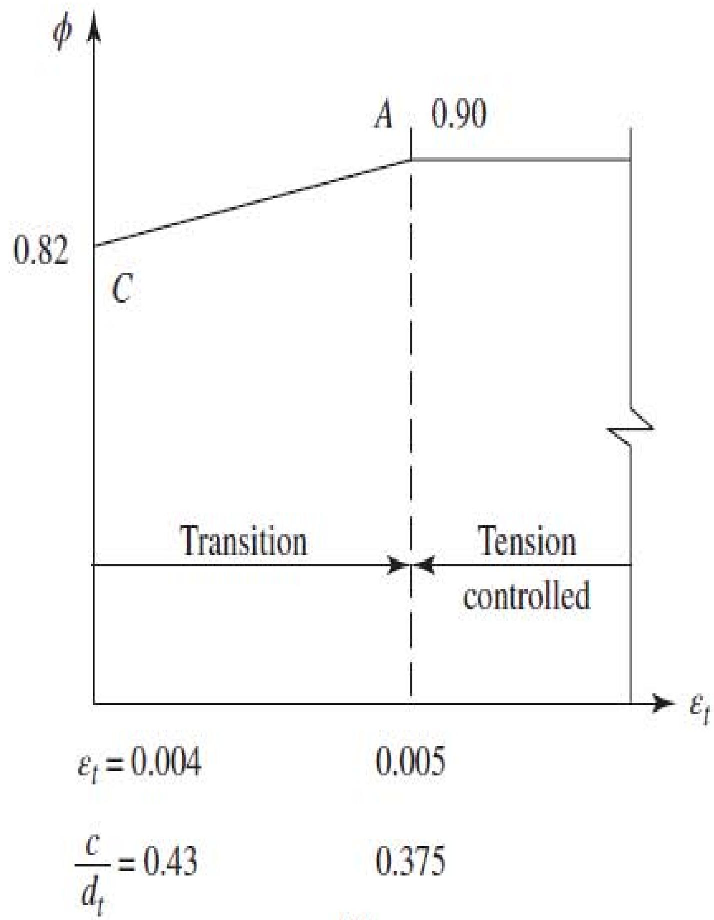
Alternatively Ø may be determined in the transition region , as a function of (c/dt) for grade 60 (fy 420 Mpa) steel as follows:

$$\emptyset = 0.75 + 0.15\left(\frac{1}{c/dt} - \frac{5}{3}\right) \quad \ldots\ldots\ldots\ldots\ldots \text{For Spiral Members}$$

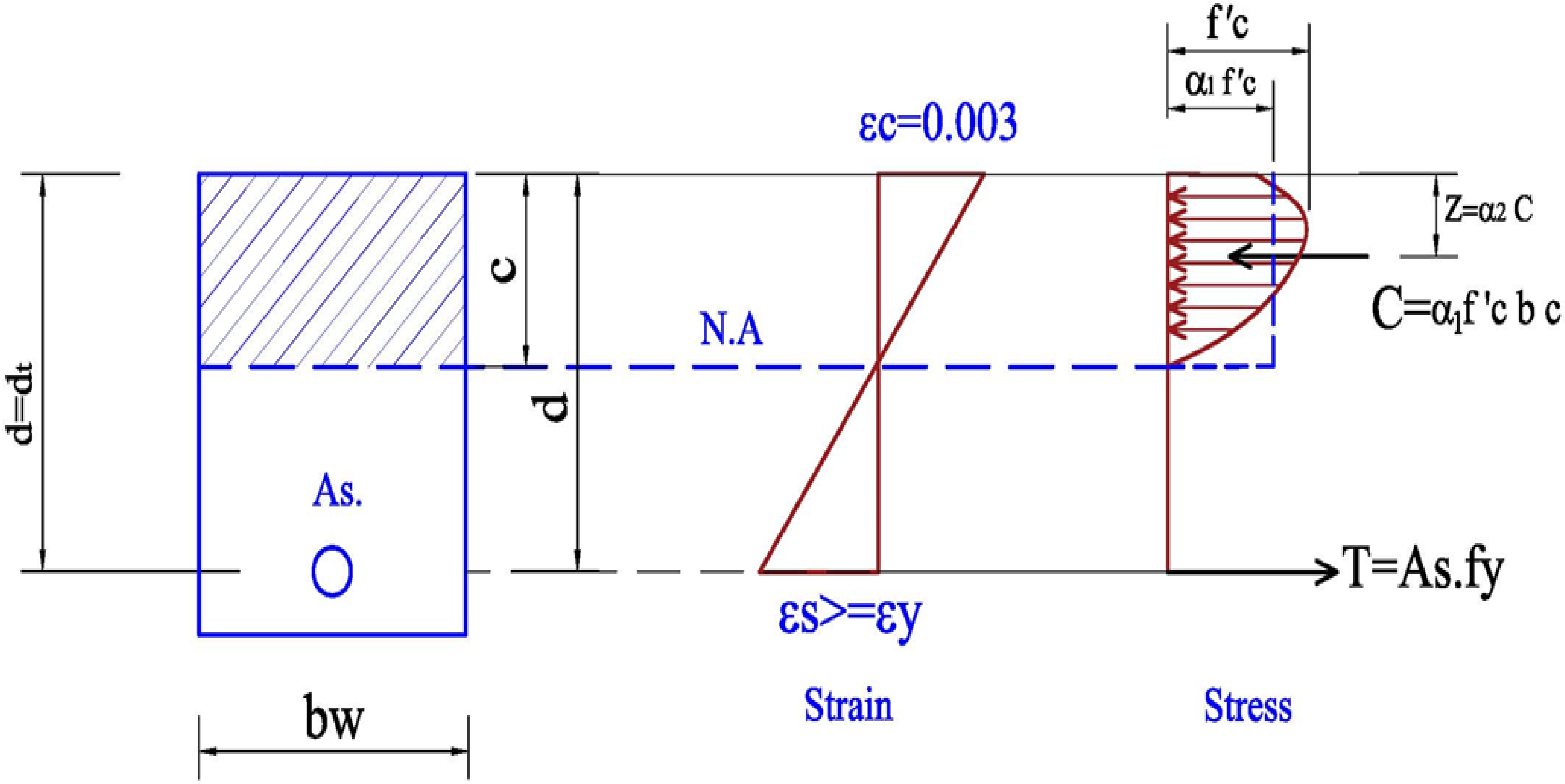
$$\emptyset = 0.65 + 0.15\left(\frac{1}{c/dt} - \frac{5}{3}\right) \quad \ldots\ldots\ldots\ldots\ldots \text{For Other Members}$$


φ
φ = 0.75 + (εt – 0.002)(50)
A
0.90
C
Spiral
0.75
B
φ = 0.65 + (εt – 0.002)(250/3)
0.65
Other
Compression
controlled
Transition
Tension
controlled
εt = 0.002
c/dt = 0.600
εt = 0.005
c/dt = 0.375
Spiral φ = 0.75 + 0.15 [1/(c/dt) – 5/3]
Other φ = 0.65 + 0.25 [1/(c/dt) – 5/3]

(e)


φ
A
0.90
0.82
C
Transition
Tension
controlled
εt
εt = 0.004
0.005
c/dt = 0.43
0.375

(f)


f′c
α1 f′c
εc=0.003
Z=α2 C
C=α1f ′c b c
c
d
d=dt
N.A
As.
T=As.fy
εs>=εy
bw
Strain
Stress

EQUIVALENT COMPRESSIVE STRESS DISTRIBUTION

The distribution of compressive concrete stresses at failure may be assumed to be a rectangle, trapezoid, parabola, or any other shape that is in good agreement with test results.

When a beam is about to fail, the steel will yield first if the section is under reinforced, and in this case the steel is equal to the yield stress. If the section is over reinforced, concrete crushes first and the strain is assumed to be equal to 0.003, which agrees with many tests of beams and columns. A compressive force, C, develops in the compression zone and a tension force, T, develops in the tension zone at the level of the steel bars. The position of force T is known because its line of application coincides with the center of gravity of the steel bars. The position of compressive force C is not known unless the compressive volume is known and its center of gravity is located. If that is done, the moment arm, which is the vertical distance between C and T, will consequently be known.
In Fig. above, if concrete fails, $\varepsilon_c = 0.003$, and if steel yields, as in the case of a balanced section, fs = fy.
The compression force C is represented by the volume of the stress block, which has the non-uniform shape of stress over the rectangular hatched area of b*c. This volume may be considered equal to $C = b\ c\ (\alpha_1\ f'c)$, where $\alpha_1\ f'c$ is an assumed average stress of the non-uniform stress block.
The position of compression force C is at a distance z from the top fibers, which can be considered as a fraction of the distance c (the distance from the top fibers to the neutral axis), and z can be assumed to be equal to $\alpha_2 C$, where $\alpha_2 < 1$. The values of $\alpha 1$ and $\alpha 2$ have been estimated from many tests, and their values are as follows:

α_1 = 0.72 for f'c ≤ (28MPa); it decreases linearly by 0.04 for every (7MPa) greater than (28 MPa)

$$\alpha_1 = 0.72 - 0.04 \times (f'c - 28)/7$$

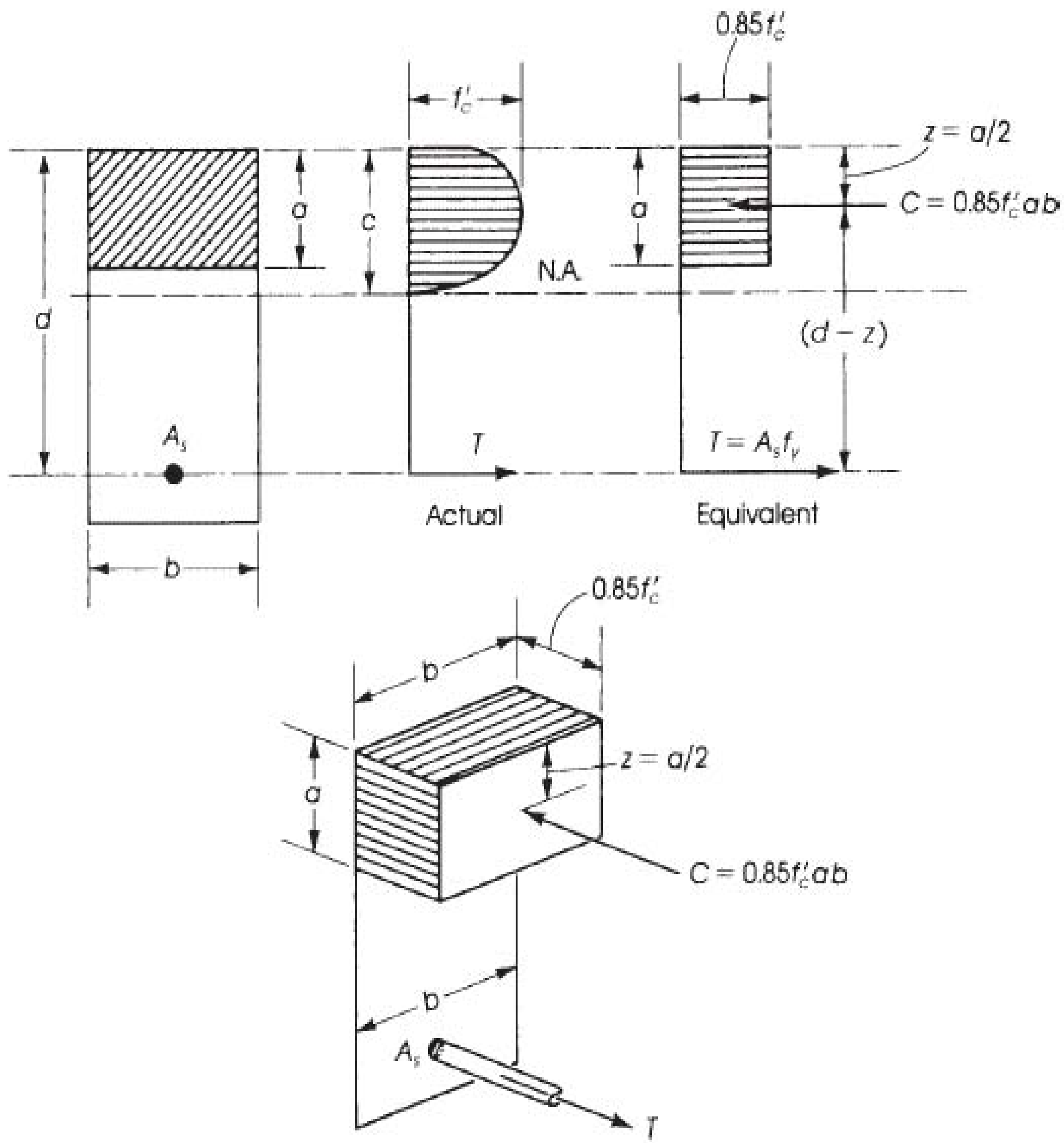
α_2 = 0.425 for f'c < (28MPa); it decreases linearly by 0.025 for every (7MPa) greater than (28MPa)

$$\alpha_2 = 0.425 - 0.025 \times (f'c - 28)/7$$

The decrease in the value of $\alpha 1$ and $\alpha 2$ is related to the fact that high-strength concretes show more brittleness than low-strength concretes.

To derive a simple rational approach for calculations of the internal forces of a section, the ACI Code adopted an equivalent rectangular concrete stress distribution, which was first proposed by C.S. Whitney and checked by Mattock and others. A concrete stress of 0.85 f'c is assumed to be uniformly distributed over an equivalent compression zone bounded by the edges of the cross section and a line parallel to the neutral axis at a distance ($a=\beta_1 c$) from the fiber of maximum compressive strain, where c is the distance between the top of the compressive section and the neutral axis. The fraction β_1 is 0.85 for concrete strengths f'c ≤ (28MPa) and is reduced linearly at a rate of 0.05 for each (7MPa) of stress greater than (28MPa) with a minimum value of 0.65.

$$\beta_1 = 0.85 - 0.05 \times \left(\frac{f'c - 28}{7}\right) \geq 0.65$$


0.85f'c
f'c
z = a/2
C = 0.85f'c ab
a
c
a
d
N.A.
(d − z)
As
T
T = As fy
Actual
Equivalent
b
0.85f'c
b
z = a/2
a
C = 0.85f'c ab
b
As
T

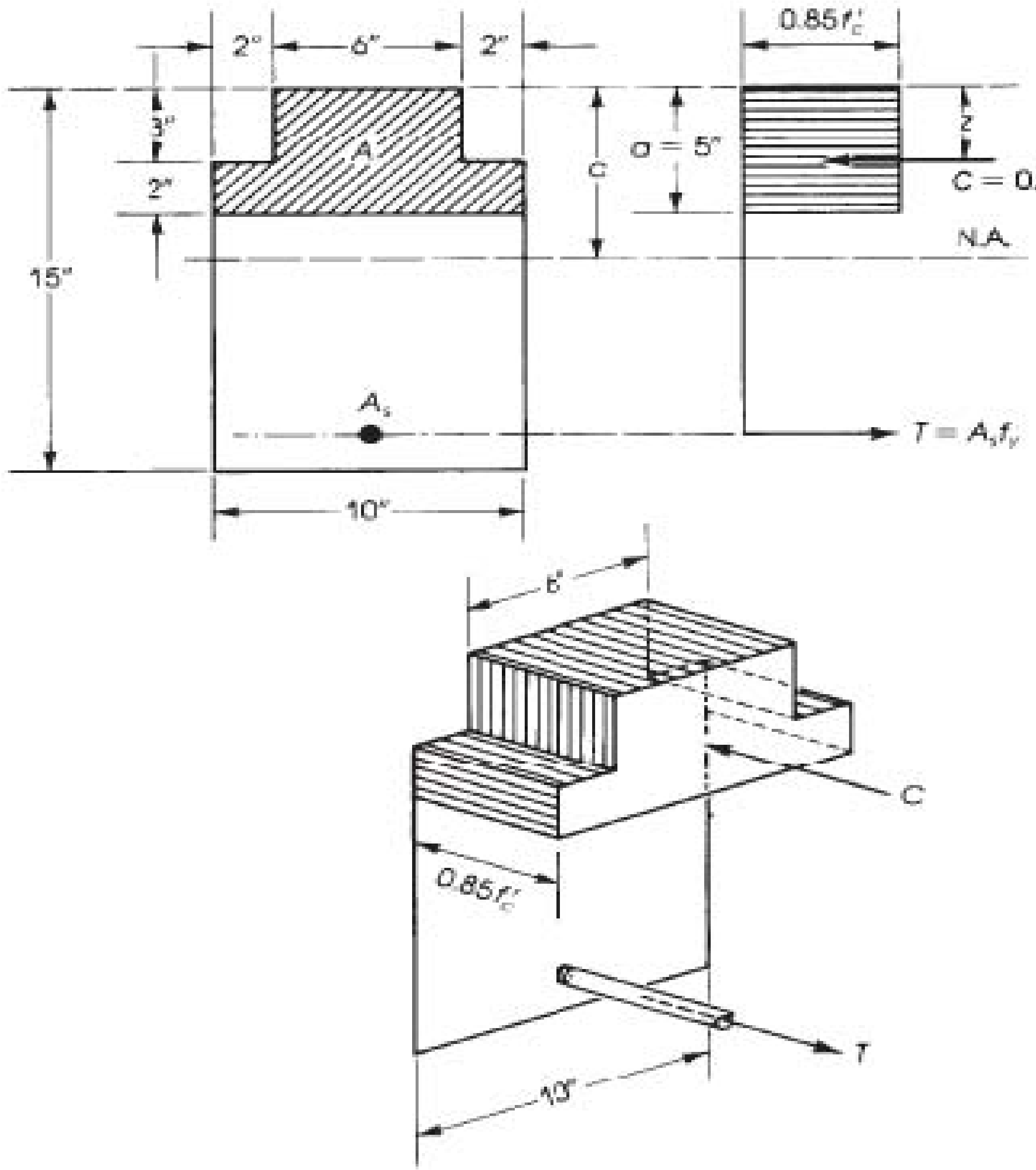


Figure 3.10 Forces in a nonrectangular section.

SINGLY REINFORCED RECTANGULAR SECTION IN BENDING

The balanced condition is achieved when steel yields at the same time as the concrete fails, and that failure usually happens suddenly. This implies that the yield strain in the steel is reached (εy =fy/Es) and that the concrete has reached its maximum strain of 0.003.

The percentage of reinforcement used to produce a balanced condition is called the balanced steel ratio, ρb. This value is equal to the area of steel, As, divided by the effective cross section bd.

$$\rho_b = \frac{As_{balanced}}{bd}$$

Where:

b = width of compression face of member

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement

Two basic equations for the analysis and design of structural members are the two equations of equilibrium that are valid for any load and any section:

1 .The compression force should be equal to the tension force; otherwise, a section will have linear displacement plus rotation:

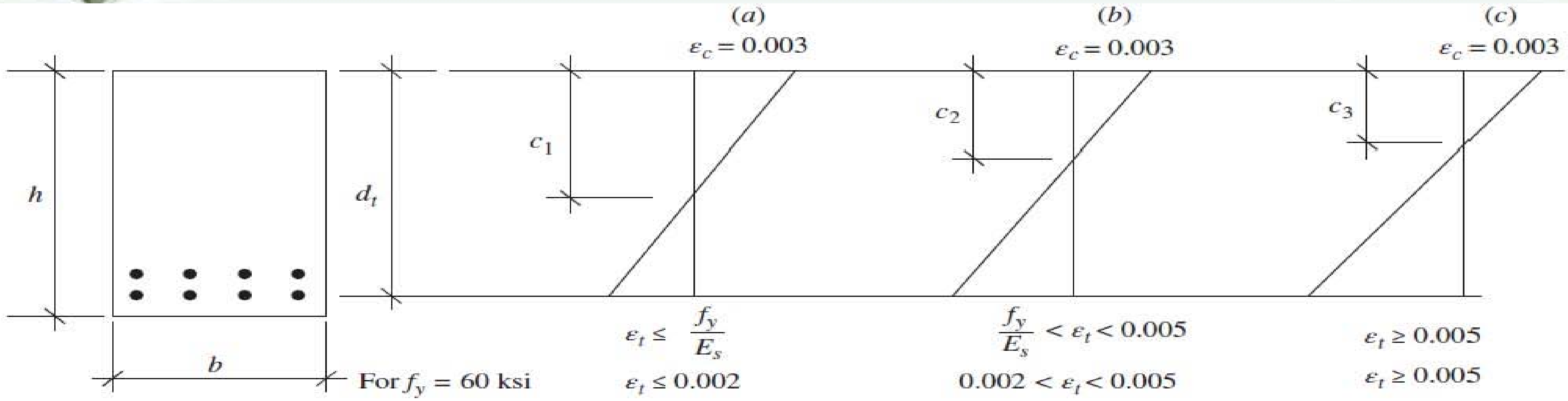
$$C = T$$

2 .The internal nominal bending moment, Mn, is equal to either the compressive force, C, multiplied by its arm or the tension force, T, multiplied by the same arm:

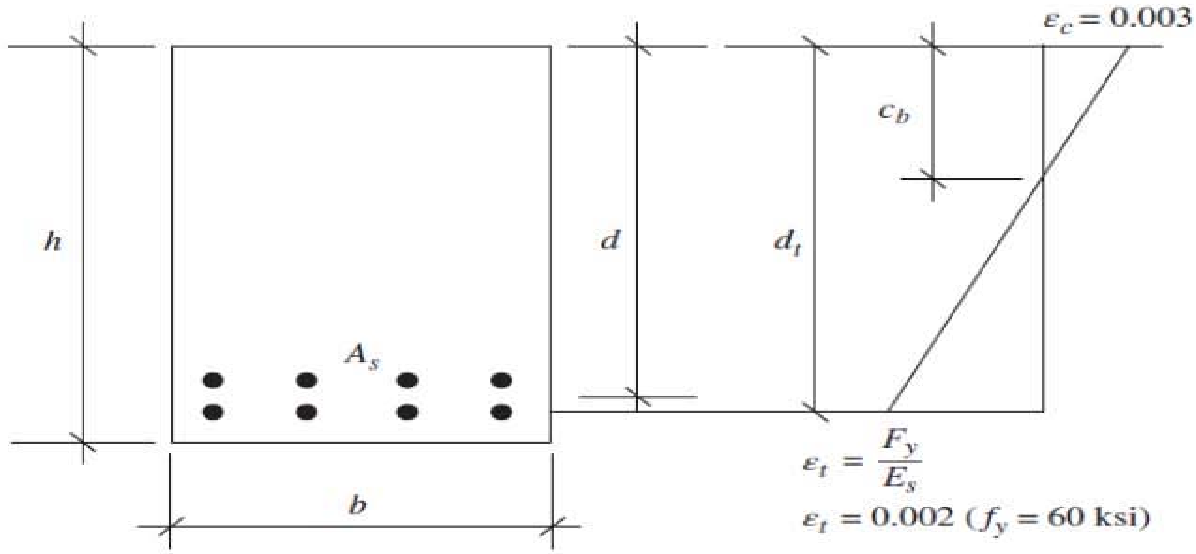
$$M_n = C(d - z) = T(d - z)$$

(Mu = ϕMn after applying a reduction factor ϕ)

The use of these equations can be explained by considering the case of a rectangular section with tension reinforcement. The section may be balanced, under reinforced, or over reinforced, depending on the percentage of steel reinforcement used.



Strain limit distribution, $c_1 > c_2 > c_3$: (*a*) compression-controlled section, (*b*) transition region, and (*c*) tension-controlled section.



d. Balanced strain section (occurs at first yield or at distance d_t).

Balanced Section

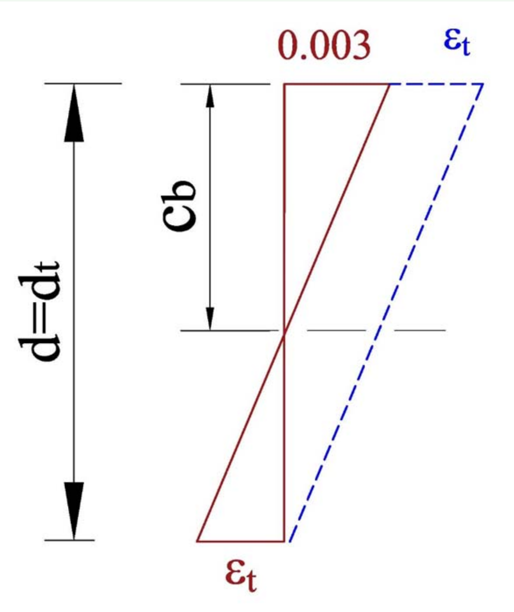
Let us consider the case of a balanced section, which implies that at maximum load the strain in concrete equals 0.003 and that of steel equals the first yield stress at distance dt divided by the modulus of elasticity of steel, fy/Es. This case is explained by the following steps.

Step 1. From the strain diagram

$$\frac{\varepsilon_t}{d_t - C_b} = \frac{0.003}{C_b} \qquad or \qquad \frac{C_b}{d_t - C_b} = \frac{0.003}{\frac{f_y}{E_s}}$$

From triangular relationships (where C_b is c for a balanced section) and by adding the numerator to the denominator,

$$\frac{C_b}{d_t} = \frac{0.003}{0.003 + \frac{f_y}{E_s}}$$

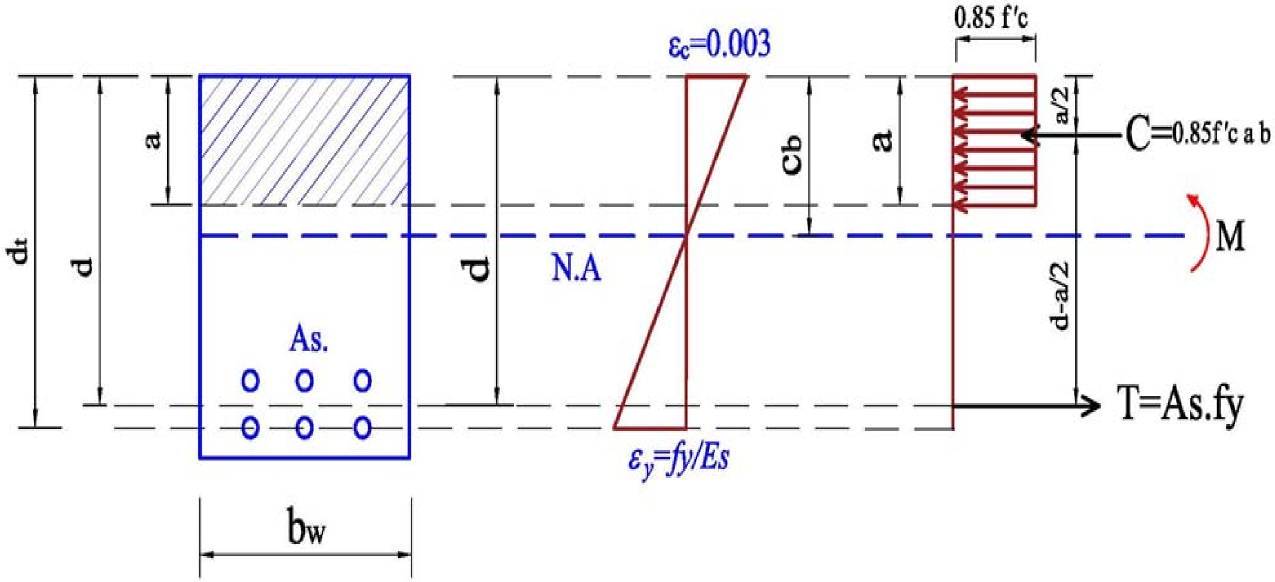


If $Es = 200000\ Mpa$
Then :

$$C_b = \left(\frac{600}{600 + fy}\right) dt \ldots\ldots (1)$$

$C = T \rightarrow 0.85\ f_c'\ b\ a = As\ fy$

$$a = \frac{As\ fy}{0.85 f_c'\ b} \quad \ldots\ldots (2)$$



β_1 C= a effective depth
While β_1= 0.85 when fc' ≤ 28 MPa

$$\rho_b = \frac{A_s\ balanced}{bd} = \frac{As_b}{bd}$$

or $As_b = \rho_b\, b\, d \ldots\ldots (3)$
Substitute in eq. (2)

$$0.85 f_c'\ b\ a = fy\ \rho_b \times b \times d$$

$$\rho_b = \frac{0.85 f_c'\ \ a}{fy \times\ d} = \frac{0.85 f_c'\ \ (\beta_1 c)}{fy \times\ d}$$

Cb from equation (1) then

$$\rho_b = \frac{0.85\ f_c' \beta_1}{fy} \left(\frac{600}{\mathbf{600 + fy}}\right) \left(\frac{\mathbf{dt}}{\mathbf{d}}\right) \ldots \ldots (4)$$

While the nominal Moment $Mn = C(d - z) = T(d - z)$ $\left(where \quad z = \frac{a}{2}\right)$

$$\therefore a = \frac{As\, fy}{0.85\, f_c'\, b}$$

$$M_n = C\left(d - \frac{a}{2}\right) = T\left(d - \frac{a}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Or:

$$M_n = A_s\, fy\left(d - \frac{a}{2}\right)$$

To get the usable design moment ϕMn, the previously calculated Mn must be reduced by the capacity reduction factor:

$$\emptyset Mn = \emptyset\, A_s fy\left(d - \frac{A_s fy}{1.7\, f_c' b}\right)$$

while $\rho_b = \frac{As}{bd}$ or $As = \rho_b bd$ then :

$$\emptyset Mn = \emptyset fy\, \rho\, bd\left(d - \frac{\rho\, bd\, fy}{1.7\, f_c' b}\right)$$

$$\emptyset Mn = \emptyset fy\, \rho\, bd^2\left(1 - \frac{\rho fy}{1.7\, f_c'}\right) \ldots\ldots (6)$$

Or $\emptyset Mn = R_u\, bd^2$

$$R_u = \emptyset \rho fy\left(1 - \frac{\rho fy}{1.7\, f_c'}\right), \qquad a = \frac{As\, fy}{0.85\, f_c'\, b} \quad \text{lead to} \quad \frac{a}{d} = \frac{\rho fy}{0.85\, f_c'} \ldots\ldots\ldots\ldots (7)$$

For more simlified, *let* $m = \dfrac{fy}{0.85\, fc'}$ then :

$$R_u = \emptyset \rho f y \left(1 - \frac{1}{2}\, \rho\, m \right) \ldots\ldots..(8)$$

Then :

$$\rho_b = \frac{0.85\, f_c' \beta_1}{fy} \left(\frac{600}{600 + fy} \right) \left(\frac{dt}{d} \right)$$

$$\rho_{\mathrm{b}} = \frac{\beta_1}{m} \left(\frac{600}{600 + fy} \right) \left(\frac{dt}{d} \right) \ldots\ldots.(9)$$

For one steel layer $\left(\frac{d}{dt} \right) = 1$

Upper Limit of Steel Percentage

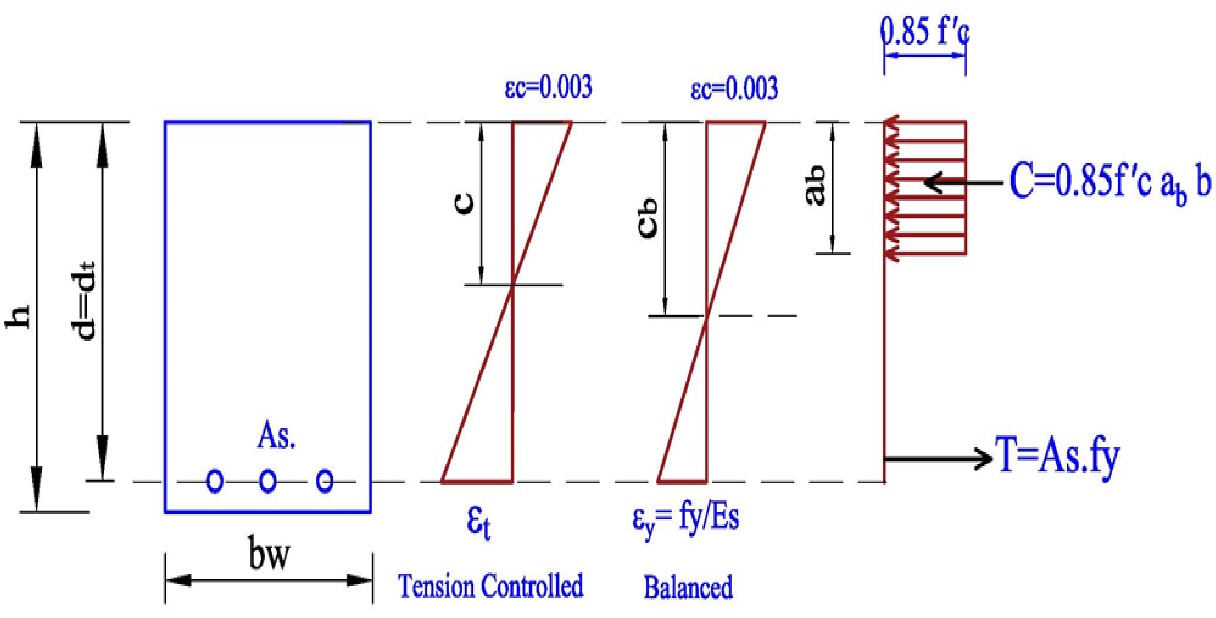
The upper limit ort he maximum steel percentage ρmax, that can be used in a singly reinforced concrete section in bending is based on then net tensile strain in the tension steel, the balanced steel ratio, and the grade of steel used. The relationship between the steel percentage ρ in the section and the net tensile strain ε_t ,is as follows:

$$\varepsilon_t = \left(\frac{0.003 + \frac{fy}{E_s}}{\frac{\rho}{\rho b}}\right) - 0.003$$

$$For\ fy\ = 420\ MPa\ and \frac{fy}{E_s} = 0.002\ then$$

$$\varepsilon t = \left(\frac{0.005}{\frac{\rho}{\rho_b}}\right) - 0.003$$

These expressions are obtained by referring to Figure shown. For a balanced section,



$$C_b = \frac{a_b}{\beta_1} = \frac{As_b.fy}{0.85\ f_c'\ b\ \beta_1} = \frac{\rho_b\ fy\ d}{0.85\ f_c'\ \beta_1}$$

Similarly for any steel Ratio ρ:

$$c = \frac{\rho\ fy\ d}{0.85\ f_c'\ \beta 1} \quad and \quad \frac{c}{c_b} = \frac{\rho}{\rho_b}$$

Divide both sides by d to get:

$$\frac{c}{d} = \frac{\rho}{\rho_b} \times \frac{c_b}{d} \ldots\ldots\ldots\ldots..(10)$$

From the triangles of the strain diagrams,

$$\frac{c}{d} = \frac{0.003}{0.003 + \ \varepsilon_t}$$

$$\varepsilon_t = \left(\frac{0.003}{\frac{c}{d}}\right) - 0.003 \ldots\ldots\ldots.(11)$$

Similarly:

$$\frac{c_b}{d} = \frac{0.003}{0.003 + fy/Es} \ldots\ldots\ldots\ldots..(12)$$

Substitute in eq. (10)

$$\frac{c}{d} = \left(\frac{\rho}{\rho_b}\right)\left(\frac{c_b}{d}\right) = \left(\frac{\rho}{\rho_b}\right)\left(\frac{0.003}{0.003 + \frac{fy}{E_s}}\right) \ldots\ldots\ldots\ldots..(13)$$

Substitute in eq. (11)

$$\varepsilon_t = \frac{0.003}{c/d} - 0.003 = \left(\frac{0.003 + \frac{fy}{Es}}{\frac{\rho}{\rho_b}}\right) - 0.003$$

$$\frac{\rho}{\rho_b} = \frac{0.003 + \frac{fy}{Es}}{0.003 + \varepsilon_t} \quad \ldots\ldots..(14)$$

$$\rho = \left(\frac{0.003 + \frac{fy}{Es}}{0.003 + \varepsilon_t}\right)\rho_b$$

$For\ fy = 420\ , Es = \ 200\ GPa, \qquad fy/E_s = \ 0.002$

$$\frac{\rho}{\rho_b} = \frac{0.0051}{0.003 + \varepsilon_t} \ldots\ldots$$

$The\ limit\ for\ tension\ to\ control\ is\ \varepsilon t \geq \ 0.005\ according\ to\ ACI. For\ \ \varepsilon_t = 0.005, becomes:$

$$\frac{\rho}{\rho_b} = \frac{0.0051}{0.008} = \frac{5.1}{8} = 0.6375$$

$\rho \leq 0.6375\ \rho_b$ Tension Control

For design purpose ε_t=0.005 and :

$\rho \leq \rho_{max}\ and\ \boldsymbol{\phi}\ = 0.9$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{E_s}}{0.008}\right)\rho_b \ldots..(15)$$

$\boldsymbol{Subistitute}\ \rho_b(\ \boldsymbol{Eq.9})\ \boldsymbol{gives}$:

$$\rho_{max} = \frac{3}{8}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right) \ldots\ldots\ldots..(16)$$

When $\rho > \rho_{max}$ section will be in transition state
then $\boldsymbol{\phi}$ will be between 0.65 and 0.9

Previously :

$$\boldsymbol{\phi} Mn = Ru\, bd^{2} \quad or \quad Mn = R_n\, bd^{2}$$

$$R_u = \emptyset \rho f y\left(1 - \frac{\rho f y}{1.7\, f_c'}\right)$$

$$m = \frac{f y}{0.85\, f_c'}$$

$$Then\ :\ R_u = \emptyset\, \rho f y\left(1 - \frac{1}{2}\rho\, m\right)$$

For one steel layer ($\frac{d}{d_t}$) =1, fy = 420 MPa, f_c'=28 MPa , ….. And m= 17.65

$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600+fy}\right)\left(\frac{dt}{d}\right)$$

$$\rho_b = \frac{0.85}{17.65}\left(\frac{600}{600+420}\right)(1) = 0.0283$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{E_s}}{0.008}\right)\rho_b \ldots\ldots\ldots\ldots\ldots\ldots..(15)$$

$$= \left(\frac{0.003 + \frac{420}{200000}}{0.008}\right)\rho_b = 0.6375\rho_b = 0.6375 \times 0.0283 = 0.01806$$

$$Ru_{\ max} = \ \emptyset\rho_{max} fy\left(1 - \frac{1}{2}\,\rho_{max}\,m\right)$$

$$Ru_{\ max} = 0.9\ (0.01806) \times 420\ \left(1 - \frac{1}{2}\ (0.01806 \times 17.65)\right)$$

$$Ru_{\ max} = 5.74$$

That's mean when $\rho > \rho_{max}$ $\varepsilon_t < 0.005$
and ACI cod 9.3.3.1 limited that should be not less than 0.004 in transition region

To keep enough ductility for beam when $\varepsilon_t = 0.004$

$$\frac{\rho}{\rho_b} = \frac{0.003 + \frac{fy}{Es}}{0.003 + \varepsilon_t}$$

$$\frac{\rho}{\rho_b} = \frac{0.003 + 0.0021}{0.003 + 0.004}$$

Then $\rho_{max\,t,} = 0.729\rho_b$

And ∅ calculated from

$\emptyset\ t = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{30}\right)$ and when ε_t=0.004

$0.817 < \emptyset \leq 0.9$ and

$0.004 < \varepsilon_t \leq 0.005$

ρ_b= 0.0283 , fy = 420 MPa, f_c'=28 MPa , ….. and m= 17.65

$\rho_{max_, t}$= 0.729 *ρ_b= 0.0206

$$Ru_{\ max_, t} = \ \emptyset\rho_{max_, t} fy \left(1 - \frac{1}{2} \rho_{\ max_, t} m \right)$$

$$Rn_{\ max_, t} = 0.0206 \ \times 420\ (\ 1 - 0.5 \ \times 0.0206 \ \times 17.65) = 7.08$$

$$Ru_{\ max_, t} = 0.812 \ \times 7.08 = 5.75$$

This value is very close from $Ru_{\ max}$, so increase the steel over the max ratio at the transition region does not increased effectively section capacity so its preferable to add steel at compression zone instead of over the $\rho_{max_, t}$

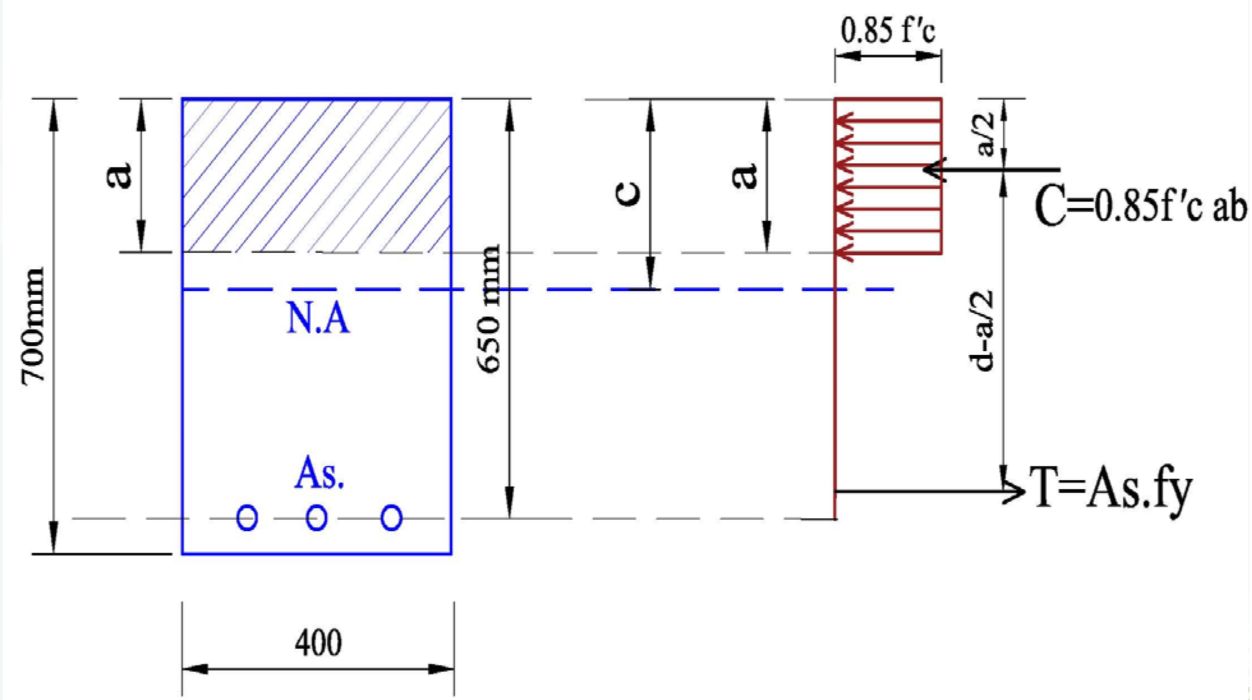
Example (1) : For the section shown below , calculate :

a- The balanced steel ratio.

b- The maximum reinforcement area allowed by ACI Code for a tension – controlled section and transition region.

c- The position of the Neutral axis and the depth of the equivalent compressive stress block for the tension – controlled section in b.

Given : $f_c' = 28$ MPa, $fy = 420$ MPa,



Solution

$$\rho_b = \frac{\boldsymbol{\beta}_1}{m}\left(\frac{600}{600+fy}\right)\left(\frac{dt}{d}\right)$$

$\boldsymbol{\beta}_1 = 0.85$ for $f_c' \leq 28$ Mpa

$$d_t = d \longrightarrow \frac{dt}{d} = 1$$

$$m = \frac{fy}{0.85\, f_c'} = \frac{420}{0.85 \times 28} = 17.65$$

$$\rho_b = \frac{0.85}{17.65}\left(\frac{600}{600+420}\right)(1) = 0.0283$$

$$A_{sb} = \rho_b \times b \times d = 0.0283 \times 650 \times 400 = 7358\, mm^2$$

b) $\varepsilon_t = 0.005$ for tension control

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{E_s}}{0.008}\right)\rho_b = \left(\frac{0.003 + \frac{420}{200000}}{0.008}\right) \times 0.0283 = 0.01804$$

$$As_{\max} = \rho_{\max} \times b \times d = 0.018043 \times 650 \times 400 = 4690.4\ mm^2$$

For Transition region , $\varepsilon_t = 0.004$

$$\rho_{max\,t} = \left(\frac{0.003 + \frac{fy}{E_s}}{0.007}\right)\rho_b = \left(\frac{0.003 + \frac{420}{200000}}{0.007}\right) \times 0.0283 = 0.0219$$

$$As_{max,t} = 0.0219 \times 400 \times 650 = 5694\ mm^2$$

$$\emptyset t = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{3}\right) = 0.65 + (0.004 - 0.002)\left(\frac{250}{3}\right) = 0.817$$

c) Block stress depth *(Tension controlled)*

$$C = T$$

$$0.85\, f_c' \times a_{max} \times b = A_{s\,max}\, fy \quad \ldots.$$

$$a_{max} = \frac{As.fy}{0.85\, f_c'\, b}\,\frac{d}{d} = \rho_{max}\, m.d$$

$$a_{\,max} = 0.01804 \times 17.65 \times 650 = 206.96mm$$

$$or \qquad cmax = \frac{a_{\,max}}{\beta 1} = \frac{206.96}{0.85} = 243.48\ mm$$

Block stress depth at Transition zone

$$a = \frac{As.fy}{0.85\,f_c'\,b}\,\frac{d}{d} = \rho_{max,t}\,m.d$$

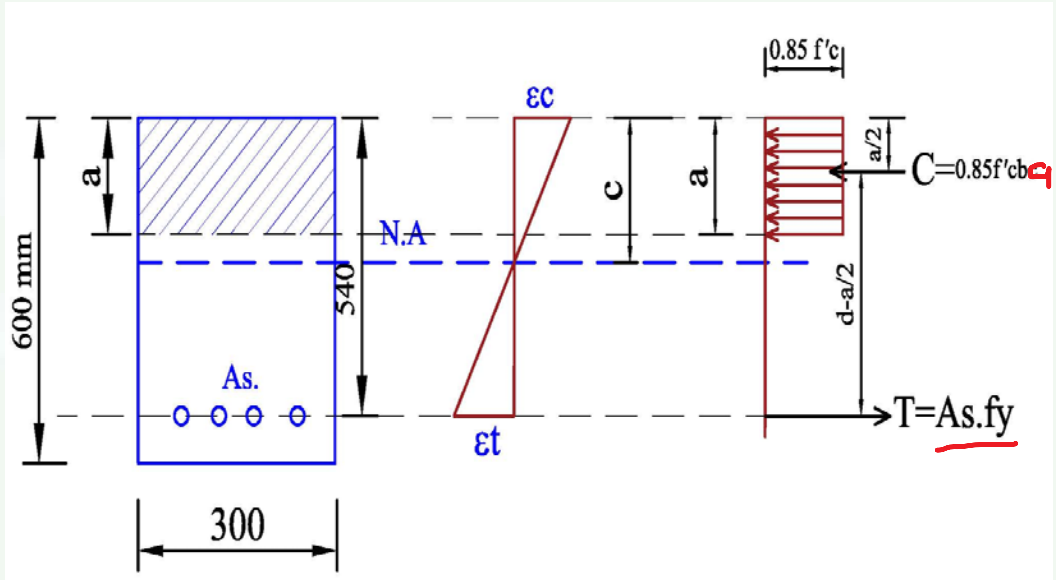
$$a_{max,t} = 0.0219 \times 17.65 \times 650 = 251.25mm$$

Or :

$$c = \frac{a_{max,t}}{\beta_1} = \frac{251.25}{0.85} = 295.6\ mm$$

Example (2) : Determine the design moment strength and the position of the neutral axis of the rectangular section shown below , if the reinforcement used is 4Ø 25 mm , Given : f'c = 28 Mpa, fy = 420 Mpa,

Solution:



$As = 4Ø\,25\,mm = 4 \times 25^2 \times \frac{\pi}{4} = 1960\,mm^2$

$\rho = \frac{As}{bd} = \frac{1960}{300 \times 540} = 0.012098$

$\rho < \rho_{max} = (0.018040\ from\ Exa.1)\quad OK$

Tension Control

$\therefore Ø = 0.9$

$C = T$

$0.85\, f_c' \times a \times b = As\, fy$

$0.85 \times 28\, a \times 300 = 1960 \times 420$

$a = 115.29mm$ Or $a = \rho\, m.d = 0.012098 \times 17.65 \times 540 = 115.29$ mm

$C = \frac{a}{\beta_1} = \frac{115.29}{0.85} = 135.64mm$

($\boldsymbol{\beta}_1 = 0.85$ for $f_c' <= 28$ Mpa)

$dt = d\ (one\ layer)$

$$\varepsilon_t = \frac{d - C}{C} \times \varepsilon_C = \frac{540 - 135.64}{135.64} \times 0.003 = 0.00894 > 0.005 \quad OK$$

Tension failure so ϕ=0.9

$$\emptyset Mn = Mu = \emptyset\, T \left(d - \frac{a}{2}\right) = \emptyset As\, fy \left(d - \frac{a}{2}\right)$$

$$= 0.9 \times 1960 \times 420 \times \left(540 - \frac{115.29}{2}\right) = 357.37 \times 10^6 N.mm = 357.37\ KN.m$$

Example (3) : Repeat Example (2) Using As = $4\emptyset\ 32\ mm$ (H.W)

Lower limit or Minimum Percentage of Steel

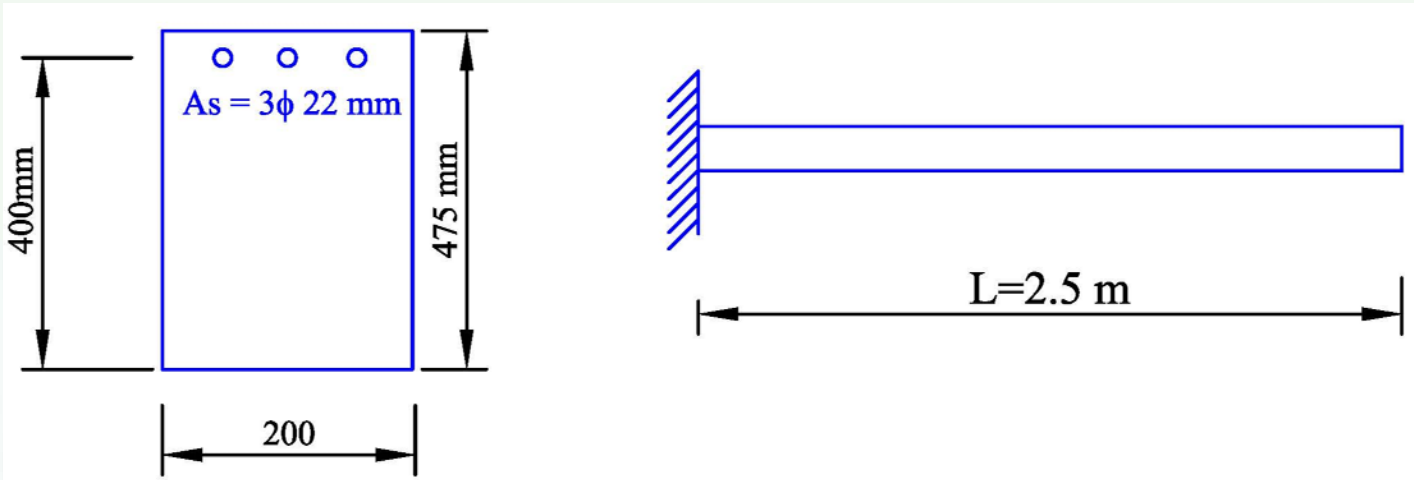
If the factored moment applied on a beam is very small and the dimensions of the section are specified (as is sometimes required architecturally) and are larger than needed to resist the factored moment , the calculation may show that very small or no steel reinforcement is required. In this case, the maximum tensile stress due to bending moment may be equal to or less than the modulus of rupture of concrete *fr*. If no reinforcement is provided, sudden failure will be expected when the first crack occurs, thus giving now warning. The ACI Code, Section 9.6.1, specifies a minimum steel area, As_{min} ,

$$As_{min} = \left(\frac{0.25\sqrt{f_c'}}{fy}\right)\mathrm{bw.d} \geq \left(\frac{1.4}{fy}\right) bw.d \ldots\ldots\ldots\ldots\ldots when\ f_c' = 31\ Mpa$$

$$\rho_{min} = \left\{\left(\frac{1.4}{fy}\right) \ldots\ldots\ldots\ldots\ldots\ldots..For\quad f_c' < 31\ Mpa\right.$$

$$\rho_{min} = \left(\frac{0.25\sqrt{f_c'}}{fy}\right) \ldots\ldots\ldots\ldots.when\ f_c' \geq 31\ Mpa$$

Example (3) : A 2.5 m Span cantilever beam has a rectangular section and reinforced as shown below , The beam carries a dead load , including its self weight of 22 KN/m and a live load of 13 KN/m , using f_c'=28 MPa, fy = 420 MPa. Check if the beam is safe to carry above load.



Solution:

1- External Load

$$Wu = 1.2\,D.L + 1.6\,L.L = 1.2 \times 22 + 1.6 \times 13 = 47.2\,KN/m$$

$$Mu = \frac{Wu\,l^2}{2} = \frac{47.2 \times 2.5^2}{2} = 147.5\,KN.m$$

2- Check ε_t As ϕ22= 380 mm^2

$$a = \frac{As.fy}{0.85\,f_c'\,b} = \frac{3 \times 380 \times 420}{0.850 \times 28 \times 200} = 100.6\,mm$$

$$c = \frac{a}{0.85} = 118.35\,mm$$

$d_t = d = 400$ mm, ϕ=0.9

$$\varepsilon_t = \left(\frac{dt - c}{c}\right)\varepsilon_c$$

$$\varepsilon_t = \left(\frac{400 - 118.35}{118.35}\right) \times 0.003 = 0.00714 > 0.005\ (\varepsilon_t)$$

Or check

$$\rho = \frac{As}{bd} = \frac{3 \times 380}{200 \times 400} = 0.01425 < \rho_{max} = 0.01804$$

3- calculate :

$$\emptyset Mn = \emptyset As\ fy\ \left(d - \frac{a}{2}\right)$$

$$\emptyset Mn\ = 0.9\ \times 3 \times 380 \times 420 \times \left(400 - \frac{100.6}{2}\right) = 150.69 KN.m$$

Other Solution

$$\rho = 0.01425 < \rho_{max} = 0.01804$$

$$m = \frac{fy}{0.85\ f_c'} = \frac{420}{0.85 \times 28} = 17.65$$

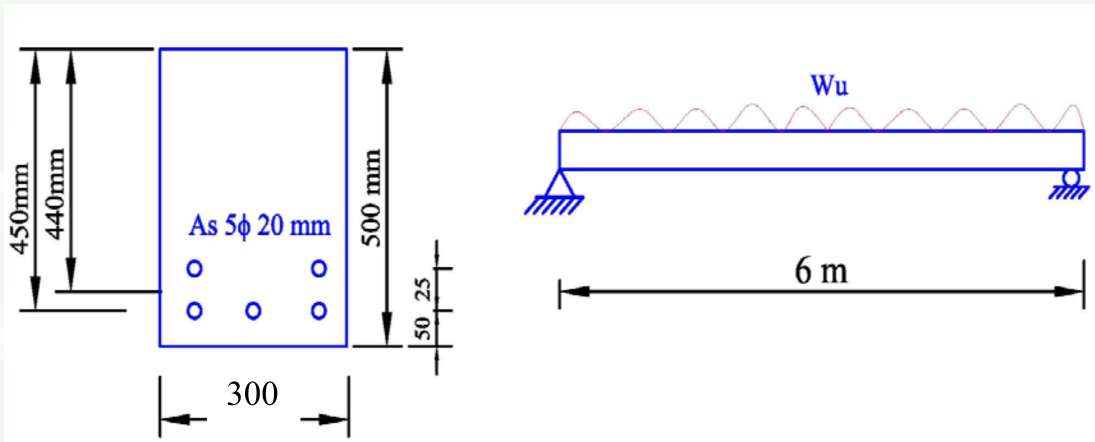
$$R = \rho fy \left(1 - \frac{1}{2}\rho\ m\right)$$

$$= 0.01425 \times 420\ (1 - 0.5 \times 0.01425 \times 17.65) = 5.23\ N/mm^2$$

$$\emptyset Mn = \emptyset \mathrm{R\ b}\ d^2$$

$$= 0.9 \times 5.23 \times 200 \times 400^2 = 150.69\ KN.m$$

Example (4) : A simply supported beam have a span of 6 m . If the cross section is shown below , f_c'=21 MPa, fy = 420 MPa determine the allowable uninform load live load on the beam assuming the dead load is due to self weight of the beam , given b= 300 mm, h= 500 and reinforced with 5Ø 20 mm (1570 $mm2$).



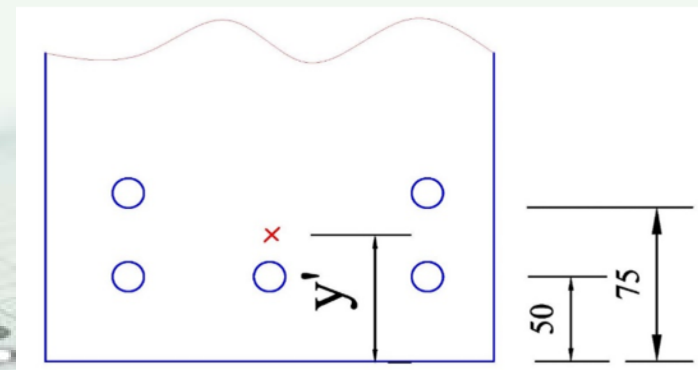
Solution

Find the centroid of steel area

$$y' = \frac{3 \times 50(\,Asb) + 2 \times 75(Asb)}{5 \times (\,Asb)} = 60mm$$

$$d_t = h - 50 = 500 - 50 = 450mm$$

$$d = h - y' = 500 - 60 = 440 \text{ mm}$$



$$\rho_b = \frac{\beta 1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right)$$

$$m = \frac{fy}{0.85\, f_c'} = \frac{420}{0.85 \times 21} = 23.53 \;\;, \beta_1 = 0.85, (\,f_c' < 28\; MPa)$$

$$\rho_b = \frac{0.85}{17.65}\left(\frac{600}{600 + 420}\right)\left(\frac{450}{440}\right) = 0.02173$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.003 + \varepsilon t}\right)\rho_b = 0.6375\, \rho_b = 0.01385$$

$$\rho = \left(\frac{As}{bd}\right) = \left(\frac{5 \times 314}{300 \times 440}\right) = 0.01189 < \rho_{max} \qquad ok \quad (\emptyset = 0.9)$$

$$\rho_{min} = \left(\frac{1.4}{fy}\right) = \left(\frac{1.4}{420}\right) = 0.003 < \rho = 0.01189 \qquad OK$$

$$\rho_{min} < \rho < \rho_{max}$$

$$\emptyset Mn = \emptyset \mathrm{R}\,\mathrm{b}\,d^2$$

$$R = \rho fy\left(1 - \frac{1}{2}\rho\, m\right)$$

$$= 0.01189 \times 420\left(1 - \frac{1}{2} \times 0.01189 \times 23.53\right) = 4.295\ MPa$$

$$\emptyset Mn = 0.9 \times 4.295 \times 300 \times 440^2 = 224.52\ KN.m$$

$$Self\ weight\ of\ beam = 0.3 \times 0.5 \times 1 \times 24 = 3.6\ KN/m$$

$$M_{Dl} = \frac{3.6 \times 6^2}{8} = 16.2\ KN.m$$

$$Mu = 1.2\ MDL + 1.6\ MLL$$

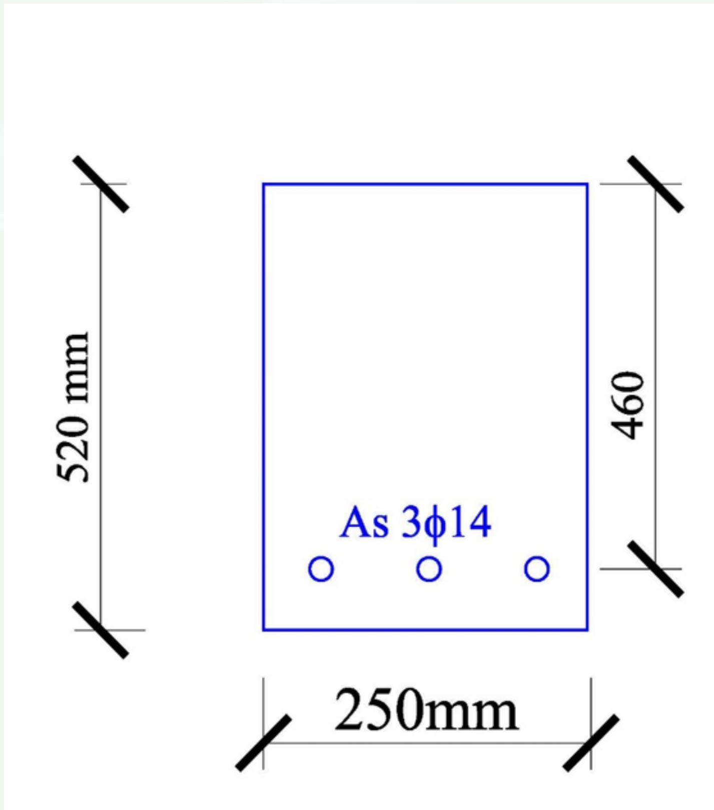
$$224.52 = 1.2 \times 16.2 + 1.6 \times M_{LL} = 128.175$$

$$M_{LL} = 128.175 = \frac{W_l \times 6^2}{8}$$

$$W_{LL} = 28.48\ KN/m$$

H.W

Example (5) : Check the design Adequacy of section below, factored moment Mu= 50 kN.m , using, f_c'=25 MPa, fy = 280 MPa



Example (6) :Determine the design moment strength of section shown below , Given f'c=28 mPa and fy=420 MPa and check the specification of the section according to ACI Code.

Solution:

$$As = 3 \times \pi \frac{25^2}{4} = 1470\, mm^2$$

$$\rho = \frac{As}{effective\ area} = \frac{As}{bd - 150 \times 100}$$

$$= \frac{As}{300 \times 500 - 150 \times 100}$$

$$Effective\ area = \ 300 \times 500 - 100 \times 150 = 135000\, mm^2$$

$$\rho = \frac{1470}{135000} = 0.01089$$

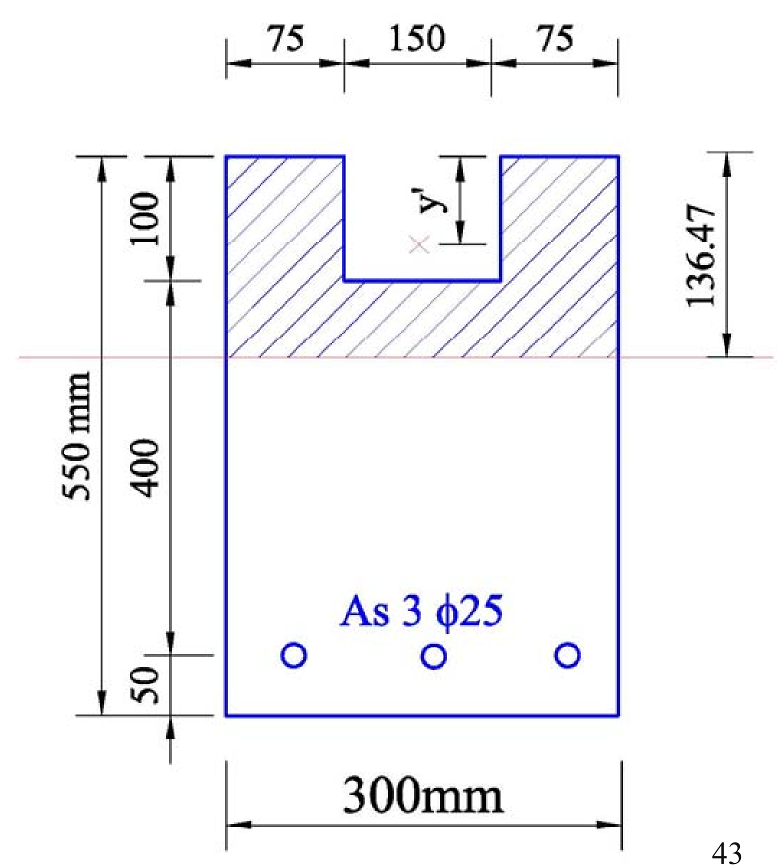
$$\rho b = \frac{\beta 1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right)$$

$dt\ = d$=500 mm

$$m = \frac{fy}{0.85\, f'c} = \frac{420}{0.85\ \times\ 28} = 17.65$$

$$\rho b = \frac{0.85}{17.65}\left(\frac{600}{600 + 420}\right) \times 17.65 = 0.0283$$

$$\rho_{max} = \left(\frac{0.003 + 420/200000}{0.003 + 0.005}\right) \times 0.0283 = 0.018041$$



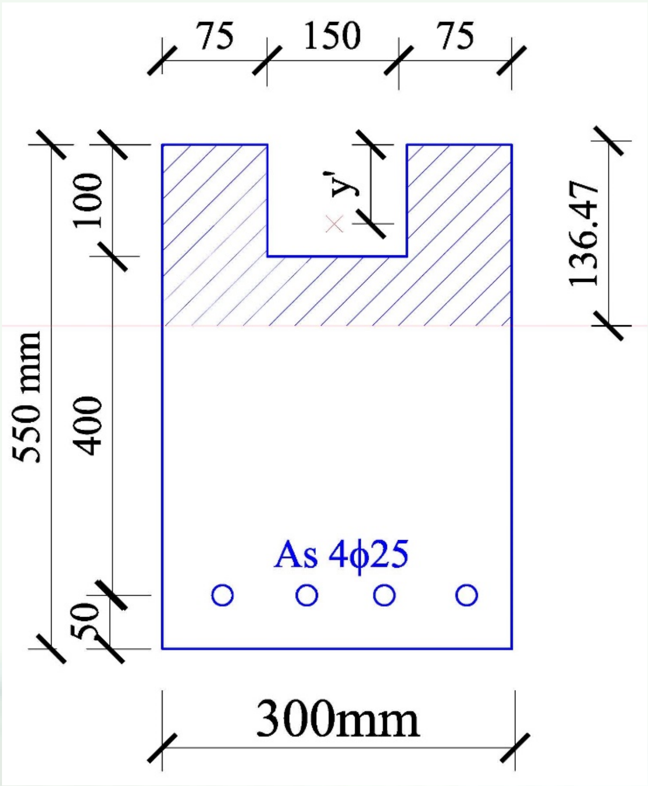
$$\rho_{min} = \left(\frac{1.4}{fy}\right) = \left(\frac{1.4}{420}\right) = 0.00333 \qquad (\,where\ f'c < 31MPa$$

$$\rho_{min} = 0.00333 < \rho = 0.01089 < \rho_{max} = 0.01804$$

$Tension\ Controlled \qquad \emptyset = 0.9$

Assume stress block depth = $a = 100\ mm$

Compression area $A_c = a \times b - 100 \times 150$



$C = T$

$0.85\ f'c\ Ac = As\ fy$

$$A_c = \left(\frac{1470 \times 420}{0.85 \times 28}\right) = 25941\ mm^2$$

$$A_c = a \times b - 100 \times 150 = a \times 300 - 150 \times 100$$

$a = 136.47mm > 100mm$

$$y' = \frac{300 \times 136.47 \times \left(\frac{136.47}{2}\right) - 150 \times 100 \times \left(\frac{100}{2}\right)}{300 \times 136.47 - 150 \times 100} = 78.78\ mm$$

The Moment Arm between C and T is :

$d - y' = 500 - 78.78 = 421.22\ mm$

$\emptyset Mn = \emptyset Asfy\ (\,d - y')$

$= 0.9 \times (1470 \times 420 \times (500 - 78.78) = 234.06\ KN.m$

Reinforced Concrete Design

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Rectangular section with compression reinforcement (Double Reinforced section)

Introduction

In concrete sections proportioned to resist the bending moments resulting from external loading on a structural member, the internal moment is equal to or greater than the external moment, but a concrete section of a given width and effective depth has a minimum capacity when ρ_{max} is used. If the external factored moment is greater than the design moment strength, more compressive and tensile reinforcement must be added.

Compression reinforcement is used when a section is limited to specific dimensions due to architectural reasons, such as a need for limited headroom in multistory buildings. Another advantage of compression reinforcement is that long-time deflection is reduced. A third use of bars in the compression zone is to hold stirrups, which are used to resist shear forces.

Two cases of doubly reinforced concrete sections will be considered, depending on whether compression steel yields or does not yield.

1- When Compression Steel Yields

Internal moment can be divided into two moments, as shown in Fig. below. Let Mu_1 be the moment produced by the concrete compressive force and an equivalent tension force in steel, As_1, acting as a basic section. Then Mu_2 is the additional moment produced by the compressive force in compression steel $A\,s'$ and the tension force in the additional tensile steel, As_2, acting as a steel section.

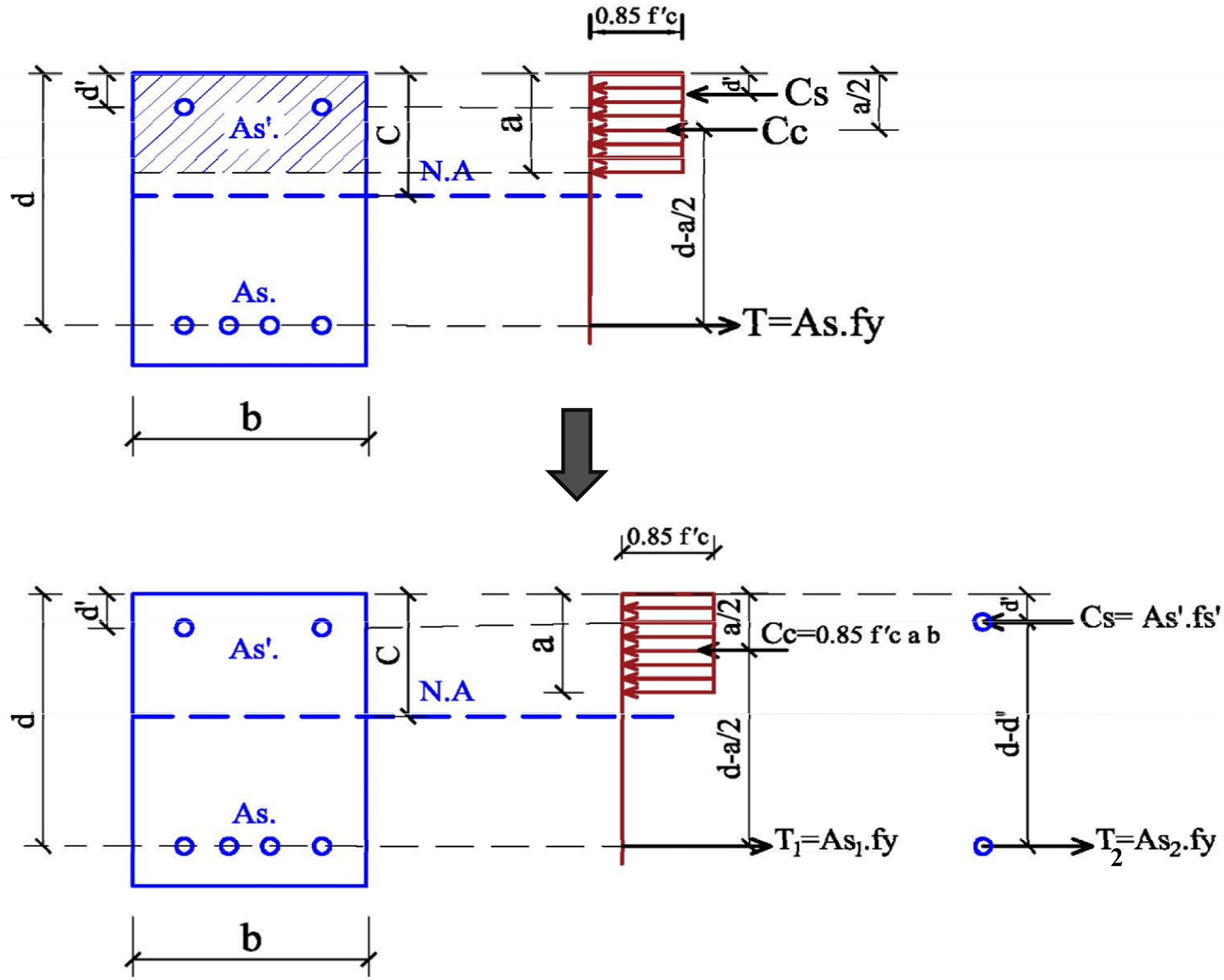
The moment M_{u1} is that of a singly reinforced concrete basic section,

$$T_1 = Cc$$

$$As_1 fy = 0.85\, f_c'\, b\, a$$

$$a = \frac{As_1 fy}{0.85\, f_c'\, b}$$

$$\emptyset Mn = \emptyset As_1 fy \left(d - \frac{a}{2} \right)$$


0.85 f'c
d'
As'.
C
a
Cs
Cc
a/2
N.A
d
d-a/2
As.
T=As.fy
b
0.85 f'c
d'
As'.
C
a
a/2
Cc=0.85 f'c a b
Cs= As'.fs'
N.A
d
d-a/2
d-d''
As.
T1=As1.fy
T2=As2.fy
b

$\emptyset M_1 = \emptyset As_1 fy\left(d - \frac{a}{2}\right)$

$\emptyset M_2 = \emptyset As_2 fy(d - d')$

For $\emptyset M_1$: −

$\rho_1 = \frac{As_1}{bd}$ *less or equal* $\rho_{\max}$ *for singl reinforcement section under tension control*

And

$fs' = fy$ then

$\emptyset M_2 = \emptyset As_2 fy(d - d')$

Or :

T_2=Cs

$As_2 . fy = As' fy \longrightarrow As' = As_2$

$\emptyset Mn = \emptyset Mn_1 + \emptyset Mn_2$

$As = As_1 + As_2 \longrightarrow As_1 = As - As'$

$$a = \frac{As_1 fy}{0.85 f'c\, b} = \frac{(As - As')fy}{0.85 f'c\, b}$$

$$\emptyset Mn = \emptyset\left[(As - As') \times fy\left(d - \frac{a}{2}\right) + As' fy(d - d')\right]$$

$$\rho_1 = \rho - \rho' \leq \rho_{max} = \left(\frac{0.003 + fy/Es}{0.003 + \varepsilon_t}\right)\rho_b$$

and when $\rho_1 = \rho - \rho' \le \rho_{max\ t}$ *then the failure case will be at transiton region*
And $\emptyset$ will be less than 0.9 for M_{u1} and $\emptyset = 0.9\ for\ M_{u2}$, so:

$$\emptyset Mn = \left[\emptyset(As - As') \times fy\left(d - \frac{a}{2}\right) + 0.9As'fy(d - d')\right]$$

Noted that: $(As - As') \le \rho_{max\ t}\, b\, d$
In the compression zone, the force in the compression steel is $Cs = A's(fy - 0.85f'c)$,taking into account the area of concrete displaced by A′s. In this case,

$T = C$
$As\ fy = C_c + C_s$
$As\ fy = 0.85\ f_c'\ a\ b + As'(fy - 0.85\ f_c')$
$As\ fy - As'fy + 0.85\ f_c' As' = 0.85\ f_c' a\ b$ $\quad$ *where* $C_c = As_1 fy = 0.85\ f_c' a\ b$ *(for the basic section)*

Divided by (b d) fy :

$$\rho - \rho'\left(1 - 0.85\frac{f_c'}{fy}\right) = \rho_1 \qquad where:\ \rho_1 \le \left(\frac{As_1}{b\ d}\right)$$

Therefore,

$$\rho_1 = \rho - \rho'\left(1 - 0.85\frac{f_c'}{fy}\right) \le \rho_{max} = \left(\frac{0.003 + fy/Es}{0.008}\right)\rho_b$$

This Eq. is more accurate than previous Eq.it is quite practical to use both equations to check the condition for maximum steel ratio in rectangular sections when compression steel yields.

The maximum total tensile steel ratio, ρ ,that can be used in a rectangular section when compression steel yields is as follows:

$$Max\ \rho \ = \ \rho_{\max} + \rho'$$

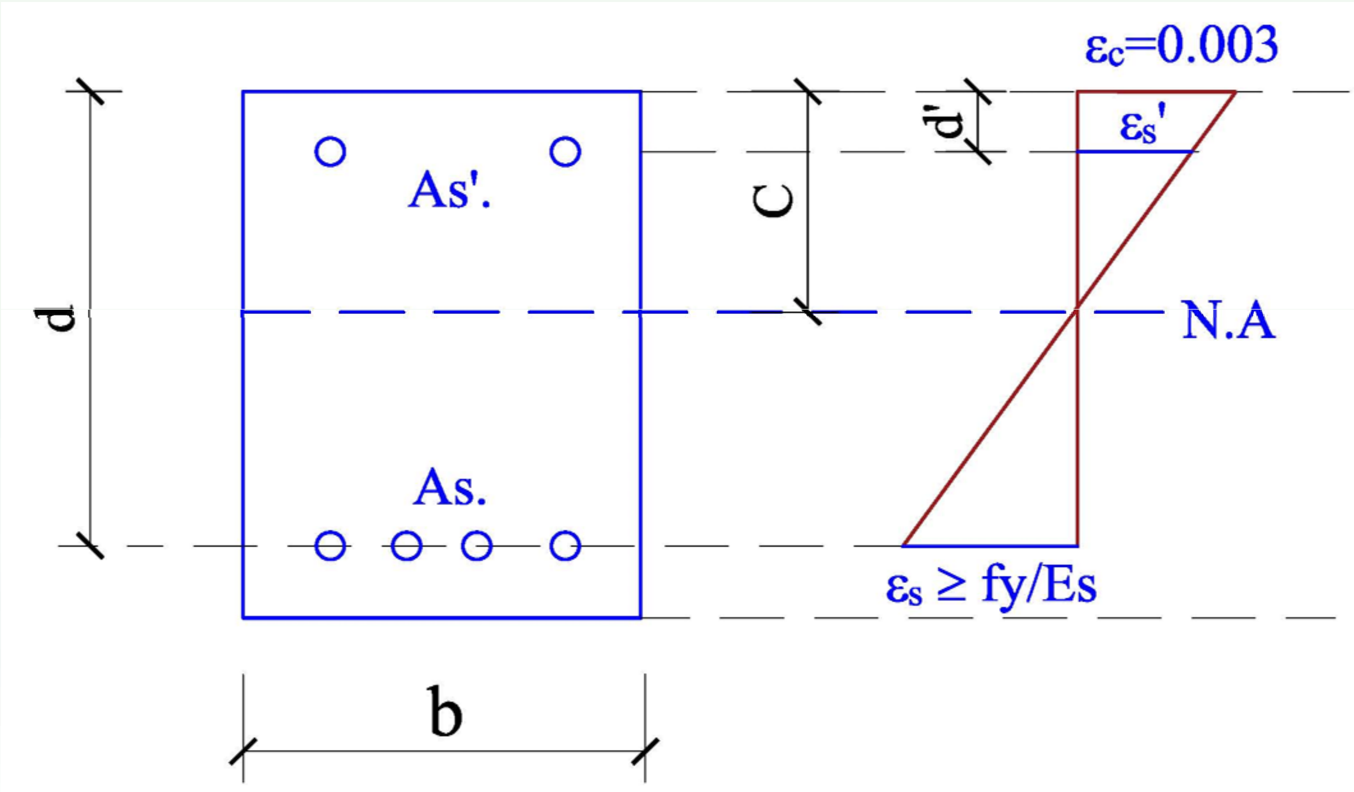
where $\rho\ max$ is maximum tensile steel ratio for the basic singly reinforced tension controlled concrete section. This means that maximum total tensile steel area that can be used in a rectangular section when compression steel yield is as follows:

$$Max\ As = b\ d\ (\rho_{\max} + \ \rho')$$

In the preceding equations, it is assumed that compression steel yields. To investigate this condition, refer to the strain diagram in Fig. Below. If compression steel yields, then :

$$\varepsilon_s' \geq \varepsilon_y = \frac{fy}{E_s}$$

$$\frac{c}{d'} = \frac{0.003}{0.003 - \frac{fy}{Es}} = \frac{600}{600 - fy} \quad \Rightarrow \quad C = \left(\frac{600}{600 - fy}\right) d'$$



$$as\ known:$$
$$As_1\,fy\ = 0.85\ f'c\ a\ b$$
$$As_1 = As - As'\ \ and\ \ \rho_1 = \rho - \rho'$$

$(As - As')fy = 0.85\, f_c'\, a\, b \qquad devided\ by\ (bd)$

$(\rho - \rho')fy = 0.85\, f_c'\, a\, b$

$$\rho - \rho' = 0.85\,\frac{f_c'}{fy}\left(\frac{a}{d}\right)$$

$$a = \beta_1 C = \beta_1\left(\frac{600}{600 - fy}\right)d'$$

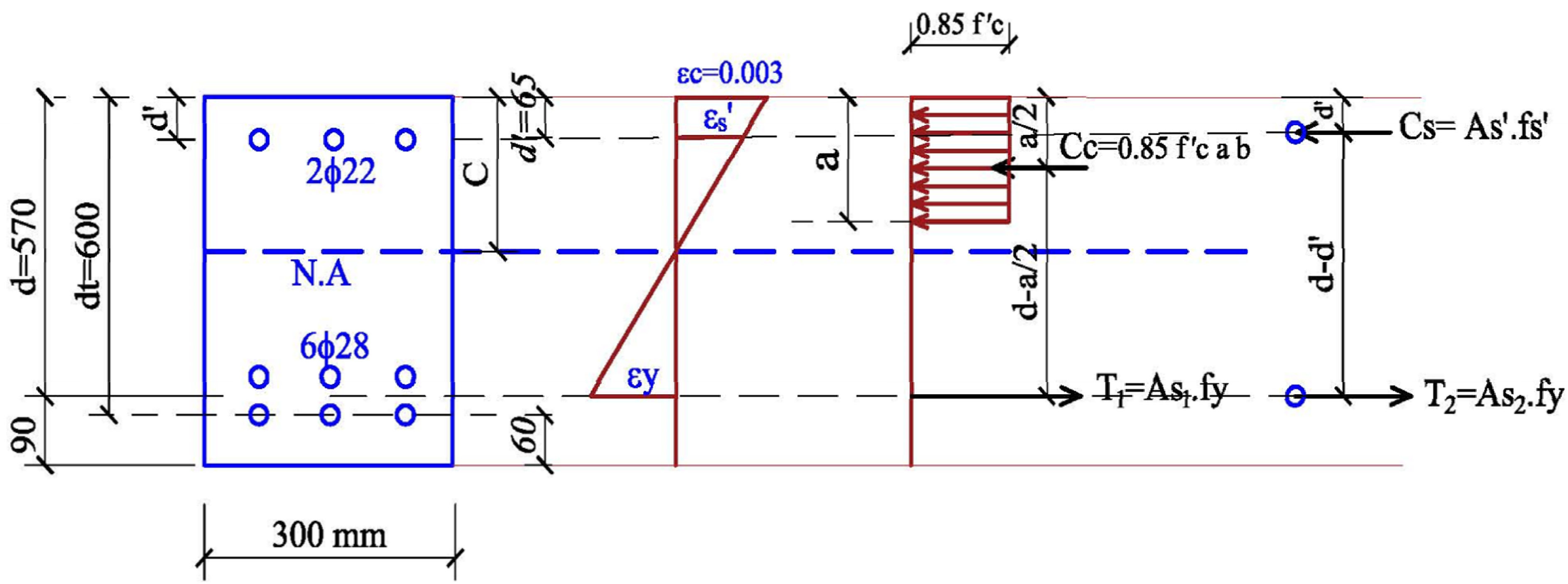
$$\rho - \rho' = 0.85\beta_1\left(\frac{f_c'}{fy}\right)\left(\frac{d}{d'}\right)\left(\frac{600}{600 - fy}\right)$$

$$\rho - \rho' \geq \frac{\beta_1\, d'}{m\, d}\left(\frac{600}{600 - fy}\right)$$

where :

$\rho - \rho'$ is the steel ratio for the single reinforced basic section $= \frac{As_1}{bd} = \frac{(As-As')}{bd}$

Example (7) : A rectangular beam section have a width of 300 mm, and an effective depth $d = 570$ mm to centroid of tension steel . Tension steel consist of $6\,\emptyset\,28\,mm$ in two layers. Compression reinforcement consist of $2\emptyset\,22\,mm$, and $d' = 50$ mm as shown below . Calculate the design moment strength of the beam , Given f_c'=28 MPa and $fy = 420\,MPa$.



Solution : Check if the compression steel yields :

$$\rho = \left(\frac{As}{bd}\right) = \left(\frac{6 \times 28^2 \times \frac{\pi}{4}}{300 \times 570}\right) = 0.02158$$

$$\rho' = \left(\frac{As'}{bd}\right) = \left(\frac{2 \times 22^2 \times \frac{\pi}{4}}{300 \times 570}\right) = 0.00444$$

1- *Check*

$$\rho - \rho' \geq \left(\frac{\beta 1 d'}{m\, d}\right)\left(\frac{600}{600 - fy}\right)$$

$$\beta_1 = 0.85\,, \qquad \text{m} = \frac{fy}{0.85\, f_c'} = \frac{420}{0.85 \times 28} = 17.65$$

$$0.01714 \geq \left(\frac{0.85 \times 50}{17.65 \times 570}\right)\left(\frac{600}{600 - 420}\right) = 0.01408$$

$Then:\ fs' = fy \qquad (O.K.)$

$2-\ Check\quad \rho - \rho' \leq \rho_{max}$

$$\rho_b = \frac{\beta 1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right) = \frac{0.85}{17.65}\left(\frac{600}{600 + 420}\right)\left(\frac{600}{570}\right) = 0.0298 \text{ or } \rho_{max} = \left(\frac{3\beta 1}{8\, m}\right)\left(\frac{dt}{d}\right)$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.003 + \varepsilon t}\right)\rho_b = \left(\frac{0.003 + \frac{420}{200000}}{0.003 + 0.005}\right) = 0.6375\rho_b = 0.019 \text{ or } \rho_{max} = \left(\frac{3\beta 1}{8\, m}\right)\left(\frac{dt}{d}\right) = 0.019$$

$\rho - \rho' = 0.01714 \leq \rho_{max}$=0.0190

Tension Controlled -Section so: $\phi = 0.9$

3- Calculate ϕ Mn

$$\emptyset Mn = \emptyset\left[(As - As') \times fy\left(d - \frac{a}{2}\right) + As' fy(d - d')\right]$$

$$a = \frac{As_1 fy}{0.85 f'c\, b} = \frac{(As - As')fy}{0.85 f'c\, b} = \frac{(3690 - 760) \times 420}{0.85 \times 28 \times 300} = 172.35\, mm$$

$$\emptyset Mn = 0.9\left[(3690 - 760) \times 420 \times \left(570 - \frac{172.3}{2}\right) + 760 \times 420 \times (570 - 50)\right] = 685.3\, kN.m$$

4- Another way to check the yield in compression steel

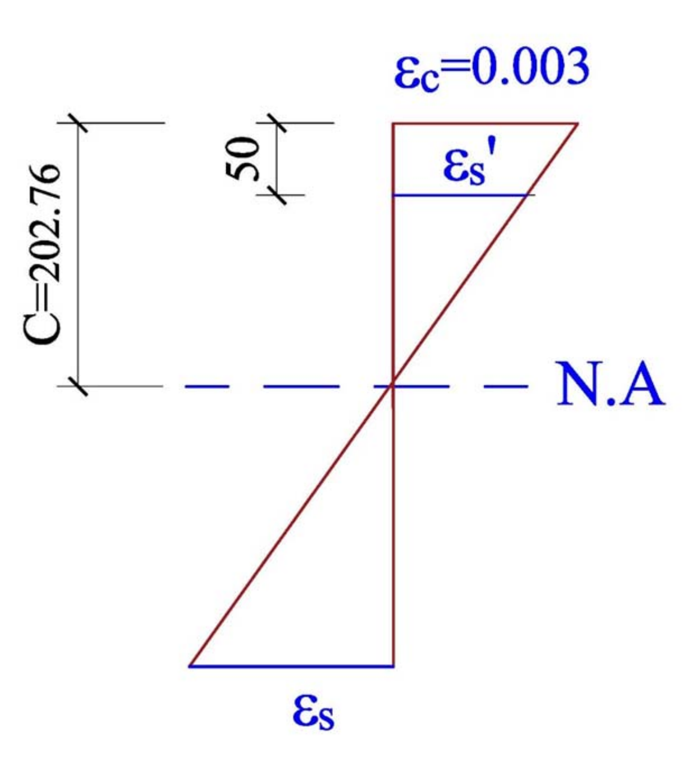
$$c = \frac{a}{\beta_1} = \frac{172.35}{0.85} = 202.76\, mm$$

$$\frac{\varepsilon s'}{\varepsilon c} = \frac{c - d'}{c}$$

$$\varepsilon s' = \frac{202.76 - 50}{202.76} \times 0.003 = 0.00226 > \varepsilon_y = 0.002$$

5- Check ε_t

$$\varepsilon_t = \left(\frac{d_t - c}{c}\right) \times \varepsilon_c = \frac{600 - 202.76}{202.76} \times 0.003 = 0.005877 > 0.005$$



6- Check The Maximum Tension steel Area for this section :

$$Max\ As = (\rho_{max} + \rho')\, bd = (0.0190 + 0.00444) \times 300 \times 570 = 4008\, mm^2$$

Reinforced Concrete Design

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Rectangular section with compression reinforcement (Double Reinforced section-II)

Steel Compression Dose Not Yield ($f'_s < fy$)

As was explained earlier, if the formula not checked,

$$\rho - \rho' \geq \left(\frac{\beta_1 d'}{m\ d}\right)\left(\frac{600}{600 - fy}\right) \dots\dots\dots\dots$$

Then compression steel does not yield. This indicates that if $\rho - \rho'$ is greater than the value of the right-hand side in above eq., So the solution can be done depend on static analysis . The stress in compression steel can be calculated in two method :

1- From Internal Forces Balance

2- direct method

3- indirect method (Iterative method)

2- direct method

$$A\,a^2 - Ba - C = 0$$

$$A = 1,$$

$$B = m\ d\left(\rho - \frac{600}{fy}\rho'\right)$$

$$C = \frac{600}{fy}\beta_1\, m\ d\ d'\rho'$$

$$a = \frac{1}{2}\left[B + \sqrt{B^2 + 4\ A\ C}\right] \qquad \text{and} \qquad C = \frac{a}{\beta_1}$$

Then Stress can be calculated :

$$fs' = 600\left(\frac{c-d'}{c}\right) \le fy$$

2- indirect method (Iterative method)

$claculate$ (a) $value$ for double reinforced section (DRRS):

$$a = \frac{As\,fy - As'\,fs'}{0.85\,f'c\,b} \qquad assume \qquad fs' = fy$$

$find$ a and c= a/β_1

$f'si = 600\left(\frac{c-d'}{c}\right) \le fy$, Compare this value $f'si$ with first one ($f's$)

If its not same then re-calculate (a) using $f'si$ and continue until obtain approximately equal $f's$ in last two step. After obtain $f's$ then can calculate the Cs and Cc

Cc= $As\,fy - A's\,fs'$ $\qquad where:\quad Cs = A's\,fs'$

$$\emptyset Mn = \emptyset\left[Cc\left(d-\frac{a}{2}\right) + Cs(\,d-d')\right]$$

$When\ f's < fy$
$then\ the\ maximum\ steel\ area\ in\ tension\ zone\ for\ rectangular\ section\ can\ be\ found$

$$Max\ As = \left(\rho_{max}\, b\, d + A's\frac{fs'}{fy}\right)$$

$$= \left(\rho_{max} + \rho'\frac{fs'}{fy}\right) bd$$

$$Max\ \rho = \frac{\max As}{bd} \le (\rho_{max} + \rho'\frac{fs'}{fy})$$

$$(\rho - \rho'\frac{fs'}{fy}) \le \rho_{max}$$

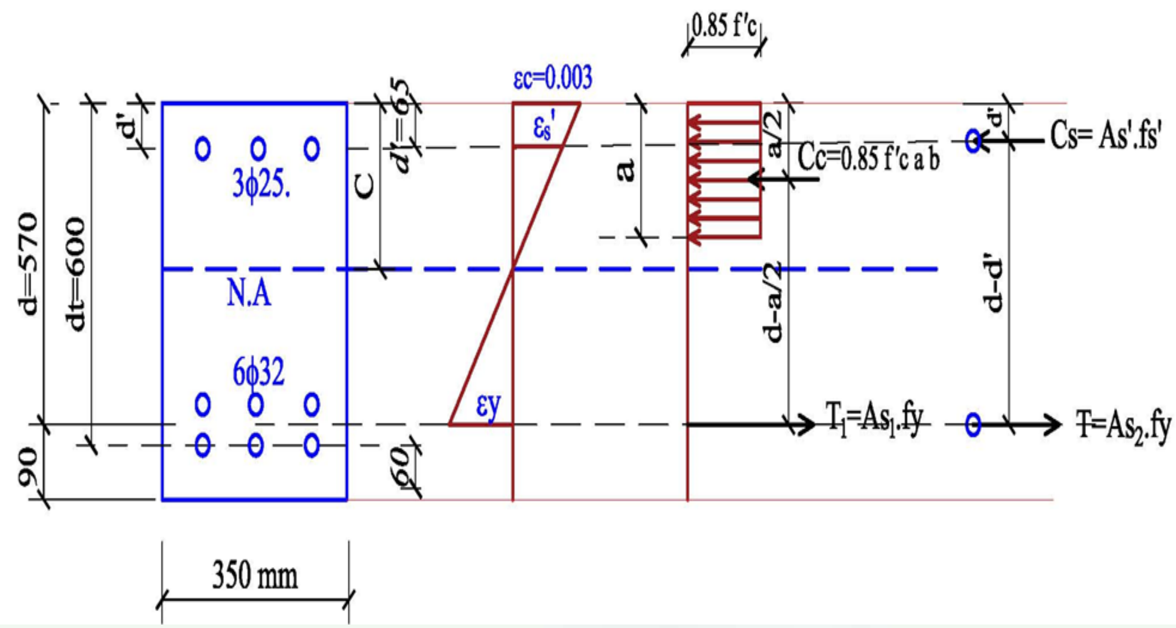
ρ_{max}: *maximum steel ratio for single beam section under tension controlled*

$$a = \frac{As\,fy - A's\,fs'}{0.85\,f'c\,b}$$

And :

$$\emptyset Mn = \emptyset\left[(Asfy - As'fs')\left(d - \frac{a}{2}\right) + As'fs'\,(d - d')\right]$$

Example (8) : Determine the design moment strength of the section shown below , using f'c= 35 Mpa , fy = 420 Mpa. $As = 6\emptyset\, 32\, mm$ (two layer) and $A's = 3\emptyset\, 25\, mm$.



Solution

1-Calculate ρ *and* ρ'

$$\rho = \left(\frac{AS}{bd}\right) = \frac{6 \times 804}{350 \times 570} = 0.02418$$

$$\rho' = \left(\frac{A's}{bd}\right) = \frac{3 \times 490}{350 \times 570} = 0.007368$$

$$m = \frac{fy}{0.85\, f_c'} = 14.12$$

$\beta 1 = 0.8$ for f'c = 35 mPa

$$\rho - \rho' \geq \left(\frac{\beta 1 d'}{m\, d}\right)\left(\frac{600}{600 - fy}\right)$$

$$= 0.02417 - 0.007368 \geq \left(\frac{0.8 \times 65}{14.12 \times 570}\right) \times \left(\frac{600}{600 - 420}\right) = 0.016812 \leq 0.0215 \qquad (\,not\ checked)$$

$\therefore \quad fs' < fy$

$$\rho_b = \frac{\beta 1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right) = \frac{0.8}{14.12}\left(\frac{600}{600 + 420}\right)\left(\frac{600}{570}\right) = 0.03508$$

$$\rho_{max} = \left(\frac{0.003 + fy/Es}{0.003 + \varepsilon t}\right)\rho b = \left(\frac{0.003 + 420/200000}{0.003 + 0.005}\right) \times 0.03508 = 0.0224$$

$\rho - \rho' < \rho_{max}$ Tension controlled ϕ=0.9

3- calculate $\emptyset Mn$ $(internal\ section\ analysis)$

$Cc = 0.85\, f'c\, a\, b$ $a = \beta_1 \times C = 0.8\, C$

$Cc = 0.85 \times 35 \times (0.8\, C) \times 350 = 8330\,C$ N

$Cs = A's\,(fs' - 0.85\, f'c)$

$$fs' = 600\left(\frac{c-d'}{c}\right) = 600\left(\frac{c-65}{c}\right)$$

$$Therefore:\quad Cs = 1470 \times \left(600 \times \left(\frac{c-65}{c}\right) - 0.85 \times 35\right)$$

$$= 882000\left(\frac{c-65}{c}\right) - 43732.5$$

$T = T_1 + T_2 = (As_1 + As_2)fy = As \times fy = 6 \times 804 \times 420 = 2026080\, N$

4- Internal Forces

$T = Cc + Cs$

$$2026082 = 8330\, C + 882000 \times \left(\frac{c-65}{c}\right) - 43732.5$$

$2026080C = 8330C^2 + 882000\, C - 65 \times 882000 - 43732.50C$

$8330C^2 - 1187812.5\, C - 57330000 = 0$

C= 180.68 mm

$a = \beta_1 \times C = 0.8 \times 180.68 = 144.54$ mm

Or

2- Using Direct Method

$$A\,a^2 - Ba - C = 0$$

$$A = 1,$$

$$B = m\,d\left(\rho - \frac{600}{fy}\rho'\right)$$

$$C = \frac{600}{fy}\beta 1\,m\,d\,d'\rho'$$

$$a = \frac{1}{2}\left[B + \sqrt{B^2 + 4\,A\,C}\,\right]$$

$$a = \frac{1}{2}\left[\left(m\,d\left(\rho - \frac{600}{fy}\rho'\right)\right) + \sqrt{\left[m\,d\left(\rho - \frac{600}{fy}\rho'\right)\right]^2 + 4\times 1.0 \times \frac{600}{fy}\beta_1\,m\,d\,d'\rho'}\,\right], \qquad C = \frac{a}{\beta}$$

$$= \frac{1}{2}\left[\left(14.12 * 570\left(0.02418 - \frac{600}{420} * 0.007368\right)\right) + \sqrt{\left[14.12 * 570\left(0.02418 - \frac{600}{420} * 0.007368\right)\right]^2 + 4 * \left(\frac{600}{420} * 0.8 * 14.12 * 570 * 65 * 0.007368\right)}\,\right]$$

$$a = 141.11\,mm \quad and \quad c = \frac{a}{\beta_1} = 176.39mm$$

Or Using

3-Indirect Method (iterative method)

both method dose not subtract the term (0.85 f'c)

-Calculate *(a)*

$$a = \frac{As\,fy - A's\,fs'}{0.85\,f'c\,b} \qquad where\ fs' = fy$$

$$a = \frac{4824 \times 420 - 1470 \times 420}{0.85\ \times 35 \times 350} = 135.29\,mm$$

$$c = \frac{a}{\beta 1} = \frac{135.29}{0.8} = 169.11mm$$

$$f's = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{169.11 - 65}{169.11}\right) = 369.33\ Mpa$$

$$a = \frac{4824 \times 420 - 1470 \times 369.33}{0.85\ \times 35 \times 350} = 142.44\,mm$$

$$c = \frac{a}{\beta 1} = \frac{142.44}{0.8} = 178.04\,mm$$

$$f's = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{178.04 - 65}{178.04}\right) = 381\ MPa$$

$$a = \frac{4824 \times 420 - 1470 \times 381}{0.85\ \times 35 \times 350} = 140.79\,mm$$

$$c = \frac{a}{\beta_1} = \frac{140.79}{0.8} = 176\,mm$$

$$f's = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{176 - 65}{178.04}\right) = 378.4\ MPa$$

almost last two value are equal

$$\frac{4824 \times 420 - 1470 \times 378}{0.85 \ \times 35 \times 350} = 141.21\, mm$$

$$c = \frac{a}{\beta_1} = \frac{141.21}{0.8} = 176.5\, mm$$

5- Calculate fs' , Cc and Cs

$$f's = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{176.5 - 65}{176.5}\right) = 379.1\, MPa$$

$$Cc = 0.85 \times \ 35 \ \times (\, 0.8\, C) \times 350 = 8330\,\text{C} \ = 8330 \times 176.5 = 1470245\ \text{N}$$

$$Cs = A's\,(\, fs' \ - 0.85\, f'c\,) = 1470(379.1 - 0.85 \times 35) = 513544\, N$$

6- Calculate ∅Mn

$$\emptyset Mn = \emptyset\left[Cc\left(d - \tfrac{a}{2}\right) + Cs(\, d - d')\right] = 0.9\left[1470245\left(570 - \tfrac{141.21}{2}\right) + 513544(\,570 - 65)\right]$$

= 894222 065 N.m

= 894.22 KN.m

7- Check that $\rho - \rho' \frac{fs'}{fy} \leq \rho_{max}$

$$0.02418 - 0.007368 \times \left(\frac{379.1}{420}\right) = 0.01753 < \rho_{max} = 0.02236 \qquad (\text{O.K.})$$

The maximum total tension steel can be used in this is calculated by :

$$Max\ As = \left(\rho_{\ max} + \rho' \frac{fs'}{fy}\right) b\ d$$

$$= \left(0.02236 + 0.007368 \times \frac{379.1}{420}\right) \times 350 \times 570 = 5787\ mm^2$$

8- Let Check εt as follow:

$C = 176.5mm \quad , \quad d_t = 600\ mm$

$$\varepsilon_t = \left(\frac{dt - c}{c}\right) \times 0.003 = \frac{600 - 176.5}{176.5} = 0.0072 > 0.005 \qquad O.K\ tension\ controll$$

Thank You……..

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Analysis of T-and I-sections

ANALYSIS OF T-AND I-SECTIONS

It is normal to cast concrete slabs and beams together, producing a monolithic structure. Slabs have smaller thicknesses than beams. Under bending stresses, those parts of the slab on either side of the beam will be subjected to compressive stresses, depending on the position of these parts relative to the top fibers and relative to their distances from the beam. The part of the slab acting with the beam is called the flange, and it is indicated in Fig. below a by area $b*h_f$. The rest of the section confining the area $(h-h_f)\ b_w$ is called the stem, or web.

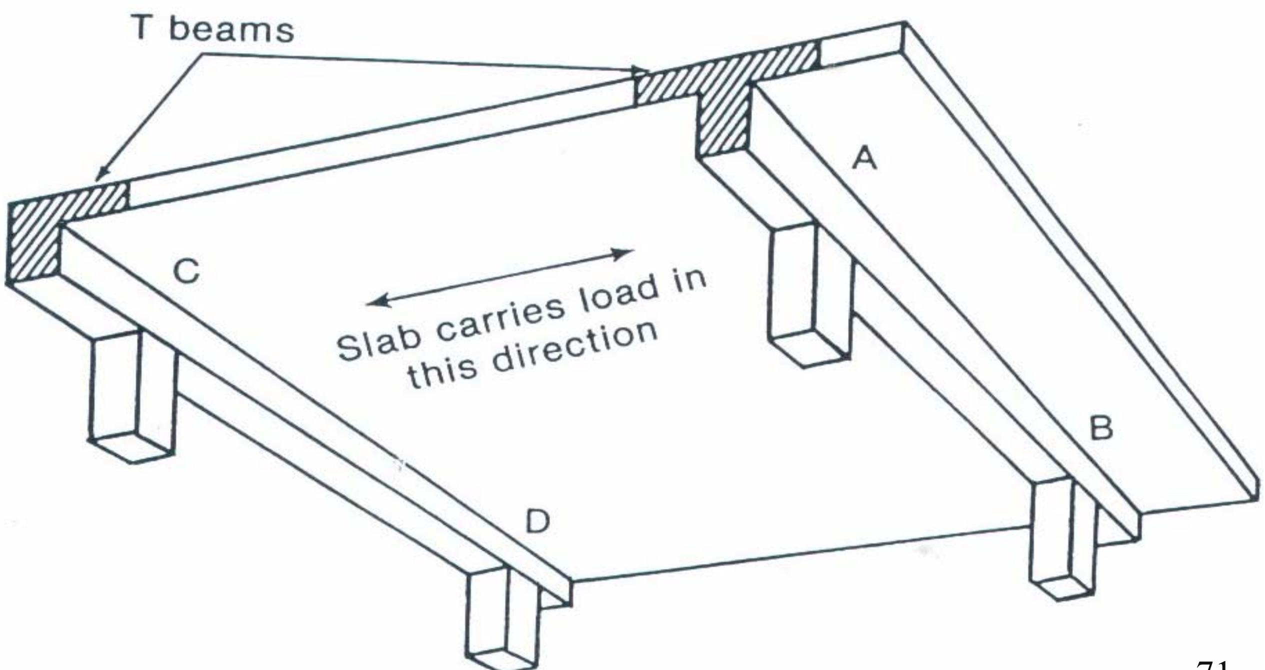
In an I-section there are two flanges, a compression flange, which is actually effective, and a tension flange, which is ineffective because it lies below the neutral axis and is thus neglected completely. Therefore, the analysis and design of an I-beam is similar to that of a T-beam.

Floor systems with slabs and beams are placed in monolithic pour.

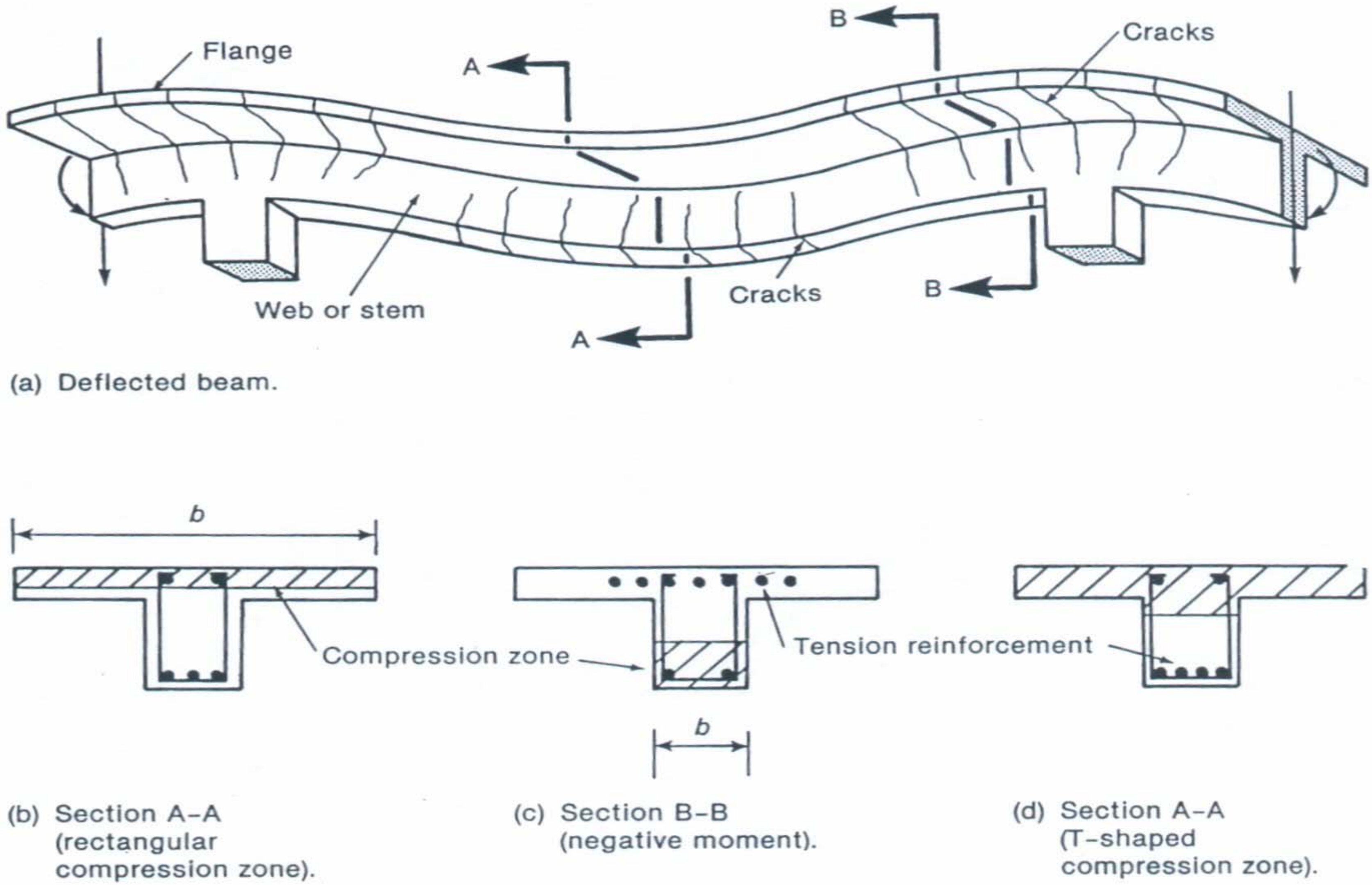
Slab acts as a top flange to the beam;

1- T-beams

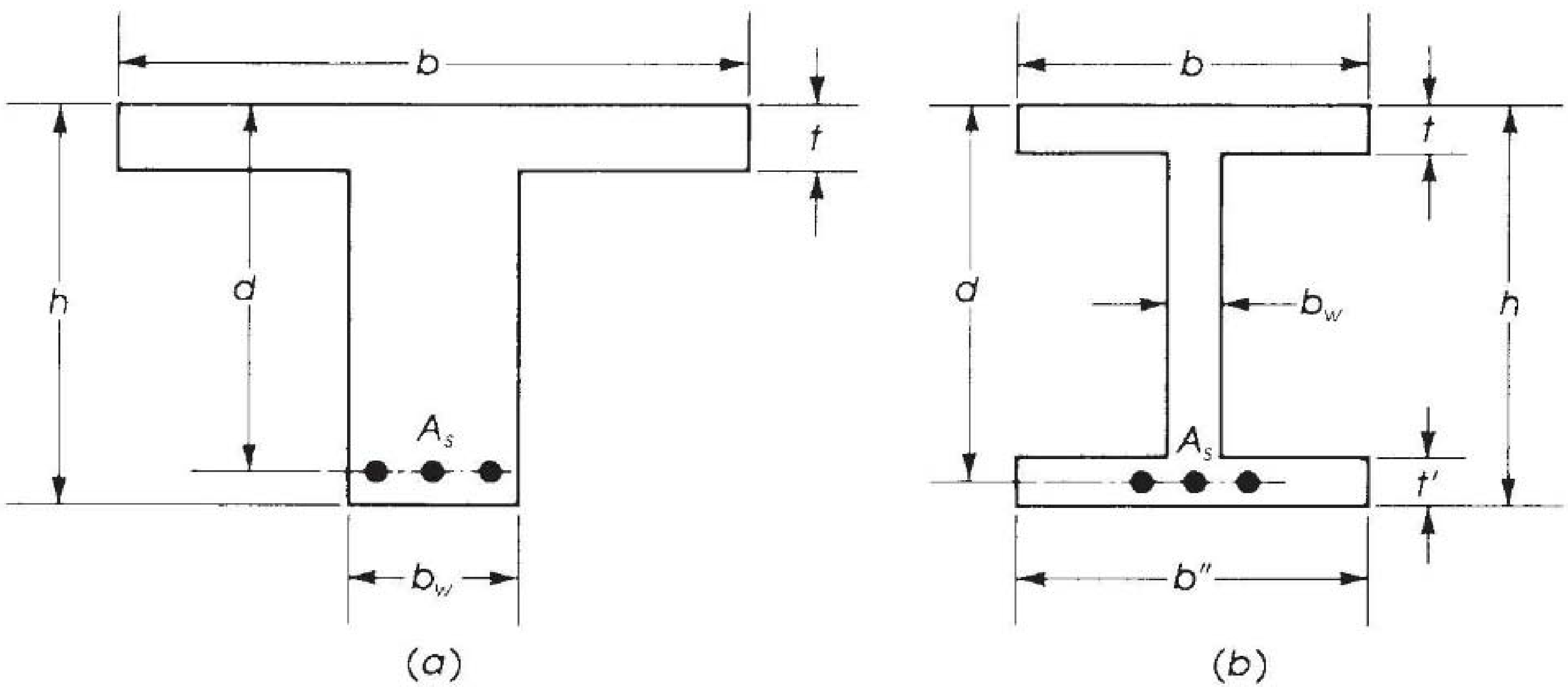
2- Inverted L(Spandrel) Beams.



Positive and Negative Moment Regions in a T-beam

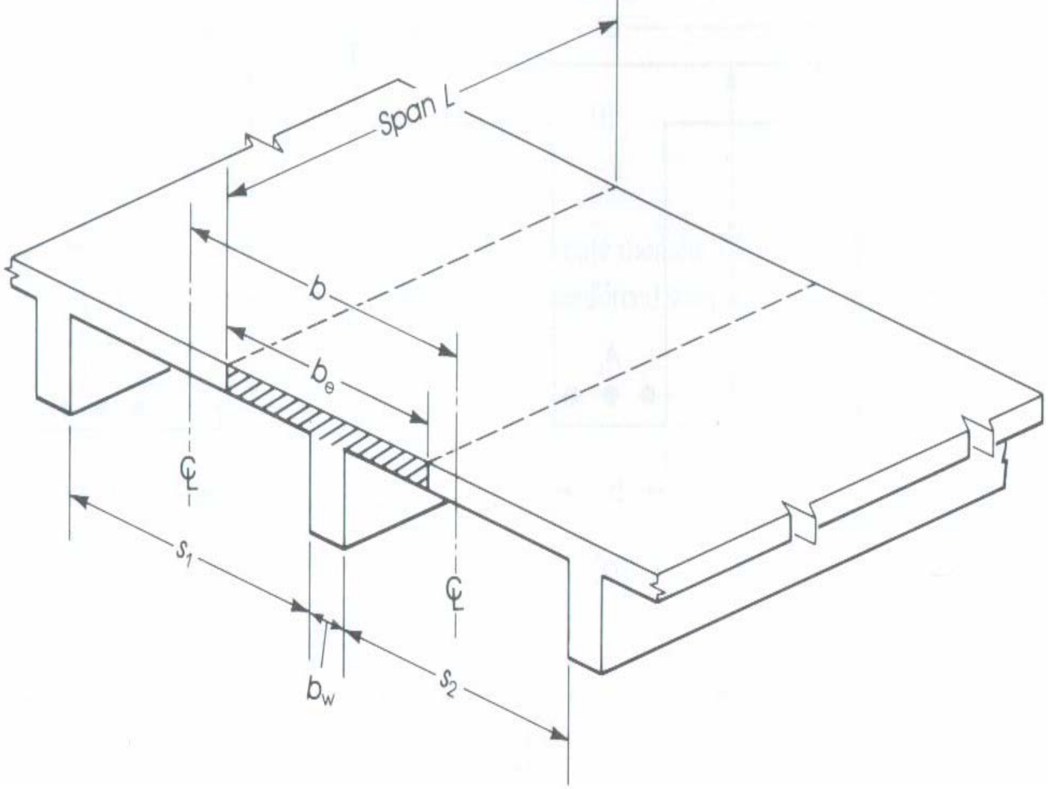


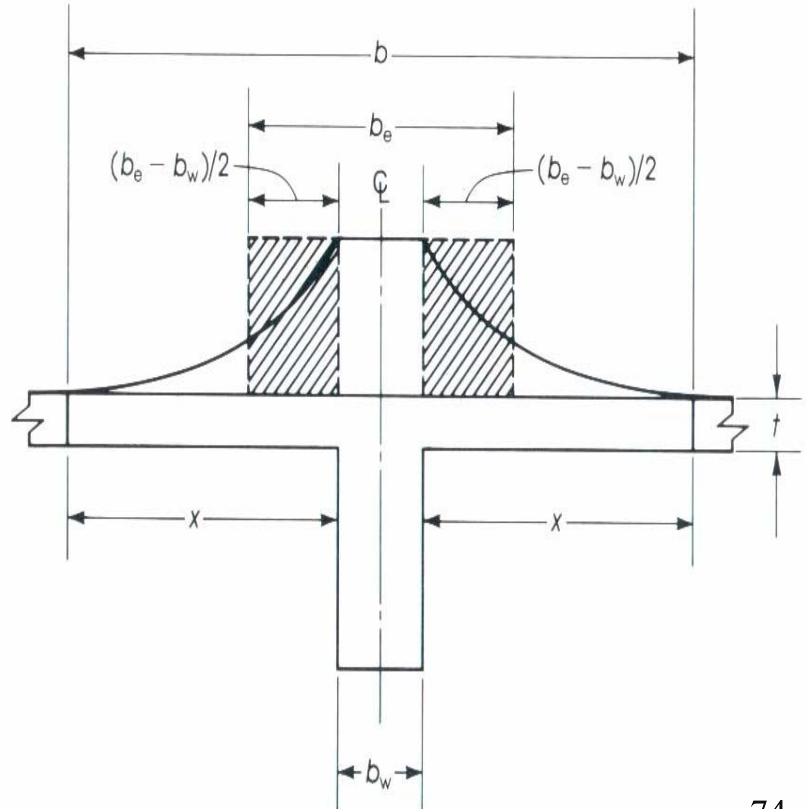
If the neutral axis falls within the slab depth analyze the beam as a rectangular beam, otherwise as a T-beam.



Effective width (b_e)

b_e is width that is stressed uniformly to give the same compression force actually developed in compression zone of width $b_{(actual)}$





1-From ACI 318, 2014 Section 6.3.2.1

T Beam Flange:

$$be \le \frac{L}{4}$$

$$be \le 16\ h_f + bw$$

$$be \le b \qquad (\text{ clear distance to next web})$$

2-From ACI 318 2014 Section 6.3.2.1

Inverted L Shape Flange

$$be \le \frac{L}{12} + bw$$

$$be \le 6\ h_f + bw$$

$$be \le b = bw + 0.5 \times (\text{ clear distance to next web})$$

3-From ACI 318 2014 Section 6.3.2.2

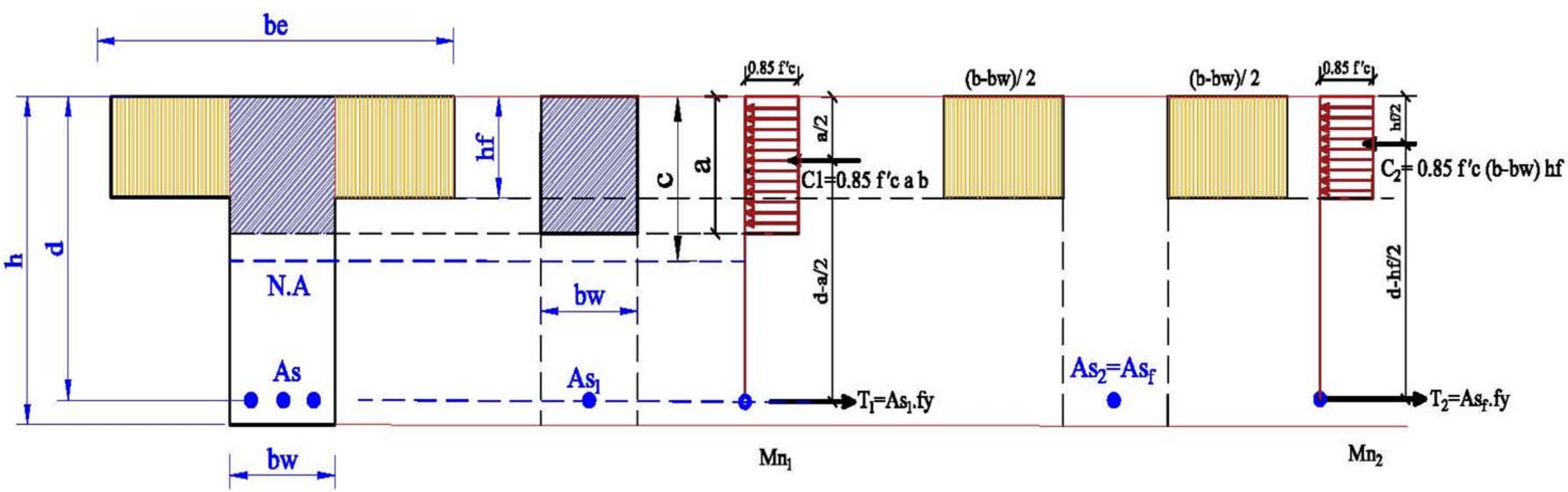
Isolated T-Beams

$$h_f \ge \frac{b_w}{2}$$

$$be \ge 4\ bw$$

The analysis of a T-section is similar to that of a doubly reinforced concrete section, considering an area of concrete (be−bw)*t as equivalent to the compression steel area A′s. The analysis is divided into two parts, as shown in Fig. below.

1. A singly reinforced rectangular basic section , bw *d ,and steel reinforcement As_1. The compressive force, C1, is equal to 0.85f′c a bw, the tensile force, T_1, is equal to $As_1 fy$, and the moment arm is equal to (d−a/2).
2. A section that consists of the concrete over hanging flange sides $2\times[(be-bw)\ h_f]/2$ developing the additional compressive force (when multiplied by 0.85f′c) and a moment arm equal to d−hf/2. If A_{sf} is the area of tension steel that will develop a force equal to the compressive strength of the overhanging flanges, then



$As_f\, fy = 0.85f'c(be - bw)h_f$

$$As_f = \frac{0.85f'c\; hf\;(be - bw)}{fy}$$

The total steel used in the T-section As is equal to $As_1 + Asf$, or:

$As_1 = As - As_f$

The T-section is in equilibrium, so $C_1 = T_1$, $C_2 = T_2$, and $C = C_1 + C_2$ and $T = T_1 + T_2$.

Considering equation $C_1 = T_1$ for the basic section, then

$As_1\, fy = 0.85f'c\; a\; b_w$ $\quad or \quad$ $(As - As_f)fy = 0.85f'c\; a\; b_w$ therefore,

$$a = \frac{(As - As_f)\; fy}{0.85f'c\; bw}$$

Note that ***bw*** is used to calculate a. The factored moment capacity of the section is the sum of the two moments Mu_1 and Mu_2:

$\emptyset Mn = Mu_1 + Mu_2$

$$Mu_1 = \emptyset As_1\, fy\left(d - \frac{a}{2}\right) = \emptyset(As - Asf)fy(d - \frac{a}{2})$$

$As_1 = (As - Asf)$ and

$$a = \frac{(As - As_f)\; fy}{0.85f'c\; bw}$$

$$Mu_2 = \emptyset Asf\; fy\;(d - \frac{h_f}{2})$$

$$\emptyset\; Mn = \emptyset[(As - Asf)fy(d - \frac{a}{2}) + Asf\; fy\;(d - \frac{h_f}{2})]$$

Considering the web section $bw \times d$, the net tensile strain (NTS), εt, can be calculated from a, c, and dt as follows:

If $c = \frac{a}{\beta_1}$ and $d_t = h - 62.5$, then $\varepsilon_t = 0.003(d_t - c)/c$. For tension-controlled section in the web, $\varepsilon_t \geq 0.005$. The design moment strength of a T-section or I- section can be calculated from the preceding equation above .It is necessary to check the following:

1.The total tension steel ratio relative to the web effective area is equal to or greater than ρ min:

$$\rho_w = \frac{As}{bw\ d} \geq \rho_{min}$$

$$\rho_{min} = \frac{0.25\sqrt{f'c}}{fy} \geq \frac{1.4}{fy}$$

2. Also, check that the NTS is equal to or greater than 0.005 for tension-controlled sections.

3.The maximum tension steel (Max As) in a T-section must be equal to or greater than the steel ratio used, As, for tension-controlled sections, with Ø =0.9.

$$Max\ As = As_f\ (Flange) + \rho_{max}\ (bw\ d)\ (web)$$

$$Max\ As = \left(\frac{1}{fy}\right)[0.85 f'c\ h_f(b - bw)\ ] + \rho_{max}(b_w\ d)$$

In steel ratios , relative to the web only, divide by bw d:

$$\rho_w = \left(\frac{As}{b_w\ d}\right) \leq \left(\rho_{max} + \frac{A_{sf}}{b_w\ d}\right)$$

$$\rho_w - \rho_f \leq \rho_{max} \text{ (web)}$$

where ρ_{max} *is the maximum steel ratio for the basic singly reinforced web section and* $\rho_f = \frac{As_f}{bw\ d}$.

Analysis of T-and I-sections

Reinforced Concrete Design

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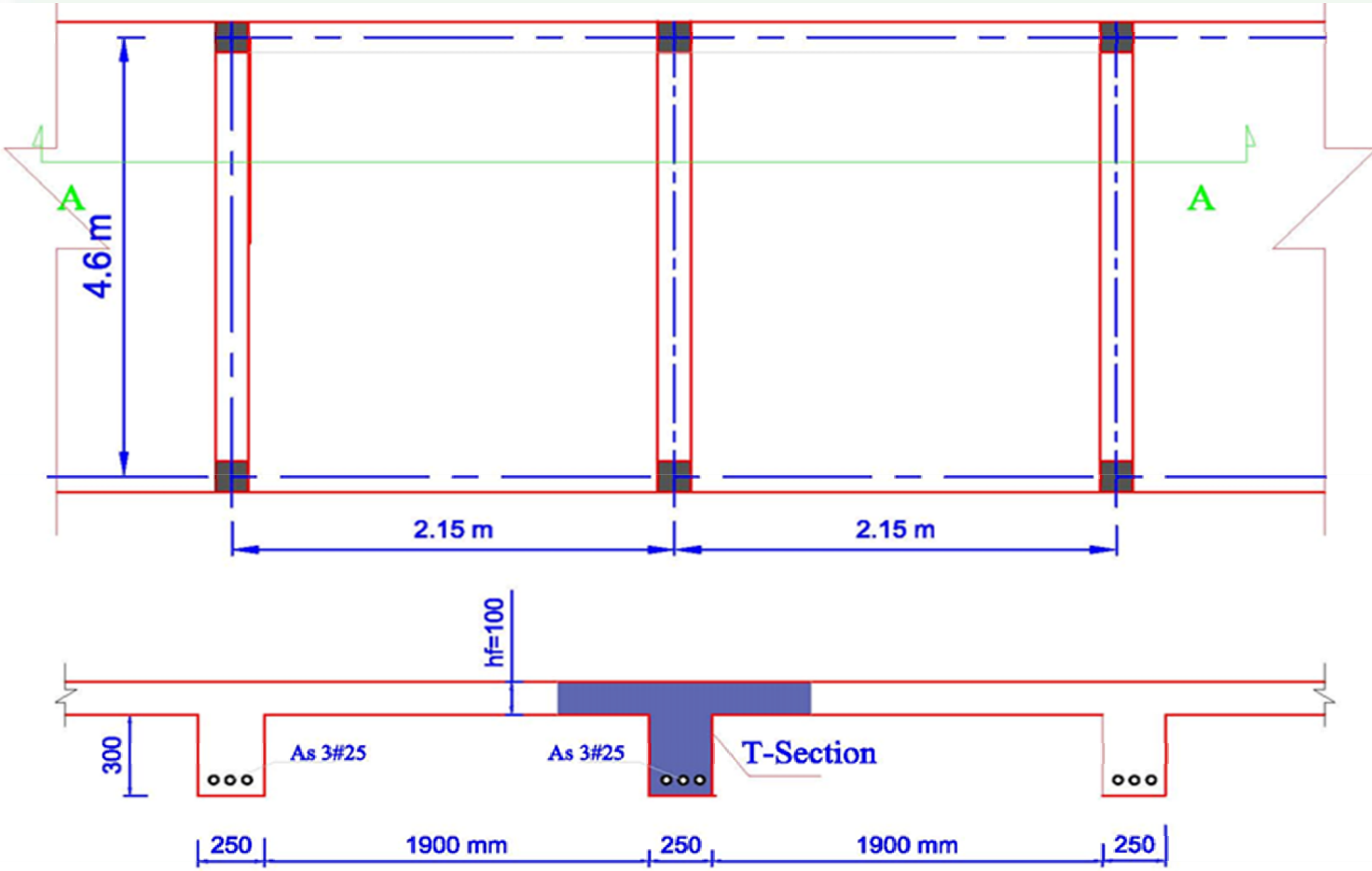
Examples -Analysis of T Sections

Example (10) :A series of reinforced concrete beams spaced at , 2.15 m on centers have a simply supported span of 4.6 m. The beams support a reinforced concrete floor slab 100 mm thick. The dimensions and reinforcement of the beams are shown in Fig. below .Using $f'c$=21 MPa and fy=420 MPa , determine the design moment strength of a typical interior beam.



Solution

1.Determine the effective flange width *be*. The effective flange width is the smallest of:

$$be = \frac{L}{4} = \frac{4.6}{4} = 1150\, mm$$

$be = 16\, h_f + bw = 16 \times 100 + 250 = 1850\, mm.$

be = Center to center of adjacent slabs = 2.15 m

Therefore be= 1150 mm

2.Check the depth of the stress block. If the section behaves as a rectangular one, then these stress block lies within the flange. In this case, the width of beam used is equal to 1150 mm.

$$a = \frac{As\, fy}{0.85 f'c\ be} = \frac{1470 \times 420}{0.85 \times 21 \times 1150} = 30.01 < h_f = 100mm$$

therefore ,it is a rectangular section.

3.Check that:

$$\rho_w = \frac{As}{b_w\, d} \geq \rho_{min} = \frac{1.4}{fy} = \frac{1.4}{420} = 0.0033 \qquad for\ f'c < 31\, MPa$$

$$\rho_w = \frac{1470}{250 \times 400} = 0.0148 > \rho_{min} = 0.0033$$

$4. Check\ \varepsilon t : a = 30.01\ mm ., C = \frac{30.01}{0.85} = 35.31\ mm ,\quad d_t = d = 400\ mm.$

$$\varepsilon_t = \frac{d_t - c}{c}\ \varepsilon_c = \frac{400 - 35.31}{35.31} \times 0.003 = 0.03098 > 0.005 \quad Ok$$

Tension Controlled and $\phi = 0.9$

5. Calculate $\emptyset\ Mn$

$$\emptyset\ Mn = \emptyset Asfy\left(d - \frac{a}{2}\right) = 0.9 \times 1470 \times 420\ (400 - \left(\frac{30.01}{2}\right) = 213.93\ KN.m$$

6.Check that As used is less than or equal to Max As

$$Max\ As = As_f + \rho_{max}(b_w\ d)$$

$$MaxAs = \left(\frac{0.85f'c\ h_f(be - bw)}{fy}\right) + \rho_{max}(b_w\ d)$$

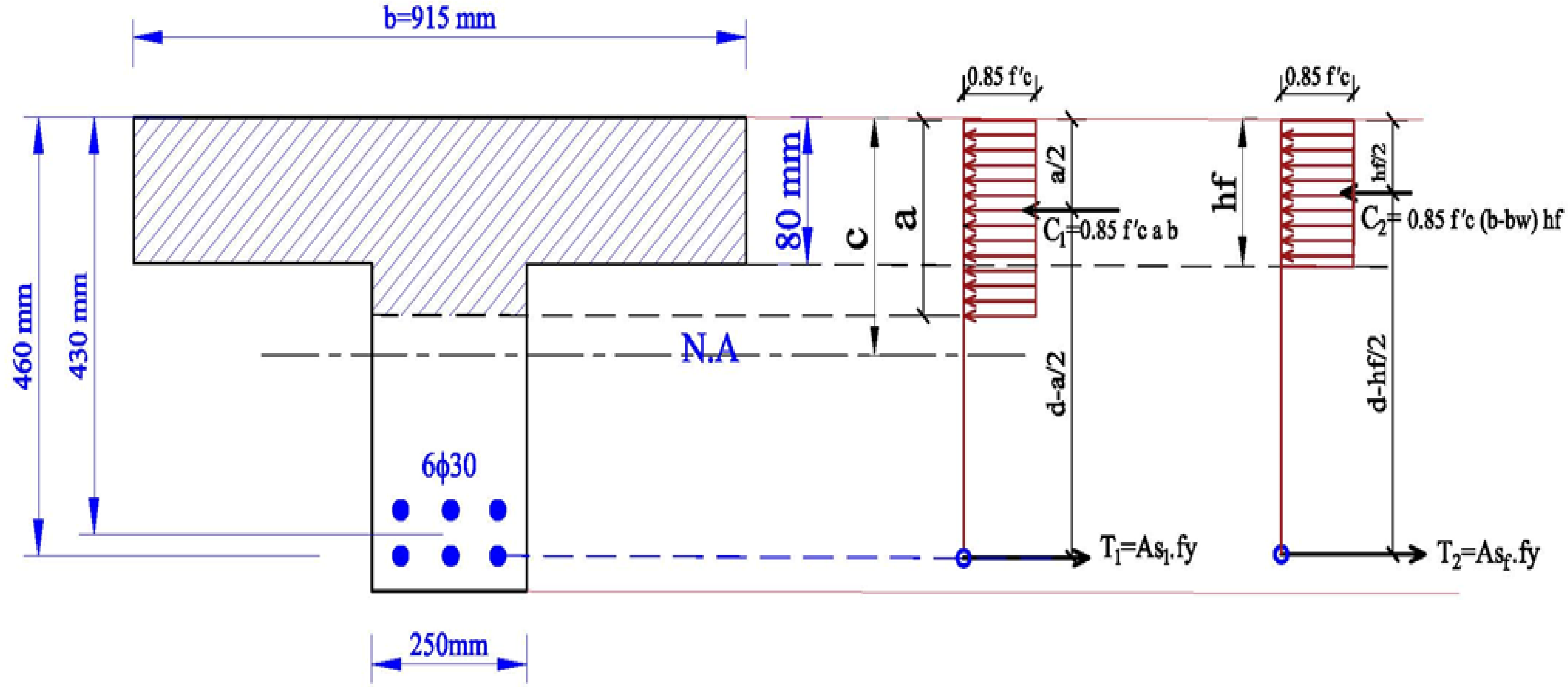
$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{d_t}{d}\right) = \frac{0.85}{23.53}\left(\frac{600}{600 + 420}\right)(1) = 0.02125$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b = \left(\frac{0.003 + \frac{420}{200000}}{0.008}\right) \times 0.02125 = 0.01354$$

$$Max\ As = \left(\frac{0.85 \times 21 \times 100(1150 - 250)}{420}\right) + 0.01354\ (250 \times 400) = 5179\ mm^2 > As\ (used)$$

$$= 1470\ mm^2 \quad O.K.$$

Example (11) :Calculate the design Moment strength of T- Section Shown below using f′c=24 MPa and fy=420 MPa , determine the design moment strength of a typical interior beam.



Solution

1- Calculate a :

$$a = \frac{As\, fy}{0.85 f'c\; be}$$

$As = \emptyset\ 30 = 706\ mm^2$

$$a = \frac{6 \times 706 \times 420}{0.85 \times 24 \times 915} = 95.31\ mm\ > h_f = 80\ mm$$

Since a $> h_f$, $it\ is\ a\ \ T -\ Section\ analysis$

2- Find As_f

$$As_f = \frac{0.85 f'c\; h_f\,(be - bw)}{fy} = \frac{0.85 \times 24 \times 80 \times (915 - 250)}{420} = 2584\ mm^2$$

$As_1 = As - As_f = 4236 - 2584 = 1652\ mm^2$

$$a = \frac{As_1\, fy}{0.85 f'c\; bw} = \frac{1652 \times 420}{0.85 \times 24 \times 250} = 136.05\ mm$$

$$c = \frac{a}{\beta_1} = \frac{136.05}{0.85} = 160.06\ mm$$

3- Check εt

$d_t = 460\ mm$

$$\varepsilon_t = \frac{d_t - c}{c} \times 0.003 = \frac{460 - 160.06}{160.06} \times 0.003 = 0.005623\ > 0.005\quad OK$$

$\emptyset = 0.9$ Tension Failure

4- Check As min.

$$As_{min} = \rho_{\min}\, bw\, d \geq \frac{1.4}{fy} b_w\, d \quad where \quad f'c \leq 31\, MPa$$

$$As_{min} = 0.0033 \times 250 \times 430 = 357.98\; mm^2$$

$Max\ As = A_{sf} + \rho_{max}\, (b_w\, d)$

$$m = \frac{fy}{0.85\, f'c} = \frac{420}{0.85 \times 24} = 20.59$$

$$\rho_{max} = 0.6375\, \rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right) = 0.6375 \times \frac{0.85}{20.59}\left(\frac{600}{600 + 420}\right)\left(\frac{460}{430}\right) = 0.01651$$

Max As= $2584 + 0.01651 \times 250 \times 430 = 4364.3\ mm^2$

$As = 4236 mm^2 < 4364.3\; mm^2$ $\quad O.K$

5. Calculate $\emptyset\ Mn$

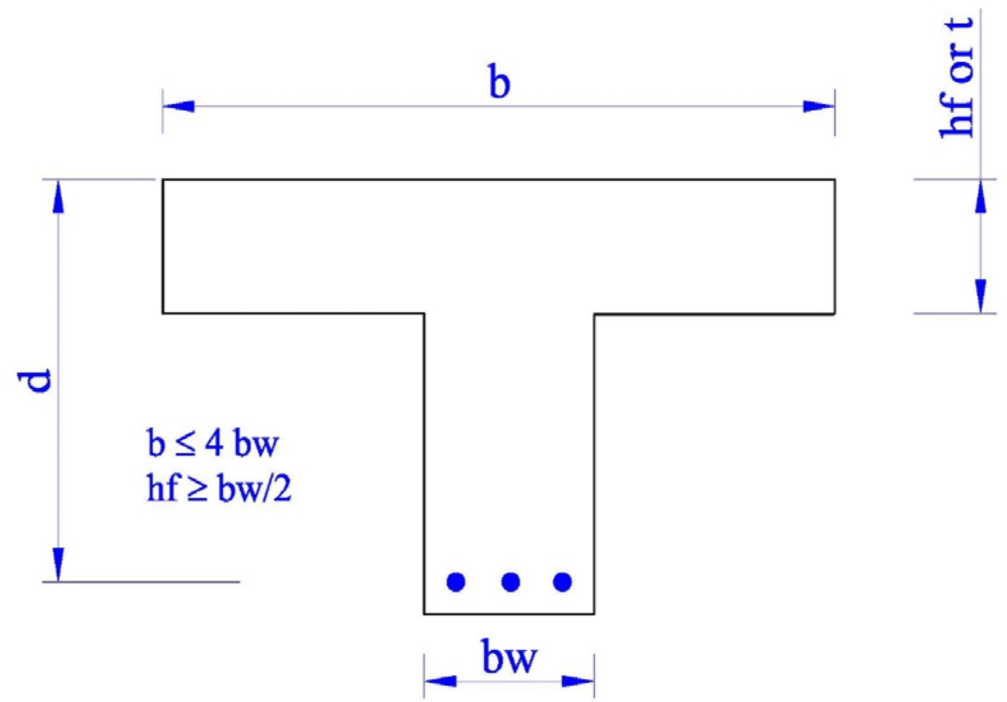
$$\emptyset\ Mn = \emptyset\left[(As - Asf) fy\left(d - \frac{a}{2}\right) + Asf\ fy\left(d - \frac{h_f}{2}\right)\right]$$

$$= 0.9\ [(4236 - 2584) \times 420\ (430 - \left(\frac{136.05}{2}\right) + 2584 \times 420(430 - \frac{80}{2}) = 606.97\ \ KN.m$$

Dimensions Of Isolated T-shaped Sections

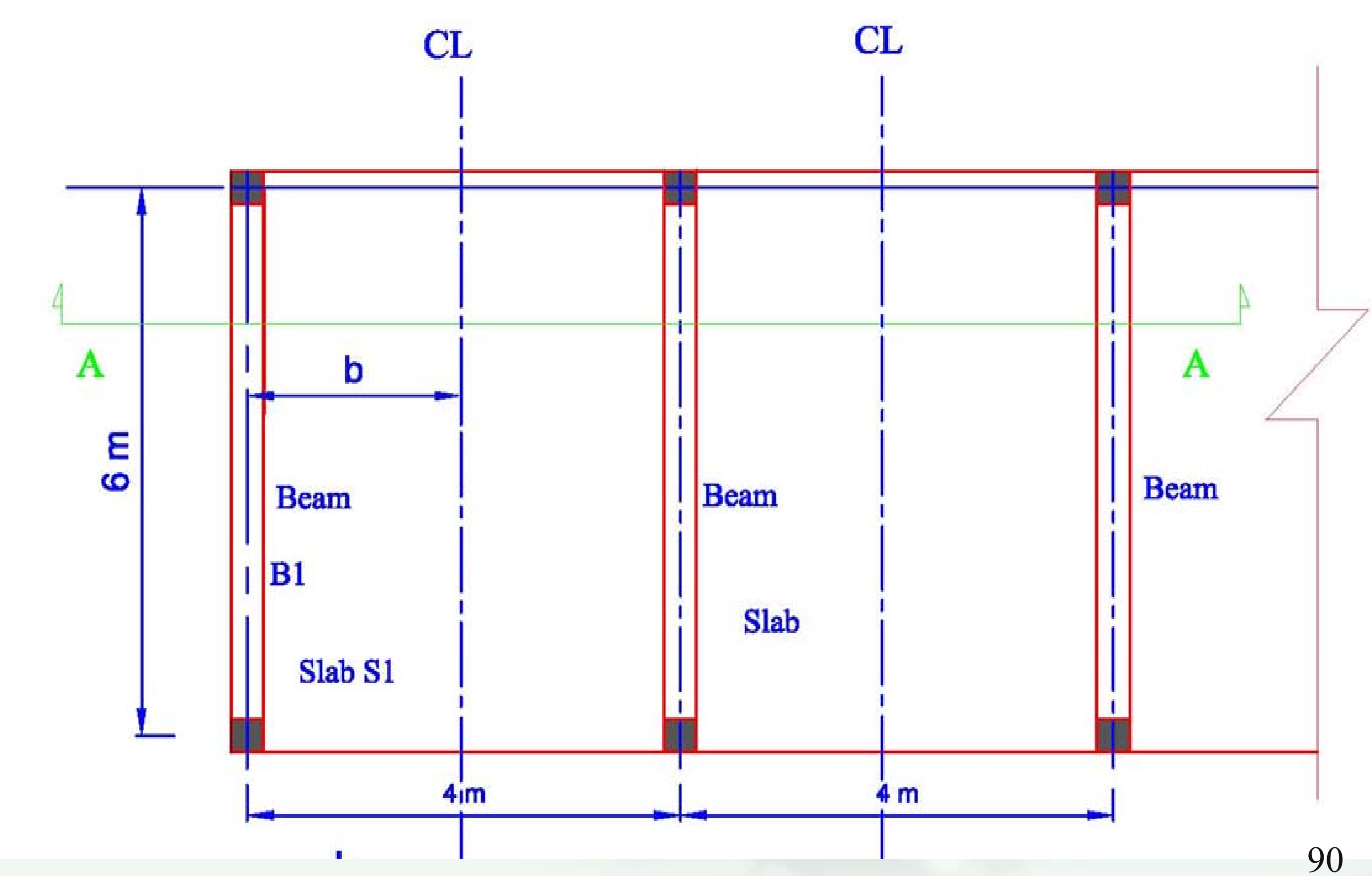
In some cases, isolated beams with the shape of a T-section are used in which additional compression area is provided to increase the compression force capacity of sections. These sections are commonly used as prefabricated units. The ACI Code, Section 6.3.2.2, specifies the size of isolated T-shaped sections as follows:

1. Flange thickness, $\boldsymbol{h_f}$, shall be equal to or greater than one-half of the width of the web ,$\boldsymbol{b_w}$.
2. Total Flange width b shall be equal to or less than four times the width of the web, $\boldsymbol{b_w}$.



Inverted L-shaped Sections

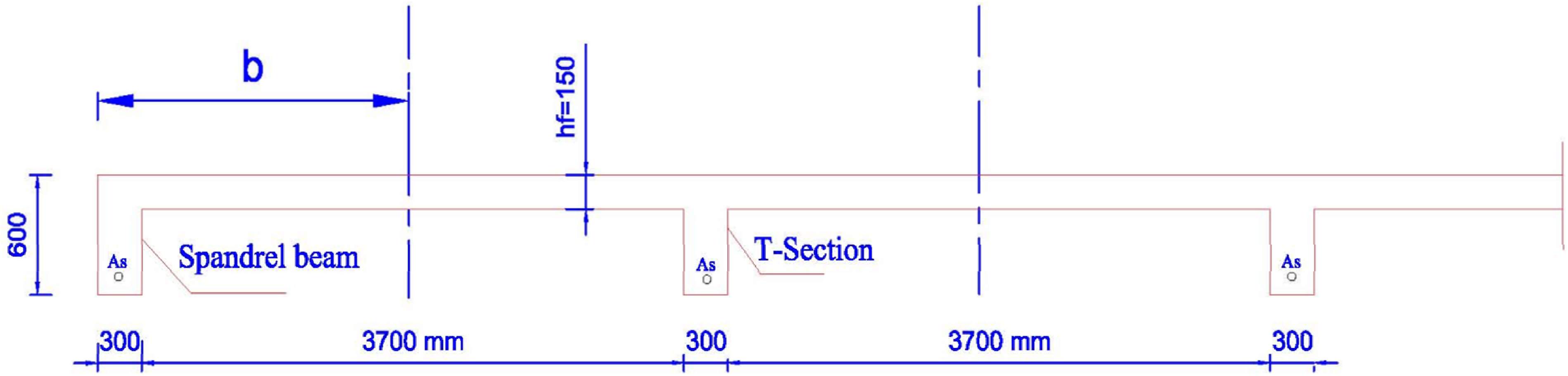
In slab beam girder Floors, the end beam is called a spandrel beam. This type of Floor has part of the slab on one side of the beam and is cast monolithically with the beam. The section is un symmetrical under vertical loading (Fig. shown below). The loads on slab S1 cause torsional moment uniformly distributed on the spandrel beam B1. The over hanging Flange width (b- bw) of a beam with the Flange on one side only is limited by the ACI Code, Section 6.3.2.1, to the smallest of the following:

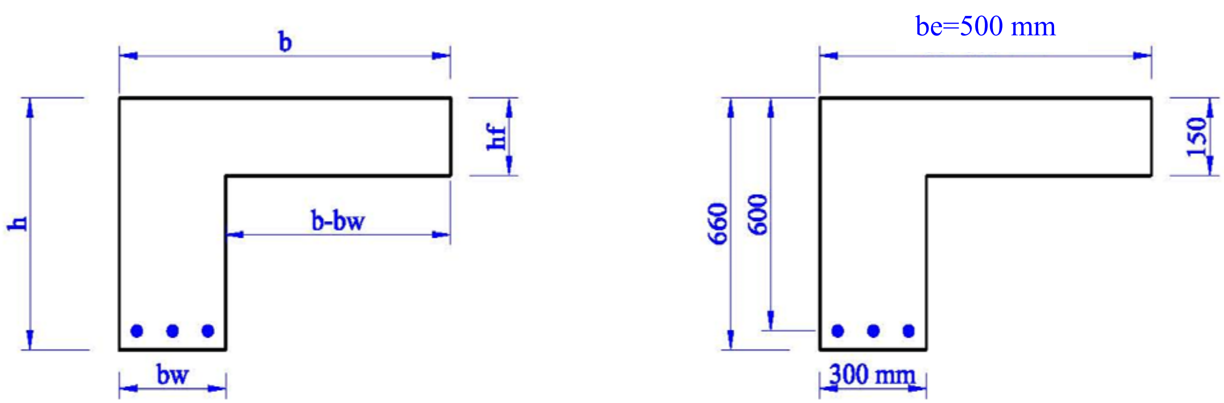


1. $be = \frac{L}{12}$ $\quad be = \frac{6000}{12} = 500\,mm$ $\quad$ (*controlled*)

2. $be = 6 \times h_f + bw.$ $\quad be = 6 \times 150 + 300 = 1200\,mm$

3. $be = b$ $\quad be = \frac{3700}{2} + 300 = 2150\,mm$





Example (12) : Calculate the design moment strength of the precast concrete section shown below using f′c= 28 MPa and fy=420 MPa .

Solution:

1.The section behaves as a rectangular section with b= 350 mm and $d = 610 - 62.5 = 547.5mm$.

Note that: the width b is that of the section on the compression side.

2. Check that $\rho = As/bd = 5 \times 615/(350 \times 547.5) = 0.01605$

$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600+fy}\right)\left(\frac{d_t}{d}\right) = \frac{0.85}{17.65}\left(\frac{600}{600+420}\right)(1) = 0.02834$$

$$\rho_{max} = \left(\frac{0.003+\frac{fy}{Es}}{0.008}\right)\rho_b = \left(\frac{0.003+\frac{420}{200000}}{0.008}\right) \times 0.02834 = 0.01807 > \rho = 0.01605$$

$$\rho_{min}\frac{1.4}{fy} = \frac{1.4}{420} = 0.00333$$

So its tension-controlled sections.

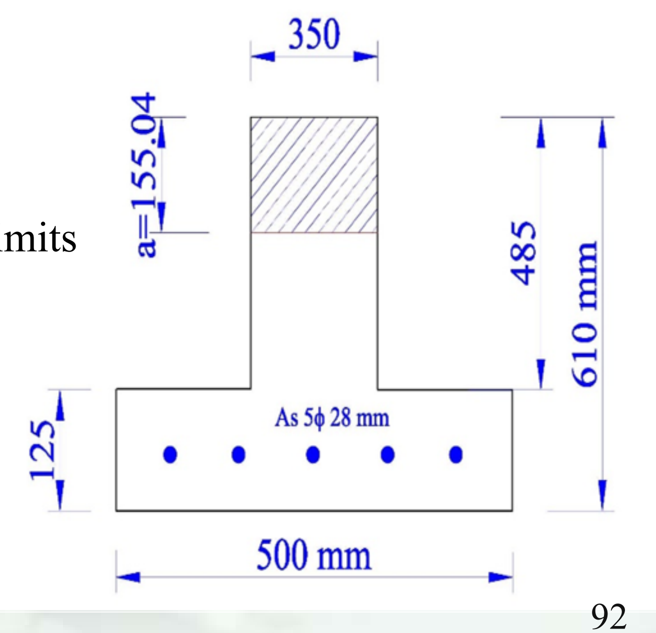
Therefore ϕ=0.9. Also $\rho > \rho_{min}$ min=0.00333.Therefore, ρ is within the limits of a tension-controlled section.

3.Calculatea (a)

$$a = \frac{As\,fy}{0.85f'c\,b} = \frac{5 \times 615 \times 420}{0.85 \times 28 \times 350} = 155.04\,mm$$

$$\emptyset\,Mn = \emptyset As\,fy\left(d - \frac{a}{2}\right) = 0.9 \times 5 \times 615 \times 420 \times \left(547.5 - \frac{155.04}{2}\right)$$

$$= 546.28\,kN.m$$



Thank You……..

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Chapter III

Flexural Design of Reinforced Concrete

Introduction

In the previous chapter, the analysis of different reinforced concrete sections was explained. Details of the section were given, and we had to determine the design moment of the section. In this chapter, the process is reversed: The external moment is given, and we must find safe, economic, and practical dimensions of the concrete section and the area of reinforcing steel that provides adequate internal moment strength.

Rectangular Sections With Tension Reinforcement Only

From the analysis of rectangular singly reinforced sections the following equations were derived for tension-controlled sections, where f′c and fy are in MPa:

$$\rho_b = \frac{0.85\, fc'}{fy}\left(\frac{600}{\mathbf{600 + fy}}\right)\left(\frac{\mathbf{dt}}{\mathbf{d}}\right)$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b \quad or \quad \rho_{max} = \frac{3}{8}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right)$$

$$m = \frac{fy}{0.85 f'c}$$

$$\beta_1 = 0.85 - 0.05\left(\frac{f'c - 28}{7}\right) \geq 0.56$$

$For\ fy = 420\ Mpa$ $\qquad \rho_{max} = 0.6375\ \rho_b$
$For\ fy = 280\ Mpa$ $\qquad \rho_{max} = 0.55\ \rho_b$
$For\ fy = 350\ Mpa$ $\qquad \rho_{max} = 0.594\ \rho_b$

$$\beta_1 = 0.85 - 0.05\frac{f'c - 28}{7} \geq 0.56$$

$$m = \frac{fy}{0.85f'c}$$

It should be clarified that the designer has a wide range of choice between a large concrete section and relatively small percentage of steel ρ , producing high ductility and a small section with a high percentage of steel with low ductility. A high value of the net tensile strain, εt, indicates a high ductility and a relatively low percentage of steel. The limit of the net tensile strain for tension-controlled sections is 0.005 ,with =0.9. The strain limit of 0.004 can be used with a reduction in ϕ . If the ductility index is represented by the ratio of the net tensile strain , εt, to the yield strain, εy=fy/Es, the relationship between εt , / b , , and εt/εy is shown in Table below for fy=420 MPa . Also, the ACI Code, Section 6.6.5.1, indicates that εt should be ≥ 0.0075 for the redistribution of moments in continuous flexural members producing a ductility index of 3.75. It can be seen that adopting $\varepsilon_t \geq 0.005$ is preferable to the use of a higher steel ratio, ρ / ρ b ,with ε_t =0.004 , because the increase in Mn is offset by a lower ϕ. The value of ε_t=0.004 represents the use of minimum steel percentage of 0.00333 for f′c=28 Mpa and fy=420 Mpa .This case should be avoided.

$For\ fy = 420\ Mpa$

ε_t	0.004	0.005	0.006	0.007	0.0075	0.008	0.009	0.010	0.040
ρ/ρ_b	0.714	0.625	0.555	0.500	0.476	0.454	0.417	0.385	0.117
$\varepsilon_t/\varepsilon_y$	2.0	2.5	3.0	3.5	3.75	4.0	4.5	5.0	20
ϕ	0.82	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9

The value of ε_t between $\varepsilon_t = 0.005$ and $\varepsilon_t = 0.004$ can be calculated from Eq. :

$$\phi = 0.65 + (\varepsilon t - 0.002)\left(\frac{250}{3}\right).$$

The design moment equations were derived in the previous chapter in the following forms:

$$\phi Mn = Mu = \phi R\, bd^{2}$$

$$R = \emptyset \rho f y\left(1 - \frac{1}{2}\rho\, m\right)$$

This equation have two unknown, this can be find by assumes $\rho \leq \frac{1}{2}\rho_{max}$ for and also assume value of b then we can find the value of h

For design purpose , two method can be adopted:

A- First Case

The knowns is Mu and the properties of used material and the unknowns is As , d , b

1- assume $\rho \leq \frac{1}{2}\rho_{max}$ and assume b

2- find value of R:

$$R = \emptyset \rho f y\left(1 - \frac{1}{2}\rho\, m\right) \quad \text{and} \quad m = \frac{fy}{0.85 f'c}$$

3- find the effective depth d from equation :

$$\phi Mn = Mn = \phi R\, bd^{2}$$

$$d = \sqrt{\frac{Mu}{\emptyset R\, b}}$$

4- Calculate As:

$As = \rho\, b\, d$

Then choose a suitable bar diameter numbers and calculate the total depth h considering the concrete cover (h should be choose around 10 mm)

B- Second Case

-The knowns $\boldsymbol{Mu}$ and the **dimension of section** according to the architectural requirement

- Unknown is the steel Area As

1- calculate R value

$$R = \frac{Mu}{\emptyset b d^2}$$

2- Calculate steel Ratio ρ from :

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$

Then compare value of ρmin and ρmax with value of ρ

3- calculate the As

$$As = \rho\, b\, d$$

than find the no. of bars

If $\rho > \rho_{max}$,the section should be design as **Double reinforced section**

Spacing Of Reinforcement And Concrete Cover

Specifications

Figure below shows two reinforced concrete sections. The bars are placed such that the clear spacing shall be at least the greatest of (25mm), nominal bar diameter D, and (4/3) d_{agg}.(nominal maximum size of the aggregate) , (ACI Code, Section 25.2.1). Vertical clear spacing between bars in more than one layer shall not be less than (25mm), according to the ACI Code, Section 25.2.2. Also for reinforcement of more than two layers, the upper layer reinforcement shall be placed directly above the reinforcement of the lower layer. The width of the section depends on the number , n, and diameter of bars used. Stirrups are placed at intervals; their diameters and spacing depend on shear requirements, to be explained later. At this stage, stirrups of (10mm) diameter can be assumed to calculate the width of the section. There is no need to adjust the width ,b , if different diameters of stirrups are used. The specified concrete cover for cast-in-place and pre-cast concrete is given in the ACI Code, Section 20.6.1. Concrete cover for beams and girders is equal to (38mm), and that for slabs is equal to (20mm), when concrete is not exposed to weather or in contact with the ground.

Minimum Width of Concrete Sections

The general equation for the minimum width of a concrete section can be written in the form

$$b_{min} = n \times D + (n-1) \times s + 2 \times (stirrup\ diameter) + 2 \times (concrete\ cover)$$

Where:

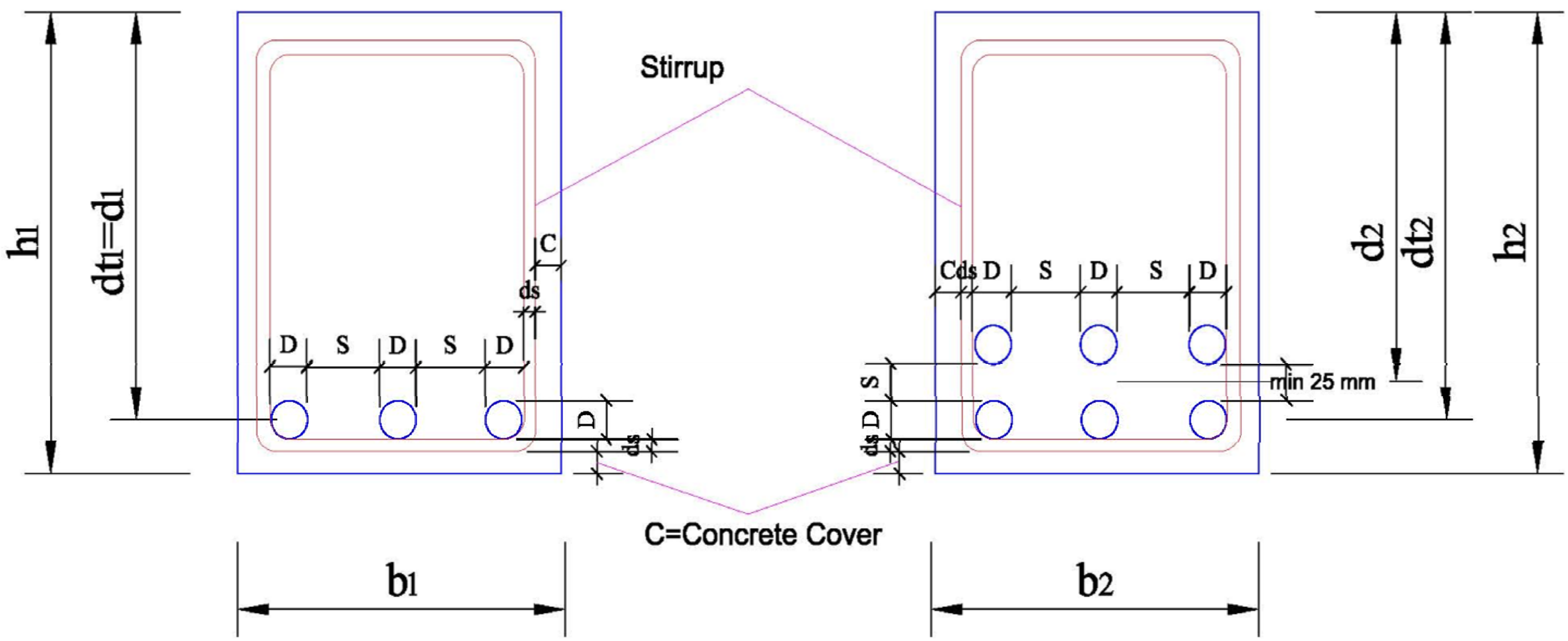
$n = number\ of\ bars$

$D = diameter\ of\ largest\ bar\ used$

$s = spacing\ between\ bars\ (equal\ to\ D\ or\ 25\ mm\ , whichever\ is\ larger)$

If the stirrup's diameter is taken equal to (10 mm) and concrete cover equals .(38mm),then

$B_{\text{min}} = n \times D + (n-1) \times s + 96$



This equation, if applied to the concrete section s in Fig, above becomes:

$b_1 = 3D + 2S + (96mm)$

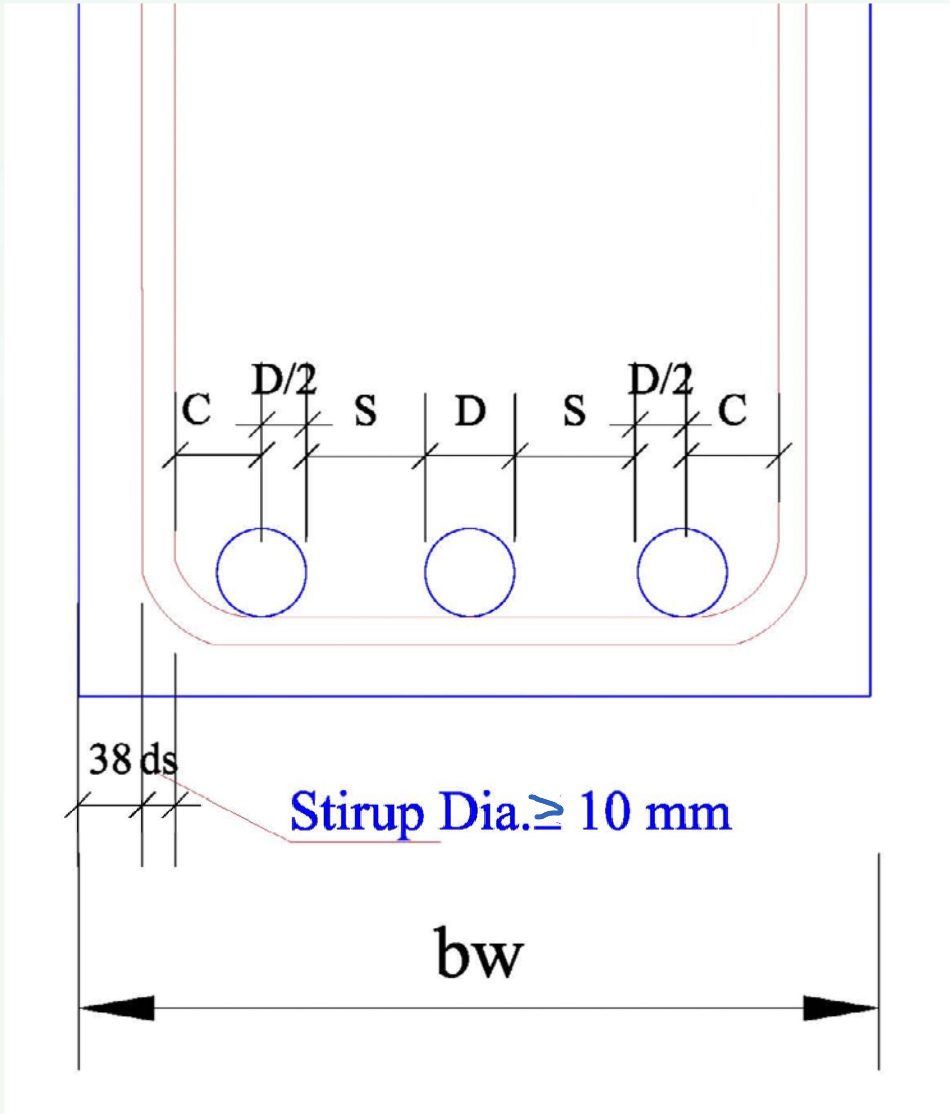
$b_2 = 3D + 2S + (96mm)$ *while for 4 bars* $b1 = 4D + 3S + (96mm)$

In fig. below , c = 20 mm when ds more than 10 mm

$$b_{min} = 2 \times 38 + 2\,ds + 2\,c + (n-1)(D+S)$$
$$b_{min} = 116 + 2ds + (n-1)\,(D+S)$$

If b is known then:

$$\text{Bar No.} = n = \frac{b - 116 - 2ds}{D+S} + 1$$



Minimum Over all Depth of Concrete Sections

The effective depth, d, is the distance between the extreme compressive fibers of the concrete section and the centroid of the tension reinforcement. The minimum total depth is equal to d plus the distance from the centroid of the tension reinforcement to the extreme tension concrete fibers, which depends on the number of layers of the steel bars .In application to the sections shown in Fig

$$h_1 = d_1 + \frac{D}{2} + ds. + 38\ mm \qquad \text{One layer}$$

$$h_2 = d_2 + \frac{25}{2} + D + ds + 38\ mm \qquad \text{Two layer}$$

When use bar diameter $\emptyset \leq 28\,mm$ then total depth calculated from :

$$h = d + 65\ mm \qquad \text{one layer}$$

Or

$$h = d + 90\ mm \qquad \text{two layer}$$

It should be mentioned that the minimum spacing between bars depends on the maximum size of the coarse aggregate used in concrete. The nominal maximum size of the coarse aggregate shall not be larger than one-Fifth of the narrowest dimension between sides of forms, or one-third of the depth of slabs, or three-fourths of the minimum clear spacing between individual reinforcing bars or bundles of bars (ACI Code, Section 26.4.2.1).

Example (1): Design a simply reinforced rectangular section to resist a factored moment of 490 KN.m using the maximum steel percentage ρ_{max} for tension-controlled sections to determine its dimension. Given: f′c=21 MPa fy=420 MPa.

Sol.

$for\ f'c = 21\ MPa\ \ then \qquad \beta_1 = 0.85$

$$m = \frac{fy}{0.85\ f'c} = 23.53, \qquad \emptyset = 0.9$$

$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right) = \frac{0.85}{23.53}\left(\frac{600}{600 + 420}\right)(1) = 0.02125$$

$$\rho_{max} = \left(\frac{0.003 + fy/Es}{0.008}\right)\rho_b = \left(\frac{0.003 + 0.0021}{0.008}\right)\rho_b = 0.6375\ \rho_b$$

ρ_{max} =0.01355

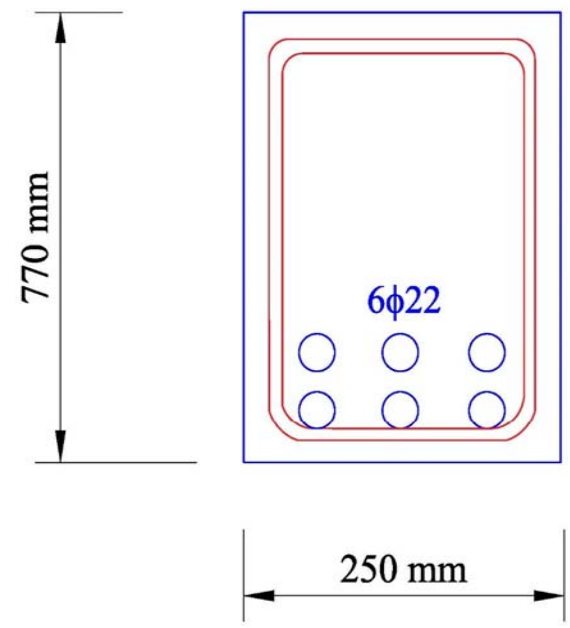
$$R = \rho fy\left(1 - \frac{1}{2}\rho\ m\right) = 0.01355 \times 420\left(1 - \frac{1}{2} \times 0.01355 \times 23.53\right) = 4.784\ MPa$$

$$Mn = \frac{Mu}{\emptyset} = Rbd^2$$

$$bd^2 = \frac{Mu}{\emptyset R} = \frac{490 \times 10^6}{0.9 \times 4.784} = 113805277\ mm^3$$

$Assume\ b\ and\ find\ d$

b mm	d mm	As mm²
250	672.18	2298.86
300	613.61	2518.25
350	568.09	2720.00
400	531	2907.83



$If\ we\ choose\ b\ =\ 250\ mm, d\ =\ 672.18\ mm$

Ø 22 mm (380 mm²).

$$No. of\ Bars = n = \frac{2298.86}{380} = 6.04 \quad Use\ 6\ Bars$$

$$Bar\ No. (n) = \frac{b\ - 116 - 2ds}{D + S} + 1$$

$$ds = 10\ mm, D\ =\ 22\ mm, S = 25\ mm$$

$$n = \frac{250\ - 116 - 2 \times 10}{22 + 25} + 1 = 3.42 = 3$$

$If\ use\ two\ layer \quad h =\ d + 90\ mm =\ 762.18\ mm \quad use\ h =\ 770\ mm \quad (increase\ the\ value\ for\ 10\ mm)$

Check the effective depth :

$$d = h - 38 - 10 - 22 - \frac{25}{2} = 770 - 38 - 10 - 22 - 12.5 = 687.5\ mm$$

$$b = 250\ mm$$

$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right)$$

$$d_t = 770 - 38 - 10 - \frac{22}{2} = 711\ \text{mm}$$

$$\rho_b = \frac{0.85}{23.53}\left(\frac{600}{600 + 420}\right)\left(\frac{711}{687.5}\right) = 0.02219$$

$$\rho_{max} = \left(\frac{0.003 + fy/Es}{0.008}\right)\rho_b = \left(\frac{0.003 + 0.0021}{0.008}\right) \times \rho_b = 0.6375\ \rho_b$$

$$\rho_{max} = 0.6375 \times 0.02219 = 0.014146$$

$$Mn = \frac{Mu}{\emptyset} = Rbd^2$$

$$\rho = \frac{As}{bd} = \frac{6 \times 380}{250 \times 687.5} = 0.01326 < \rho_{max}$$

$$R = \rho fy\left(1 - \frac{1}{2}\rho\, m\right) = 0.01326 \times 420\left(1 - \frac{1}{2}\, 0.01326 \times 23.53\right) = 4.7$$

$$Mn = 4.7 \times 250 \times 687.52 = 555.37\ KN.m$$

$$Mu = \emptyset Mn = 0.9 \times 555.37 = 499.83\ KN.m \qquad Mu = 490 KN.m \qquad OK$$

Example (2): Design a simply reinforced rectangular section with steel percentage $\rho = 0.5\ \rho_{\max}$ of previous example

Sol:

$\rho = 0.5\ \rho_{\max}$ then tension Controlled section ϕ=0.9

$\rho = 0.5 \times (0.01368)$ (previous Example Exa. (1))

$\rho = 0.00684$

$$R = \emptyset \rho f y \left(1 - \frac{1}{2}\rho\, m\right)$$

$$= 0.9 \times (0.00684) \times (420)\big(1 - 0.5 \times (0.00684) \times (23.53)\big) = 2.642$$

$$\mathrm{d} = \sqrt{\frac{Mu}{\emptyset b\ R}} = \sqrt{\frac{490 \times 106}{0.9\ b \times 2.642}}$$

Assume *b* to find *d*:

b mm	d mm	As mm²
250	907.9	1552.5
300	828.8	1700.7
350	767.3	1836.9
400	717.8	1963.8

Use $b = 300$, $then\ d = 828.2\ mm$, $As = 1700.7\ mm^2$

Use $\emptyset\ 25\ mm$

$Ab = 490\ mm^2$

$No.of\ bars = n = \frac{1700.7}{490} = 3.47\ mm$ $use\ 4\ bar$

To find the bw , how many bars can be contains :

$$n = \frac{b - 116 - 2 \times ds}{D + s} + 1 = \frac{300 - 116 - 2 \times 10}{25 + 25} + 1 = 4.28 \qquad use\ 4\ bar$$

Use One layer

Find total depth of Beam h

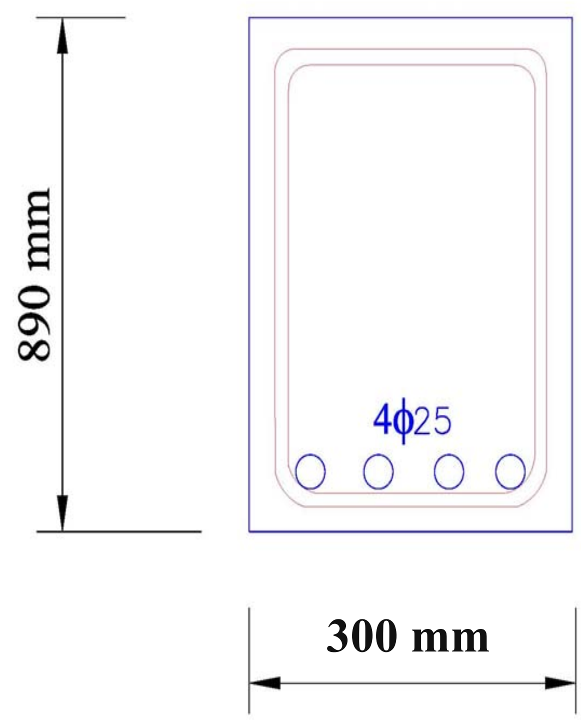
$$h = d + 38 + ds + \frac{D}{2}$$

$$= 828.8 + 38 + 10 + \frac{25}{2} = 889.3\ mm$$

Use h= 890 mm

Note that in this example (2) , the value of h use less than calculated nearest 10 mm , cause the provided steel area is larger than required area and this allow to use h less than calculated .

While in example (1) the selected h was greater than the calculated cause the provided steel area was less than required in very small a mount



$$d = 890 - 38 - 10 - \frac{25}{2} = 829.5\ mm$$

$$\rho = \frac{As}{b\ d} = \frac{4 \times 490}{300 \times 829.5} = 0.007876$$

$$R = \emptyset \rho f y \left(1 - \frac{1}{2}\ \rho\ m \right) = 0.07876\ (420)\big(1 - 0.5\ (0.007876)(23.53)\big) = 3.0\ MPa$$

$$Mn = 3\ \times 300\ \times (829.5)^2 = 619.26\ KN.m$$

$$\emptyset Mn = Mu = 0.9 \times 619.26 = 557.33\ KN.m > Mu = 490\ KN.m$$

Example (3): Find the necessary reinforcement for a given section that has a width of 250 mm and a total depth of 500mm ,if it is subjected to an external factored moment of 222 KN. m. Given: f'c= 28 mPa and fy= 420 mPa .

Solution

Assume one layer of steel

$$d = h - 65\,mm = 500 - 65 = 435\,mm$$

$$R = \frac{Mn}{bd^2} = \frac{Mu}{\emptyset bd^2} = \frac{222 \times 10^6}{0.9 \times 250 \times 435^2} = 5.214$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$

$$m = \frac{fy}{0.85\,f'c} = \frac{420}{0.85 \times 28} = 17.65$$

$$\rho = \frac{1}{17.65}\left(1 - \sqrt{1 - \frac{2 \times 17.65 \times 5.214}{420}}\right) = 0.01419$$

$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right) = \frac{0.85}{17.65}\left(\frac{600}{600 + 420}\right)(1) = 0.028328$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b = \frac{0.0051}{0.008} \times \rho_b = 0.6375\,\rho_b = 0.6375 \times 0.028328 = 0.018059 > \rho = 0.01419$$

Tension Controlled section $\emptyset = 0.9$

$As = \rho \times b \times d = 0.01419 \times 250 \times 435 = 1543.1\ mm^2$

Use Ø 20 mm (A_b= 314 mm^2)

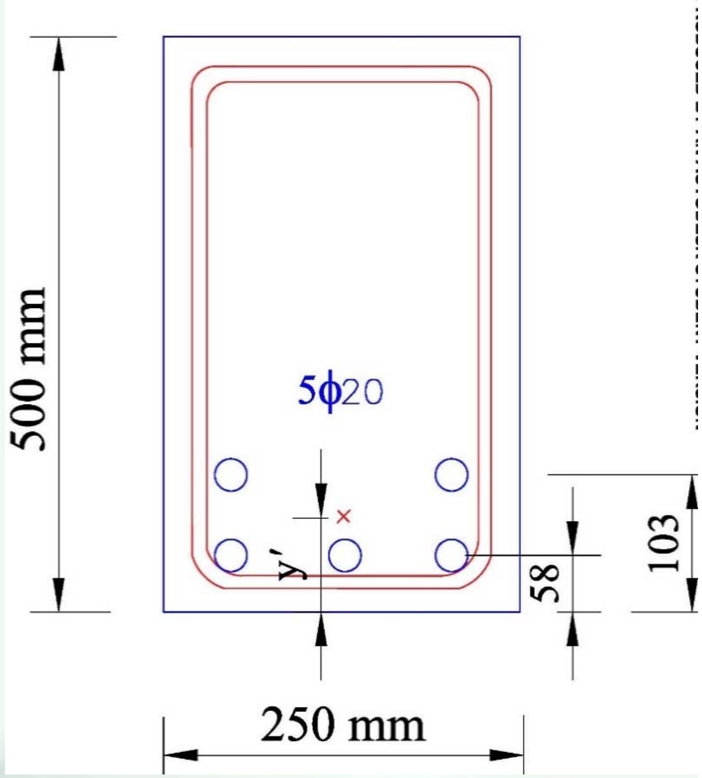
$No.of\ bars = \frac{1543.1}{314} = 4.91 \qquad Use\ 5Ø\ 20\ mm$

Check spacing between bars

$n = \frac{b - 116 - 2 \times ds}{D + s} + 1 = \frac{250 - 116 - 2 \times 10}{20 + 25} + 1 = 3.53 \quad use\ 3\ bars$

Need two layers

Or increased the steel bar area



$$y' = \frac{2 \times 314 \times 103 + 3 \times 314 \times 58}{5 \times 314} = 76\ mm$$

$d = h - 76 = 500 - 76 = 424\ mm$

$\rho = \frac{5 \times 314}{bd} = \frac{5 \times 314}{250 \times 424} = 0.01481$

$R = \rho fy \left(1 - \frac{1}{2}\rho m\right) = 0.01481 \times 420 \left(1 - \frac{1}{2}(0.01481 \times 17.65)\right) = 5.407$

$ØMn = Mu = ØRbd^2 = 0.9 \times 5.407 \times 250 \times 4242 = 218.72\ KN.m \quad < Mu\ applied$

Note: we can start solution by assuming two layer and d=h-90 mm

Thank You……….

Reinforced Concrete Design

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Rectangular Sections With Compression Reinforcement

A singly reinforced section has its moment strength when ρ_{max} of steel is used. If the applied factored moment is greater than the internal moment strength, as in the case of a limited cross section, a doubly reinforced section may be used, adding steel bars in both the compression and the tension zones. Compression steel will provide compressive force in addition to the compressive force in the concrete area.

The procedure for designing a rectangular section with compression steel when M_u, $f'c$, fy , b, d, and d' are given can be summarized as follows:

When Mu > $\emptyset$ Mn $_{max}$

1- calculate $\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600+fy}\right)\left(\frac{dt}{d}\right)$

and calculate $\rho_{max} = \left(\frac{0.003+\frac{fy}{Es}}{0.008}\right)\rho_b$ or calculate $A_{s1} = \rho_1 bd$ (maximum steel area as singly reinforced).

where $\rho_1 = 0.75\,\rho_{max}$ to ρ_{max} , and As_1 and its preferable to use $\rho_1 = 0.75\,\rho_{max}$ using to produces moment equal to Mn_1

$$R = \rho_1 fy\left(1 - \frac{1}{2}\rho\, m\right) \quad and \quad Mn_1 = Rbd^2 \; or \; Mu_1 = \emptyset Rbd^2$$

2. Calculate $M_{u2} = M_u - M_{u1}, or\ Mn_2 = Mn - Mn_1$, the moment to be resisted by compression steel.
3. Calculate the As_2 in tension zone where ;

$$As = As_1 + As_2 \quad \text{and} \quad As_2 = \frac{Mn_2}{fy(d-d')}$$

4. Calculate the compression stress at the compression steel and check the condition :

$$\rho_1 = \rho - \rho' \geq \left(\frac{\beta_1 d'}{m\, d}\right)\left(\frac{600}{600 - fy}\right)$$

It the condition is checked then: $fs' = fy$
And If not then $fs' < fy$ and $fs' \text{calculated from formula}$:

$$fs' = 600\left(1 - \frac{\beta_1 d'}{\rho_1\, m\, d}\right) \le fy$$

$$fs' = 600\left(\frac{C - d'}{C}\right) = 600\left(1 - \frac{d'}{C}\right)$$

$$a = \rho_1\, m\, d \quad and \quad C = \frac{a}{\beta_1}$$

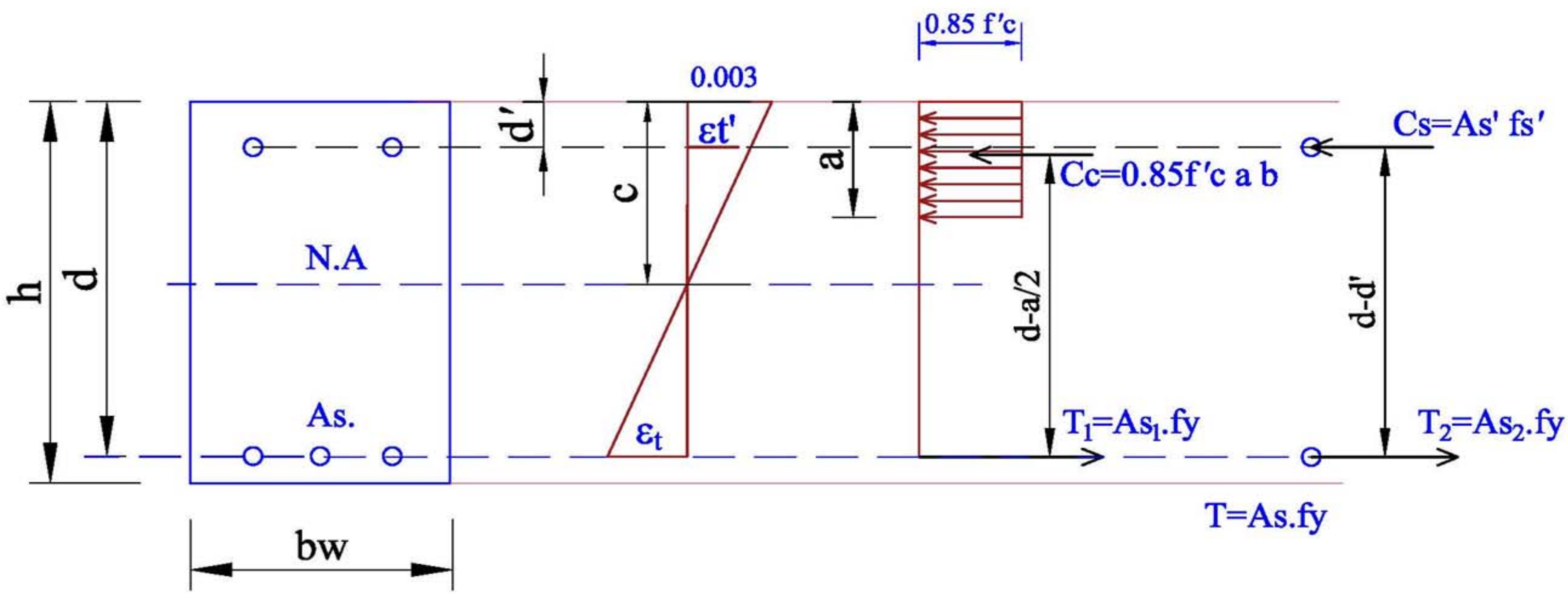
$$\therefore\ C = \frac{\rho_1\, md}{\beta_1}$$

$$\therefore fs' = 600\left(1 - \frac{\beta_1 d'}{\rho_1\, m\, d}\right)$$

In case of $fs' = fy$ use $As' = As_2$

$and\ fs' < fy$ use $As' = As_2 \times \left(\frac{fy}{fs'}\right)$

5. Choose the Tension steel bar diameter and compression steel bar whether can arrange in single layer



Example (4): A beam section is limited to a width $b = 250$mm. and a total depth $h = 550$ mm and has to resist a factored moment of 307 KN.m. Calculate the required reinforcement. Given: $f'_c = 21$ mPa and $f_y = 350$ mPa. $d' = 65$ mm.

Solution

Determine the design moment strength that is allowed for the section as singly reinforced based on tension-controlled conditions;

Assume (Two layer of steel) (assume ϕ 28 mm)

Then $d = h - 90 = 550 - 90 = 460mm$

$$dt = 460 + \frac{25}{2} + \frac{28}{2} = 486.5mm$$

1− calculate $\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right)$

$$m = \left(\frac{fy}{0.85\, f'c}\right) = \frac{350}{0.85 \times 21} = 19.61 \qquad \text{and} \qquad \beta_1 = 0.85$$

$$\rho_b = \frac{0.85}{19.61}\left(\frac{600}{600 + 350}\right)\left(\frac{486.5}{460}\right) = 0.02896$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b = \left(\frac{0.003 + \frac{350}{200000}}{0.008}\right) \times 0.02896 = 0.01719 \quad or \quad \rho_{max} = \frac{3}{8} \times \frac{\beta_1}{m}\left(\frac{dt}{d}\right) = 0.01719$$

$$Mu = \emptyset R b d^2$$

$$R = \frac{307 \times 10^6}{0.9 \times 250 \times 460^2} = 6.448$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right) = \frac{1}{19.61}\left(1 - \sqrt{1 - \frac{2 \times 19.61 \times 6.448}{350}}\right) = 0.02413 > \rho_{max} = 0.01719$$

Then Design the section as Double Reinforced Section (D.D.R.S)

2– Assume $\rho_1 = 0.75\,\rho_{max}$

$\rho_1 = 0.75 \times 0.01719 = 0.01289$

$As_1 = \rho_1\, b\, d = 0.01289 \times 250 \times 460 = 1482.6\ mm^2$

$R = \rho_1 fy\left(1 - \frac{1}{2}\rho\, m\right) = 0.01289 \times 350\left(1 - \frac{1}{2} \times 0.01289 \times 19.61\right) = 3.94$

$Mn_1 = Rbd^2 = 3.94 \times 250 \times 460^2 = 208.43\ KN.m$

3 – $Mn_2 = Mn - Mn_1 = \left(\frac{307}{0.9}\right) - 208.43 = 132.68\ kN.m$

4- Calculate the Total Tension Steel

$$As_2 = \frac{Mn_2}{fy\,(d - d')} = \frac{132.68 \times 10^6}{350\,(460 - 65)} = 959.7\ mm^2$$

$As = As_1 + As_2 = 1482.6 + 959.7 = 2442.3\ \text{mm}^2$

5- Check the stress in compression steel

$$\rho_1 = \rho - \rho' \not\geq \left(\frac{\beta_1 d'}{m\,d}\right)\left(\frac{600}{600 - fy}\right) = \left(\frac{0.85 \times 65}{19.61 \times 460}\right)\left(\frac{600}{600 - 350}\right) = 0.014699$$

$f's < fy = 350\text{MPa}$ NOT O.K

$$f's = 600\left(1 - \frac{\beta_1 d'}{\rho_1 m d}\right) = 600\left(1 - \frac{0.85 \times 65}{0.01289 \times 19.61 \times 460}\right) = 314.9\ MPa$$

$$As' = As_2 \times \left(\frac{fy}{fs'}\right) = 959.7 \times \left(\frac{350}{314.9}\right) = 1066.7\ mm^2$$

$For\ \emptyset 25\ mm\ (Ab = 490\ mm^2)$

$Use\ 5\ \phi\ 25\ mm = (5 \times 490 = 2450\ mm^2) > As\ required = 2442.3\ mm^2$

$For\ As'\ Use\ 3\ \phi\ 22\ mm = (3 \times 380 = 1140\ mm^2) > As'$ required = 1066.7 mm^2

6- Check no of Bars in one layer

use stirrup diameter ds = 10 mm

$$n = \frac{b - 116 - 2 \times ds}{D + s} + 1 = \frac{250 - 116 - 2 \times 10}{25 + 25} + 1 = 3.28 = 3$$

$$d' = 38 + 10 + \frac{22}{2} = 59\ mm$$

$$dt = 550 - 38 - 10 - \frac{25}{2} = 489.5\ mm$$

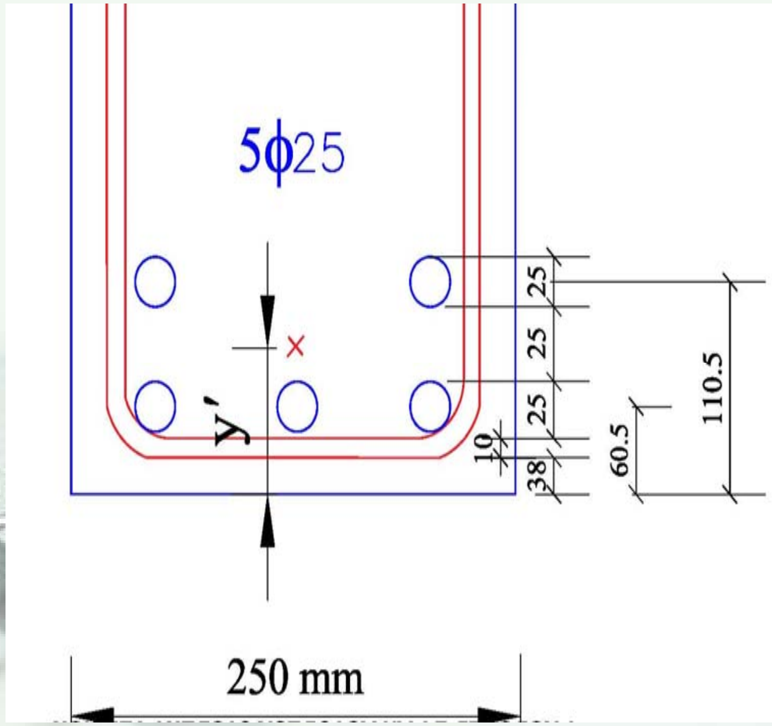
$$d = h - y'$$

$$y' = \frac{3 \times 490 \times 60.5 + 2 \times 490 \times 110.5}{5 \times 490} = 80.5\ \text{mm}$$

$$d = 550 - 80.5 = 469.5\ mm$$

$$\rho_b = \frac{0.85}{19.61}\left(\frac{600}{600 + 350}\right)\left(\frac{489.5}{469.5}\right) = 0.02854$$

$$\rho_{max} = \left(\frac{0.003 + \frac{350}{200000}}{0.008}\right) \times 0.02854 = 0.01695$$



$$\rho = \frac{As}{bd} = \frac{2450}{250 \times 469.5} = 0.020873$$

$$\rho' = \frac{As'}{bd} = \frac{1140}{250 \times 469.5} = 0.00971$$

Check again the stress in compression steel

$$\rho_1 = 0.01116 = \rho - \rho' \ngeq \left(\frac{\beta_1 d'}{m\ d}\right)\left(\frac{600}{600 - fy}\right) = \left(\frac{0.85\ \times 59}{19.61 \times 469.5}\right)\left(\frac{600}{600 - 350}\right) = 0.01307$$

Then : $f's < fy = 350\text{MPa}$

Check the failure at the tension steel :

$\rho - \rho' < \rho_{max}$

$\rho - \rho' = 0.020873 - 0.00971 = 0.01116 < \rho_{max} = 0.01695$ O.K

To find the $f's$ use the direct method where: (also can use other method to find a and c)

$A\ a^2 - Ba - C = 0$

$A = 1,$

$$B = m\ d\left(\rho - \frac{600}{fy}\rho'\right)$$

$$C = \frac{600}{fy}\beta_1\, m\ d\ d'\rho'$$

$$a = \frac{1}{2}\left[B + \sqrt{B^2 + 4\ A\ C}\right] \qquad C = \frac{a}{\beta}$$

$$B = 19.61 \times 469.5 \left(0.020873 - \frac{600}{350} \times 0.01116\right) = 16.03$$

$$C = \frac{600}{350} \times 0.85 \times 19.61 \times 469.5 \times 59 \times 0.01116 = 8833.4$$

$$a = \frac{1}{2}\left[16.03 + \sqrt{16.03^2 + 4 \times 1 \times 8833.4}\right] = 102.3mm$$

$$C = \frac{a}{\beta} = \frac{102.3}{0.85} = 120.35\ mm$$

$$fs' = \left(\frac{c - d'}{c}\right)\left(\frac{120.35 - 59}{120.35}\right) = 305.87\ MPa$$

$$\emptyset Mn = \emptyset\left[(Asfy - As'fs')\left(d - \frac{a}{2}\right) + As'fs'(d - d')\right]$$

$$= 0.9\left[(2450 \times 350 - 1140 \times 305.87) \times \left(469.5 - \frac{102.3}{2}\right) + 1140 \times 305.87 \times (469.5 - 59)\right]$$

$$= 320.4\ KN.m > Applied\ Mu = 307\ KN.m \qquad O.K$$

Another method to calculate *a and c*

$$f's = 600\left(1 - \frac{\beta 1\, d'}{\rho_1\, m\, d}\right) = 600\left(1 - \frac{0.85 \times 59}{0.011163 \times 19.61 \times 469.5}\right) = 307.2\ MPa$$

$$fs' = 600\left(\frac{c - d'}{c}\right)$$

$$307.2C = 600\,C - 35400$$

$$C = 120.9\ mm$$

$$a = 102.7\ mm \quad ok$$

Example (5): A beam section is limited to a width b = 300mm. and a total depth h = 500 mm and is subjected to a factored moment of 405 kN.m. Determine the necessary reinforcement. Given: f'_c= 28 MPa and f_y= 420 mPa, d'=65 mm.

Solution

1- Design the section considering single reinforced section (assume two layer of steel)

$d = h - 90 = 500 - 90 = 410\,mm$

$$R = \frac{Mn}{b \times d^2} = \frac{405 \times 10^6}{0.9 \times 300 \times 4102} = 8.923$$

$$m = \frac{fy}{0.85\,f'c} = \frac{420}{0.85 \times 28} = 17.65$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$

$$\rho_b = \frac{0.85}{17.65}\left(\frac{600}{600 + 420}\right) = 0.028329$$

$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b$$

$$= \left(\frac{0.003 + 0.0021}{0.008}\right)\rho_b = 0.6375 \times 0.028329 = 0.01806$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right) = \frac{1}{17.65}\left(1 - \sqrt{1 - \frac{2 \times 17.65 \times 8.923}{420}}\right) = 0.028329 > \rho_{max} = 0.01806$$

The Beam section should be design as D.D.R S

assume ρ_1 vary from 0.75 $\rho_{\max}$ to $\rho_{\max}$

$$Use\ \rho_1 = 0.9\,\rho_{\max} = 0.9 \times 0.01806 = 0.016254$$

$$As_1 = \rho_1 \times b \times d = 0.016254 \times 300 \times 410 = 1999.24\ mm^2$$

$$\rho_1 = 0.016254 = \rho - \rho' \not\geq \left(\frac{\beta_1 d'}{m\ d}\right)\left(\frac{600}{600 - fy}\right) = \left(\frac{0.85 \times 65}{17.65 \times 410}\right)\left(\frac{600}{600 - 420}\right) = 0.0254$$

So the $fs' < fy$

Check the stress in steel at compression zone from Formula:

$$f's = 600\left(1 - \frac{\beta d'}{\rho_1\ m\ d}\right) = 600\left(1 - \frac{0.85 \times 65}{0.016254 \times 17.65 \times 410}\right) = 318.16\ MPa$$

$$As' = As_2\left(\frac{fy}{fs'}\right)$$

$$As_2 = \frac{Mn_2}{fy(d - d')}$$

$$Mn_2 = Mn - Mn_1$$

$$Mn_1 = Rbd^2$$

$$R = \rho_1 fy\left(1 - \frac{1}{2}\rho_1 m\right)$$

$$R = 0.016254 \times 420\left(1 - \frac{1}{2} \times 0.016254 \ \times 17.65\right) = 5.848$$

$$Mn_1 == 5.848 \times 300 \times 410^2 = 294.9\ kN.m$$

$$Mn_2 = \frac{Mu}{\emptyset} - Mn_1 = \frac{405}{0.9} - 294.9 = 155.1\ KN.m$$

$$As_2 = \frac{Mn_2}{fy(d-d')} = \frac{155.18106}{420(410-65)} = 1070.4\ mm^2$$

$$As' = As_2 \times \left(\frac{fy}{fs'}\right) = 1070.4 \times \left(\frac{420}{381.16}\right) = 1179.5\ mm^2$$

$$As = As_1 + As_2 = 1999.24 + 1070.4 = 3070mm^2$$

As , Use 5 Ø 28 mm = 3075 mm^2

As' , Use 2 Ø 28 mm = 1230 mm^2

$$n = \frac{b - 116 - 2\ \times ds}{D+S} + 1 = \frac{300 - 116 - 2\ \times 10}{28+25} + 1 = 3.09$$

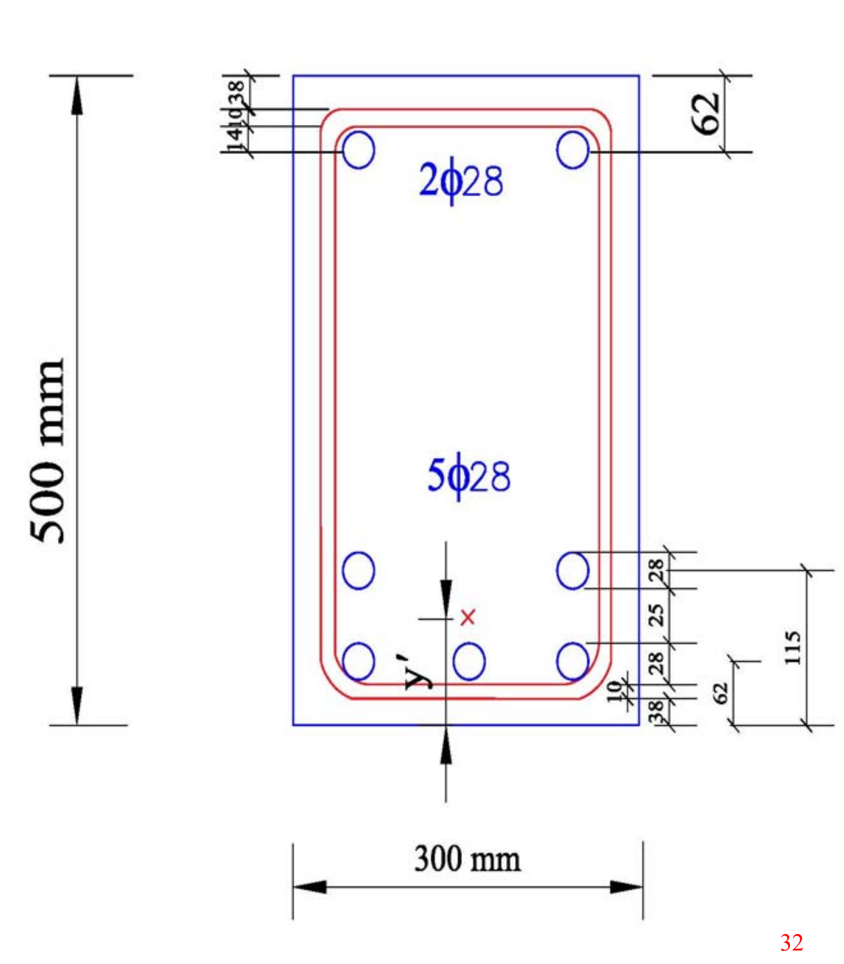
$$y' = \frac{2 \times (615) \times 115 + 3 \times (615) \times 62}{5\ \times 615} = 83.2\ mm$$

$$d' = 38 + 10 + \frac{28}{2} = 62\ mm$$

$$dt = 500 - 38 - 10 - \frac{28}{2} = 438mm$$

$$d = 500 - y' = 416.8\ mm$$

$$\rho_b = \frac{438}{416.8} \times 0.028329 = 0.02977$$



$\rho_{\max} = 0.6375\,\rho_b = 0.6375 \times 0.0297 = 0.01898$

$$\rho = \frac{3075}{300 \times 416.8} = 0.02459$$

$$\rho' = \frac{As'}{bd} = \frac{1230}{300 \times 416.8} = 0.009837$$

Check the stress in compression steel :

$$\rho_1 = 0.01475 = \rho - \rho' \ngeq \left(\frac{\beta_1 d'}{m\,d}\right)\left(\frac{600}{600 - fy}\right) = \left(\frac{0.85 \times 62}{17.65 \times 416.8}\right)\left(\frac{600}{600 - 420}\right) = 0.023879$$

So the $fs' < fy = 420\ MPa$

To find the value of fs' there is two method , Direct Method and Indirect Method

1- Direct Method

$A\,a^2 - Ba - C = 0$

$A = 1,$

$$B = m\,d\left(\rho - \frac{600}{fy}\rho'\right)$$

$$C = \frac{600}{fy}\beta_1\, m\, d\, d'\rho'$$

$$a = \frac{1}{2}\left[B + \sqrt{B^2 + 4\,A\,C}\,\right],$$

$$C = \frac{a}{\beta}$$

Find the constant ;

$$B = 17.65 \times 411.5\left(0.02459 - \frac{600}{420} * 0.009837\right) = 76.53$$

$$C = \frac{600}{420} \times 0.85 \times 17.65 \times 411.5 \times 62 \times 0.009837 = 5378.85$$

$$a = \frac{1}{2}\left[76.53 + \sqrt{76.53^2 + 4 \times 1 \times 5378.85}\right] = 120.989mm$$

$$C = \frac{a}{\beta} = \frac{120.989}{0.85} = 142.34\ mm$$

$$f's = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{142.34 - 62}{142.34}\right) = 338.7\ mPa$$

2- The In direct Method

$find\ a\ when\ fy = 420\ MPa$

$$a = \frac{As\ fy - As'fs'}{0.85f'c\ b} = \frac{3075 \times 420 - 1230 \times 420}{0.85 \times 28 \times 300} = 108.23\ mm \qquad (1st\ attempt)$$

$$C = \frac{108.23}{0.85} = 127.34\ mm$$

$$f's = 600\left(\frac{c - d'}{c}\right) = 600\left(\frac{127.34 - 62}{127.34}\right) = 307.9\ MPa \quad < 420\ Mpa$$

$$a = \frac{As\ fy - As'fs'}{0.85f'c\ b} = \frac{3070 \times 420 - 1230 \times 307.9}{0.85 \times 28 \times 300} = 127.55\ \text{mm} \qquad (2nd\ attempt)$$

and C= 150 mm

$$fs' = 600\left(\frac{150 - 62}{150}\right) = 352MPa$$

$$a = \frac{As\,fy - As'fs'}{0.85f'c\,b} = \frac{3070 \times 420 - 1230 \times 352}{0.85 \times 28 \times 300} = 119.95 \text{ mm} \qquad and\ C = 141.12\ mm \qquad (3rd\ attempt)$$

$$fs' = 600\left(\frac{141.12 - 62}{141.12}\right) = 336.4MPa$$

$$a = \frac{As\,fy - As'fs'}{0.85f'c\,b} = \frac{3070 \times 420 - 1230 \times 336.4}{0.85 \times 28 \times 300} = 122.6 \text{ mm} \qquad and\ C = 144.28\ mm \qquad (4th\ attempt)$$

$$fs' = 600\left(\frac{144.28 - 62}{144.28}\right) = 342.16.\ MPa$$

$$a = \frac{As\,fy - As'fs'}{0.85f'c\,b} = \frac{3070 \times 420 - 1230 \times 342.16}{0.85 \times 28 \times 300} = 121.6 \text{ mm} \qquad and\ C = 143.1\ mm \qquad (5th\ attempt)$$

$$fs' = 600\left(\frac{143.1 - 62}{143.1}\right) = 340.1MPa$$

$$\emptyset Mn = \emptyset\left[(Asfy - As'fs')\left(d - \frac{a}{2}\right) + As'fs'(d - d')\right]$$

$$= 0.9\left[(3075 \times 420 - 1230 \times 338.7) \times \left(416.8 - \frac{121.4}{2}\right) + 1230 \times 338.7 \times (416.8 - 62)\right]$$

$$= 413.4 \text{ KN.m} > Applied\ Moment\ \ Mu = 405\ KN.m \qquad OK$$

Reinforced Concrete Design

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LOGO

FLEXURAL DESIGN OF T- BEAM CONCRETE SECTION

Introduction

T-Beams RC floors normally consist of slabs and beams that are cast monolithically. The two act together to resist loads and because of this interaction, the effective section of the beam is a T or L section. T-section for interior beams L-section for exterior beams.

Normally, the thickness of slab varies between 100 mm and 200 mm and the web width its from 200 mm to 400 mm and its often known. Effective depth and As reinforcement quantity will be calculated. When effective stress block depth less than hf of slab thickness that's lead to design the Beam as a rectangular section while with a greater than hf , the section will be true T- section

Two Known Case for Design Procedures :

1- d is known and As should be calculated

A-Check the section is behave like rectangular section or T section . Assume a = hf and calculate the moment produce by the two flanges :

$$Mn_f(\,flange) = \emptyset\, 0.85\, f'c\; b.h_f\;(\,d - \frac{h_f}{2})$$

B- if the applied moment $Mu \;>\; Mn_f$ then :

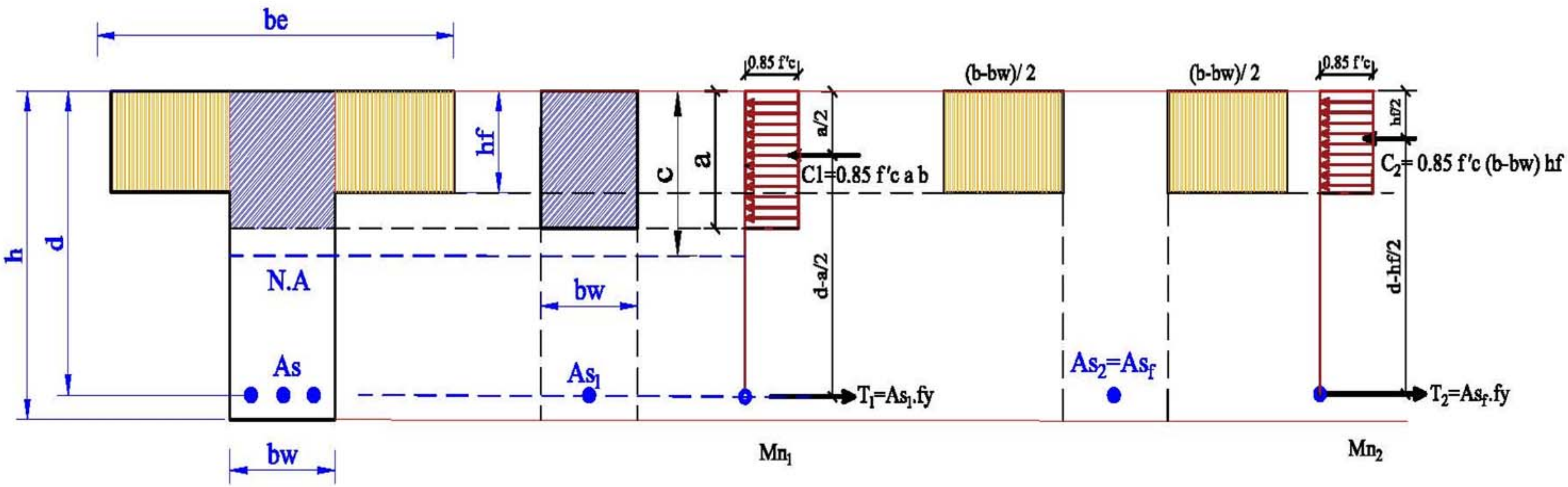
$a \;>\; h_f$ section should be design as T- Section

and if the applied moment $Mu \;<\; Mn_f$ then :

$a \;<\; hf$ and the section should be design as rectangular section ($b\; d$)

$$R = \frac{Mu}{\emptyset bd^{\,2}}$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$


be
h
d
hf
N.A
As
bw
bw
As1
c
a
0.85 f'c
a/2
C1=0.85 f'c a b
d-a/2
T1=As1.fy
Mn1
(b-bw)/ 2
(b-bw)/ 2
As2=Asf
0.85 f'c
hf/2
C2= 0.85 f'c (b-bw) hf
d-hf/2
T2=Asf.fy
Mn2

$As = \rho\, bd > As_{min}$

In T- section case calculate :

$Asf = \dfrac{(b - bw)h_f}{m}$ same $(Asf.fy = 0.85\, f'c.(b - bw)h_f)$

$Mu_2 = \emptyset\, Asf\, fy\left(d - \dfrac{h_f}{2}\right)$

$Mu = Mu_1 + Mu_2$

$R = \dfrac{Mu_1}{\emptyset bd^2}$, $\rho = \dfrac{1}{m}\left(1 - \sqrt{1 - \dfrac{2mR}{fy}}\right)$

$As_1 = \rho\, bd$ and Total $As = As_1 + As_2$

2- When As and d is Unknown:

A- Assume $a = hf$ then we can calculated the steel area at tension zone with equal the compression force for flange

$As_{ft} = \dfrac{b\,hf}{m}$ or $(As_{ft}.fy = 0.85\, f'c\, b\, hf)$

B- calculate d depending on calculated Asf and the Applied Mu

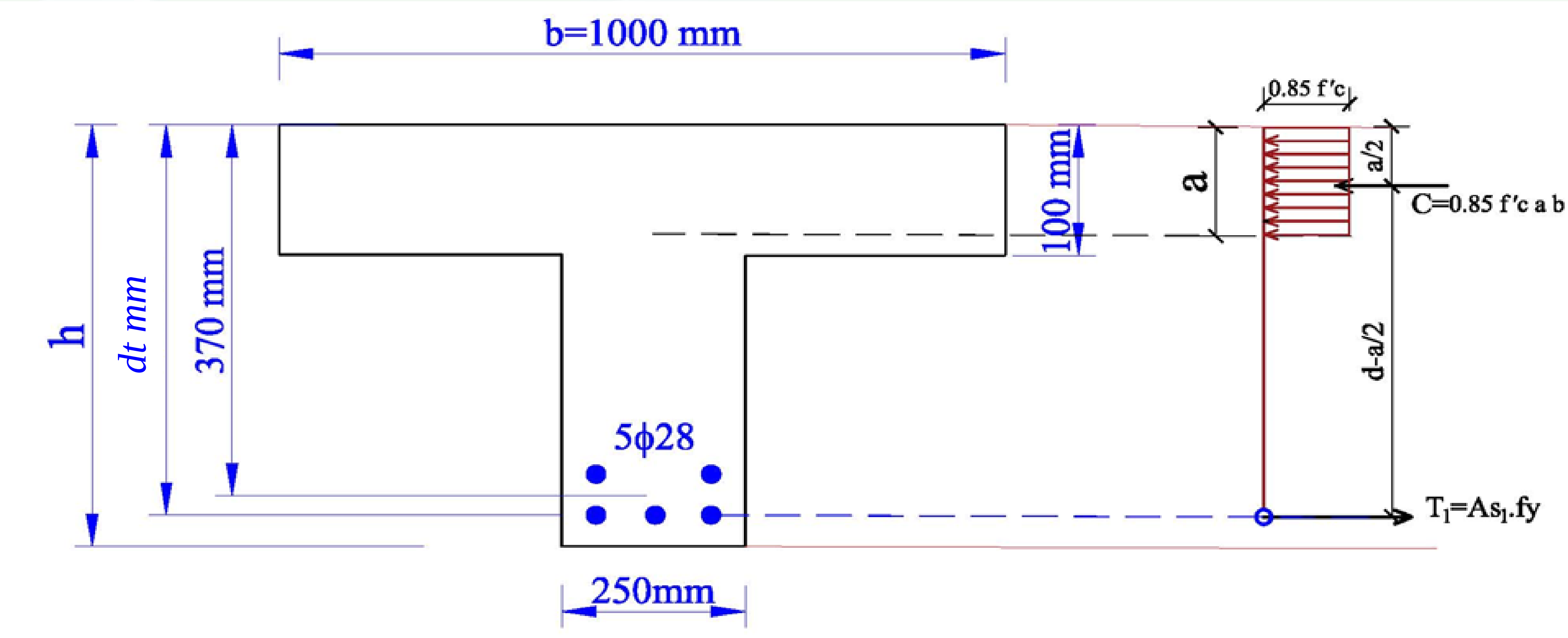
$Mu = \emptyset\, Asft\, fy\left(d - \dfrac{h_f}{2}\right)$

$$or \quad d = \frac{Mu}{\emptyset\, Asft\, fy} + \frac{h_f}{2}$$

If d is a suitable then :

$h = d + 90$ (for two layer) and
$h = d + 65$ (for one layer)

Example (6): The T-beam section Shown below has a width *bw* = 250 mm, a flange width be =1000 mm , a flange thickness = 100 mm and effective depth *d* = 370 mm. Determine the necessary reinforcement if the applied factored moment Mu= 380KN.m . Given: f'_c = 21 MPa and f_y = 420 Mpa.



1- Check the neutral axis depth

$$assume: \quad a = hf = 100\,mm$$

$$\emptyset Mn = \emptyset\,(\,0.85\,f'c)be\,hf\left(d - \frac{hf}{2}\right) = 0.9 \times 0.85 \times 21 \times 1000 \times 100\left(370 - \frac{100}{2}\right) = 514.08KN.m \;> 380KN.m$$

$$\therefore the\ section\ design\ as\ a\ Rectangular\ section\ with\ b = be = 1000\,mm$$

$$R = \frac{Mu}{\emptyset bd^2} = \frac{380 \times 10^6}{0.9 \times 1000 \times 370^2} = 3.084$$

$$m = \frac{fy}{0.85\,f'c} = \frac{420}{0.85 \times 21} = 23.53$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \times 23.53 \times 3.084}{420}}\right) = 0.008118$$

$$As = \rho\,bd = 0.008118 \times 1000 \times 370 = 3003.75\,mm^2$$

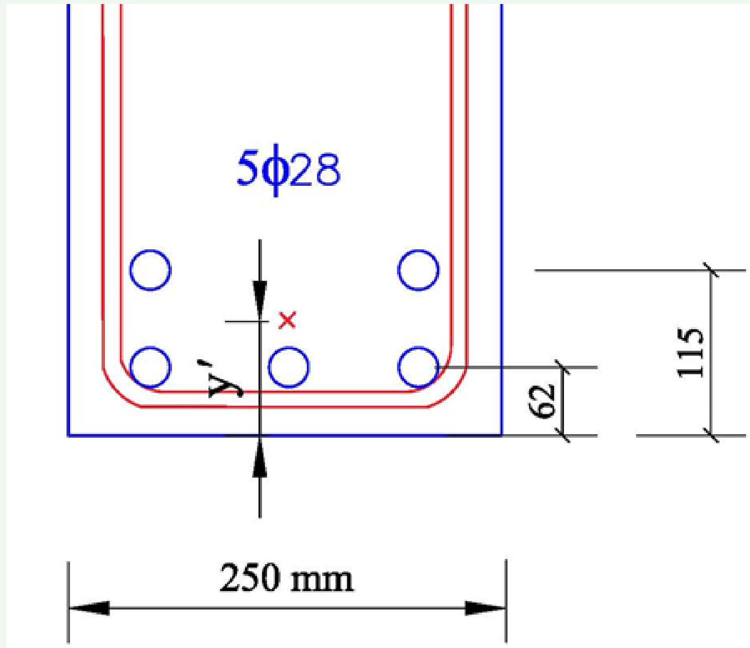
$$a = \rho\,m\,d = 0.008118 \times 23.53 \times 370 = 70.68\,mm < hf = 100\,mm$$

$$Total\ As = 5 \times 615 = 3075\,mm^2$$

$$\rho_w = \frac{3075}{250 \times 370} = 0.0332 > \rho_{min} = \frac{1.4}{420} = 0.0033$$

$$Max\ As = \frac{(b - bw)h_f}{m} + \rho_{\max}\ bw\ d$$

$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + 420}\right)\left(\frac{dt}{d}\right) = \frac{0.85}{17.65}\left(\frac{600}{600 + 420}\right)\left(\frac{dt}{d}\right)$$



$$y' = \frac{2 \times (615) \times 115 + 3 \times (615) \times 62}{5\ \times 615} = 83.2\ mm$$

$$h = 370 + y' = 370 + 83.5 = 453.5\ mm$$

$$dt = 453.5 - 38 - 10 - \frac{28}{2} = 391.5\ mm$$

$$\rho_b = \frac{0.85}{23.53}\left(\frac{600}{600 + 420}\right)\left(\frac{391.5}{370}\right) = 0.02248$$

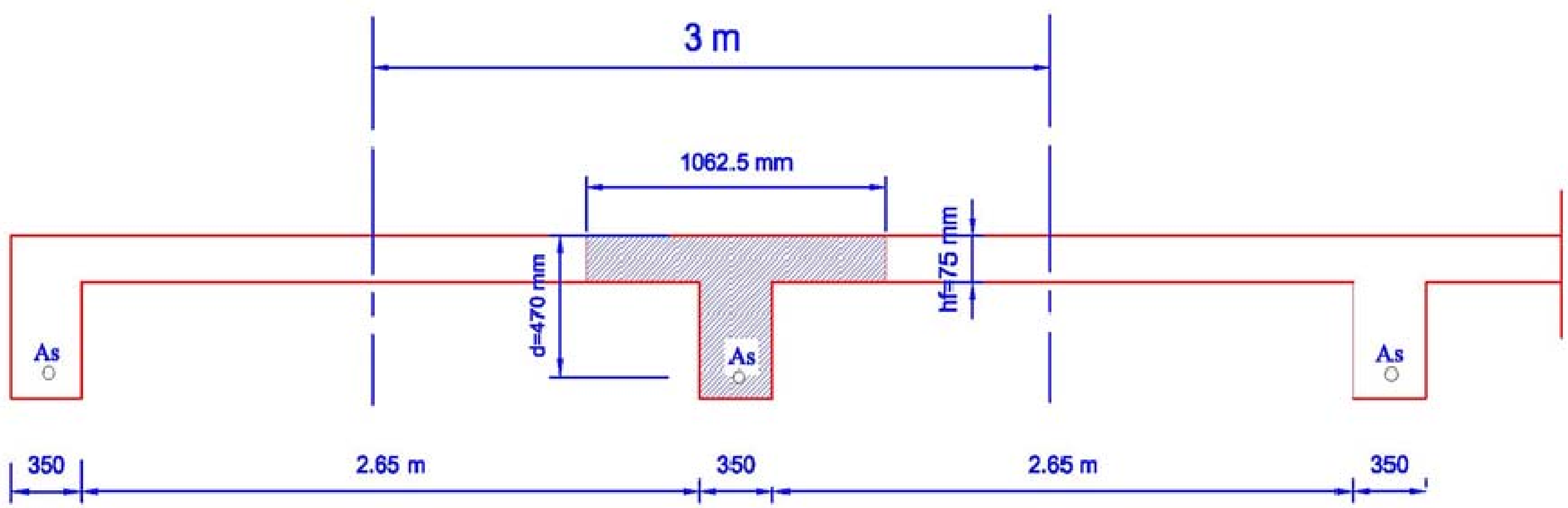
$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b = 06375 \times 0.02248 = 0.01433$$

$$Max\ As = \frac{(1000 - 250) \times 100}{23.53} + 0.01433 \times 250 \times 370 = 4513\ mm^2 > As = 3075\ mm^2$$

$$c = \frac{a}{\beta_1} = \frac{70.68}{0.85} = 83.15\ mm$$

$$\epsilon_t = \left(\frac{dt - c}{c}\right) \times 0.003 = \left(\frac{391.5 - 83.15}{83.15}\right) \times 0.003 = 0.0113 > 0.005\quad OK \qquad \emptyset = 0.9\ \ T.C$$

Example (7): The Floor system shown below consist of 75 mm slab thickness supported by 4.25 m span beam spaced 3 m on center. The beam have a web width bw = 350 mm and an effective depth d= 470 mm. Calculate the necessary reinforcement for a typical section interior beam if the factored applied moment Mu= 575 KN.m . $Given$: $f'_{c} = 21\ MPa$ and $f_y = 420\ mPa$, .



Solution :

find the effective be:

$1 - be = 16hf + bw = 16 \times 75 + 350 = 1550\,mm$

$2 - be = \frac{L}{4} = \frac{4250}{4} = 1062.5\,mm$

$3 - be\ = \ b\ (\,center\ to\ center\ adjacent\ panels) = \ 3000\,mm$

∴ be = 1062.5 mm

1-Design section as Rectangular Section :

$$R = \frac{Mu}{\emptyset bd^2} = \frac{575 \times 106}{0.9 \times 1062.5 \times 470^2} = 2.722$$

$$m = \frac{fy}{0.85\,f'c} = \frac{420}{0.85 \times 21} = 23.53$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \times 23.53 \times 2.722}{420}}\right) = 0.007069$$

$$a = \rho\ m\ d = 0.007069 \times 23.53 \times 470 = 78.18mm > hf\ = 75\ mm$$

∴ Design as T- Section

$$calculate\quad Asf = \frac{(b - bw)h_f}{m} = \frac{(1062.5 - 350) \times 75}{23.53} = 2271\ mm^2$$

$$Mu_2 = \emptyset Asf\ fy\left(d - \frac{hf}{2}\right) = 0.9 \times 2271 \times 420 \times \left(470 - \frac{75}{2}\right) = 371.27\ KN.m$$

$$\therefore Mu_1 = Mu - Mu_2 = 575 - 371.27 = 203.73\ KN.m$$

$$R = \frac{Mu}{\emptyset bd^2} = \frac{203.73 \times 10^6}{0.9 \times 350 \times 470^2} = 2.928$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \times 23.53 \times 2.928}{420}}\right) = 0.007662$$

$$As_1 = \rho\ bd = 0.007662 \times 350 \times 470 = 1260.35\ mm^2$$

Total As= As_1+As_2=1260.35+2271=3531.35 mm^2

Use 6 Ø 28 mm = 3690 mm^2

$$a = \rho\ m\ d = 0.007662 \times 23.53 \times 470 = 84.73\ mm$$

$$c = \frac{a}{\beta 1} = \frac{84.73}{0.85} = 99.68\ mm$$

$$\epsilon t = \left(\frac{dt - c}{c}\right) \times 0.003 = \left(\frac{496.5 - 99.68}{99.68}\right) \times 0.003 = 0.01194 > 0.005\quad OK$$

$$dt = 470 + \frac{25}{2} + \frac{28}{2} = 496.5\text{ mm}$$

$$\therefore \emptyset = 0.9 \qquad T.C$$

$$\rho_{min} = \frac{1.4}{fy} = 0.0033 < \rho\quad OK$$

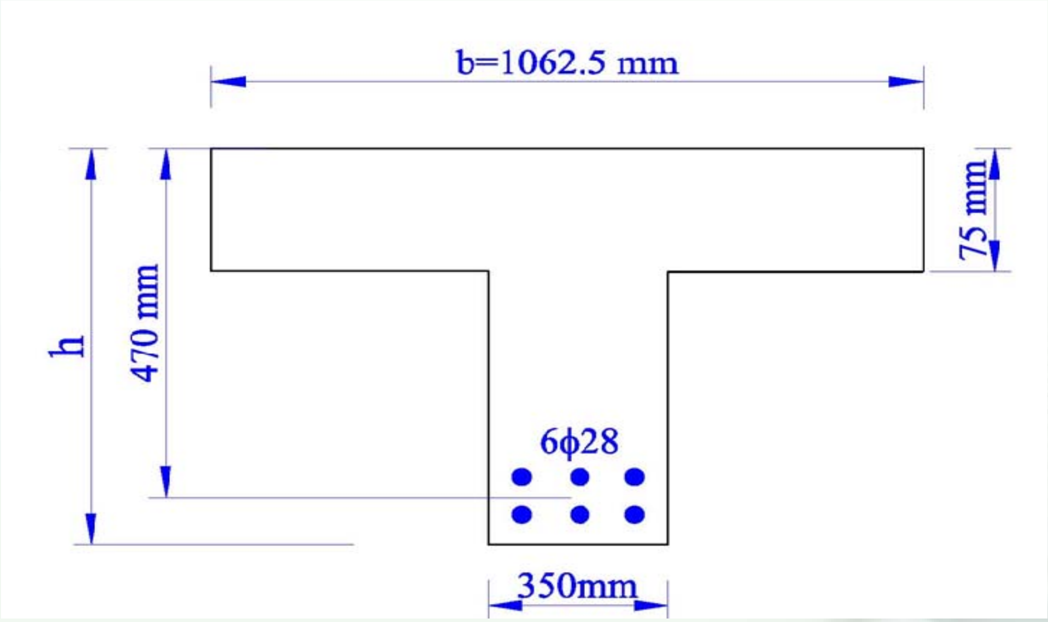
$$Max\ As = \frac{(b - bw)h_f}{m} + \rho_{\max}\ bw\ d$$

$$\rho_b = \frac{0.85}{23.53}\left(\frac{600}{600 + 420}\right)\left(\frac{496.5}{470}\right) = 0.02245$$

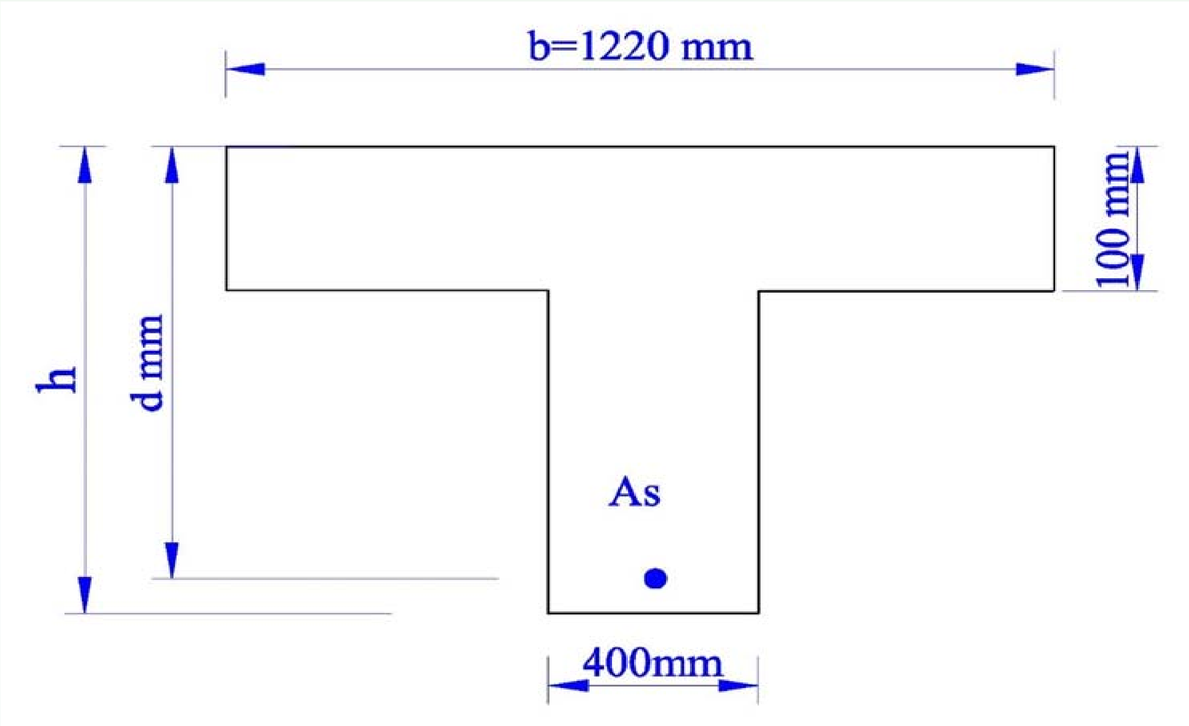
$$\rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b$$

$$= \left(\frac{0.003 + 0.0021}{0.008}\right)\rho_b = 0.6375 \times 0.02245 = 0.01431$$

$$Max\ As = \frac{(1032.5 - 350) \times 75}{23.53} + 0.01431 \times 350 \times 470 = 4625\ mm^2 > 3690\ mm^2 \quad OK$$



Example (7): In slab beam, The flange width was determine to be =1220 mm , the web width was bw=400 mm , and the slab thickness was hf= 100 mm . Design T- section to resist an external factored moment Mu= 1100 kN.m . Given: $f'_c = 21\ MPa$ and $f_y = 420\ mPa$, .



Solution

d is unknown

So choose $a = h_f = 100\ mm$

$T = C$

$As_{ft}\, fy = 0.85\, f'c\ b\ hf$

$$As_{ft} = \frac{0.85\, f'c\ b\ hf}{fy} = \frac{0.85 \times 21 \times 1220 \times 100}{420} = 5185\ mm^2$$

now calculate d from:

$$Mu = \emptyset Mn = \emptyset Asft\ fy\left(d - \frac{h_f}{2}\right) = 0.9 \times 5185 \times 420 \times \left(d - \frac{100}{2}\right)$$

$d = 661.24\ mm$

1- If we choose $d > 661.24\ mm$ (say 800 mm), in this case $a < hf$ and the section will be design as Rectangular section

$$R = \frac{Mu}{\emptyset bd^2} = \frac{1100 \times 10^6}{0.9 \times 11220 \times 800^2} = 1.565$$

$$m = \frac{fy}{0.85\, f'c} = \frac{420}{0.85 \times 21} = 23.53$$

$$\rho = \frac{1}{m}\left(1-\sqrt{1-\frac{2mR}{fy}}\right) = \frac{1}{23.53}\left(1-\sqrt{1-\frac{2\times 23.53\times 1.565}{420}}\right) = 0.003906$$

$As = \rho bd = 0.003906 \times 1220 \times 800 = 3812\ mm^2$

Use $8\ \emptyset\ 25\ mm\ (8 \times 490 = 3920\ mm^2)$

$$\rho w = \frac{3920}{400\times 800} = 0.01225 > \rho_{min} = \frac{1.4}{420} = 0.0033$$

$\rho_{max} = 0.6375\ \rho b$

$$dt = d + \frac{25}{2} + \frac{25}{2} = 825\ mm$$

$$\rho_{max} = 0.6375 \times \frac{0.85}{23.53}\left(\frac{600}{600+420}\right)\left(\frac{825}{800}\right) = 0.01397$$

$Max\ As = As_f + \rho_{\max}\ bw\ d$

$$Max\ As = \frac{(b-bw)h_f}{m} + \rho_{\max}\ bw\ d = \frac{(1220-400)\times 100}{23.53} + 0.01397\times 400\times 800 = 7955\ mm^2 > 3920\ mm^2 \quad OK$$

2- If we choose $d < 661.24\ mm$, (say 800 mm) in this case $a > hf$ and the section will be design as T − section

$$Calculate\ \ As_f = \frac{(b-bw)h_f}{m} = \frac{(1220-400)\times 100}{23.53} = 3484.9\ mm^2$$

$$Mu_2 = \emptyset Asf\ fy\left(d - \frac{h_f}{2}\right) = 0.9\times 3484.9\times 420\times\left(600-\frac{100}{2}\right) = 724.51\ KN.m$$

$\therefore Mu_1 = Mu - Mu_2 = 1100 - 724.51 = 375.49\ KN.m$

$$R = \frac{Mu}{\emptyset bd^2} = \frac{375.49 \times 106}{0.9 \times 400 \times 600^2} = 2.8973$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right) = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \times 23.53 \times 2.8973}{420}}\right) = 0.007573$$

$As_1 = \rho\, bd = 0.007573 \times 400 \times 600 = 1817.5\ mm^2$

Total As= $As_1 + As_2$=1817.5+3484.9=5302.4 mm^2

Use 8 Ø 30 mm = 5648 mm^2

$$n = \frac{b - 116 - 2 \times ds}{D + s} + 1 = n = \frac{400 - 116 - 2 \times 10}{30 + 25} + 1 = 5.8 \cong 5$$

$$dt = d + \frac{30}{2} + \frac{25}{2} = 627.5\ mm$$

$$a = \rho\, m\, d = 0.007573 \times 23.53 \times 600 = 106.9\ mm$$

$$c = \frac{a}{\beta_1} = \frac{106.9}{0.85} = 125.8\ mm$$

$$\epsilon_t = \left(\frac{dt - c}{c}\right) \times 0.003 = \left(\frac{627.5 - 125.8}{125.8}\right) \times 0.003 = 0.01196 > 0.005 \quad (OK)$$

$\therefore \emptyset = 0.9 \qquad T.C$

Thank You……….

Reinforced Concrete Design

Analysis and Design of One Way Concrete Slab

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Civil Engineering Department

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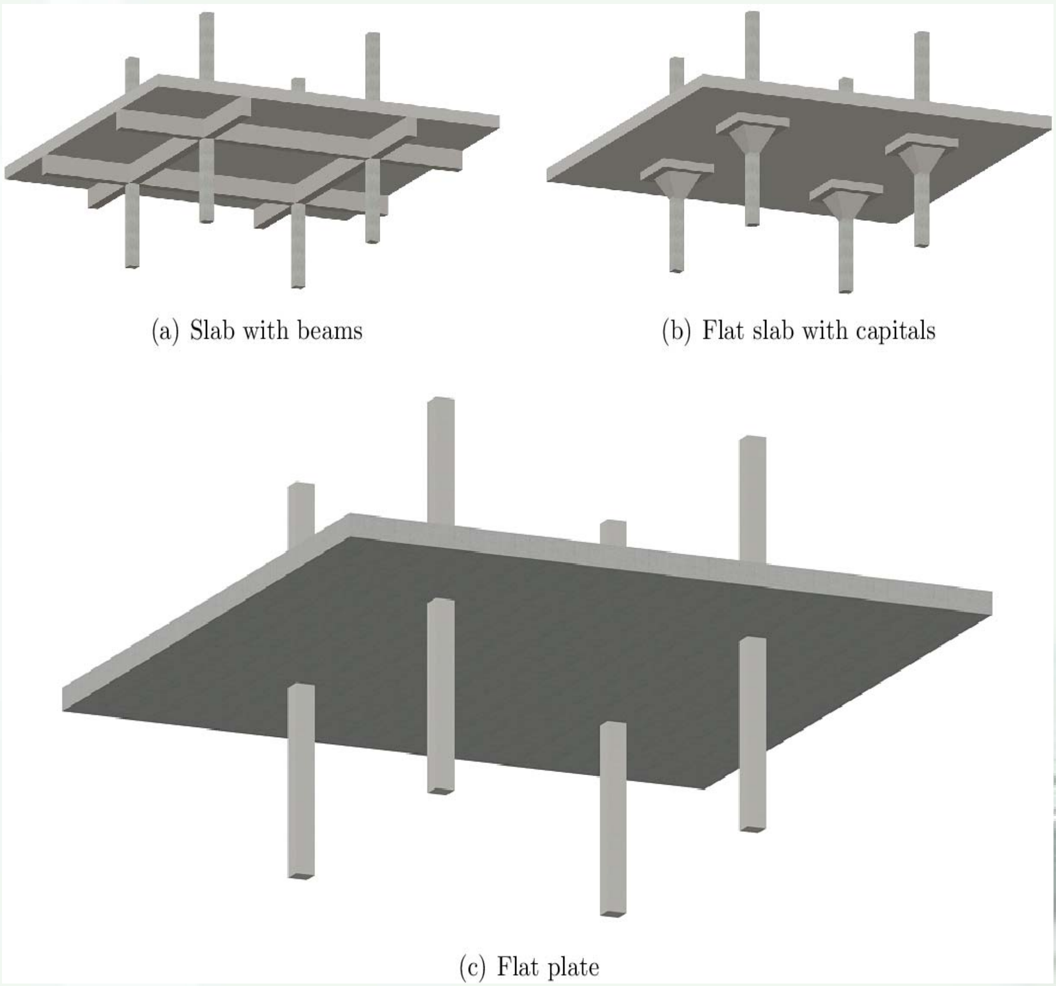
LOGO

The Type Of Slab

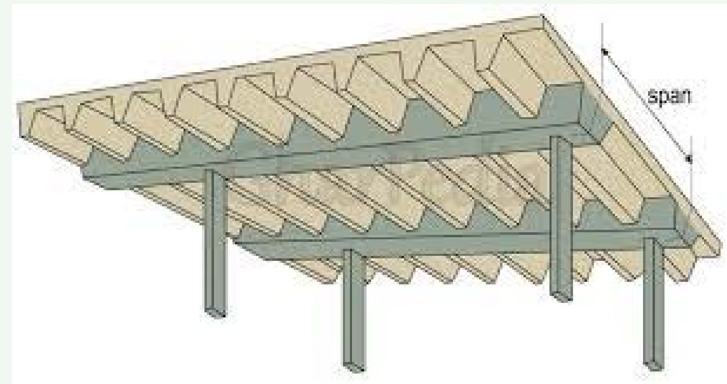
1-One-way slabs 2- Two-way floor systems   3-Flat slab   4- Waffle slab   5- Ribbed Slab

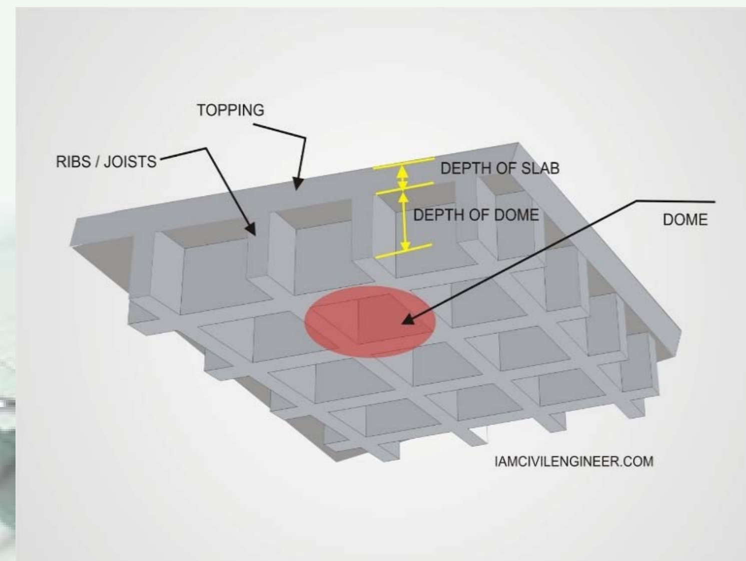


(a) Slab with beams

(b) Flat slab with capitals

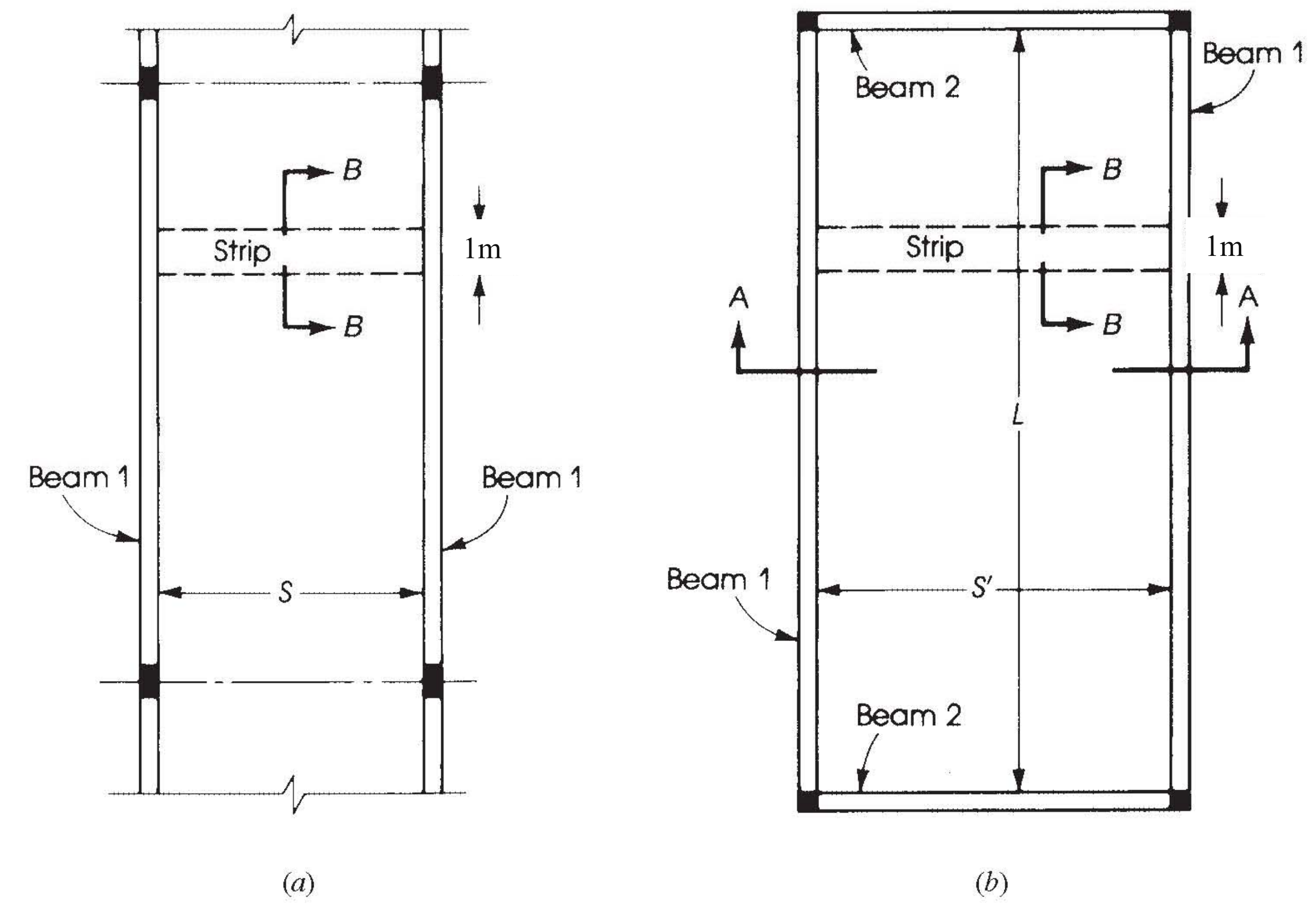
(c) Flat plate

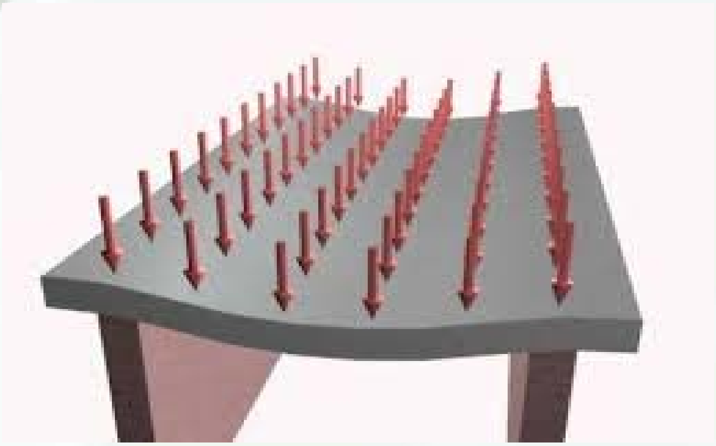


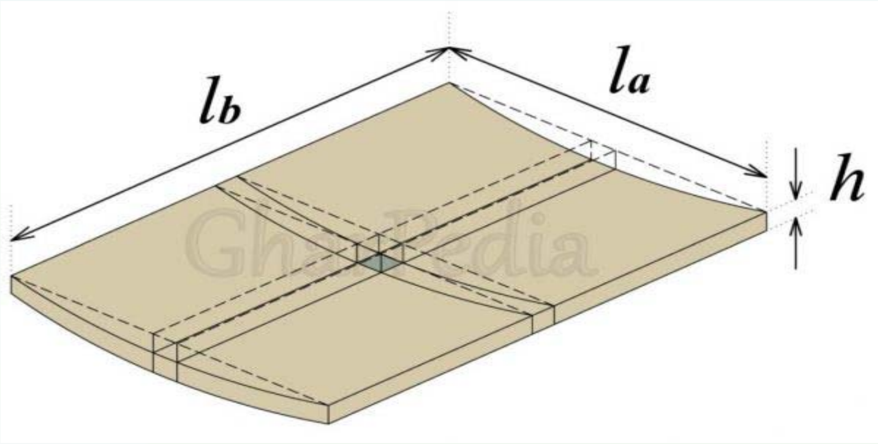


2. One Way Slab:

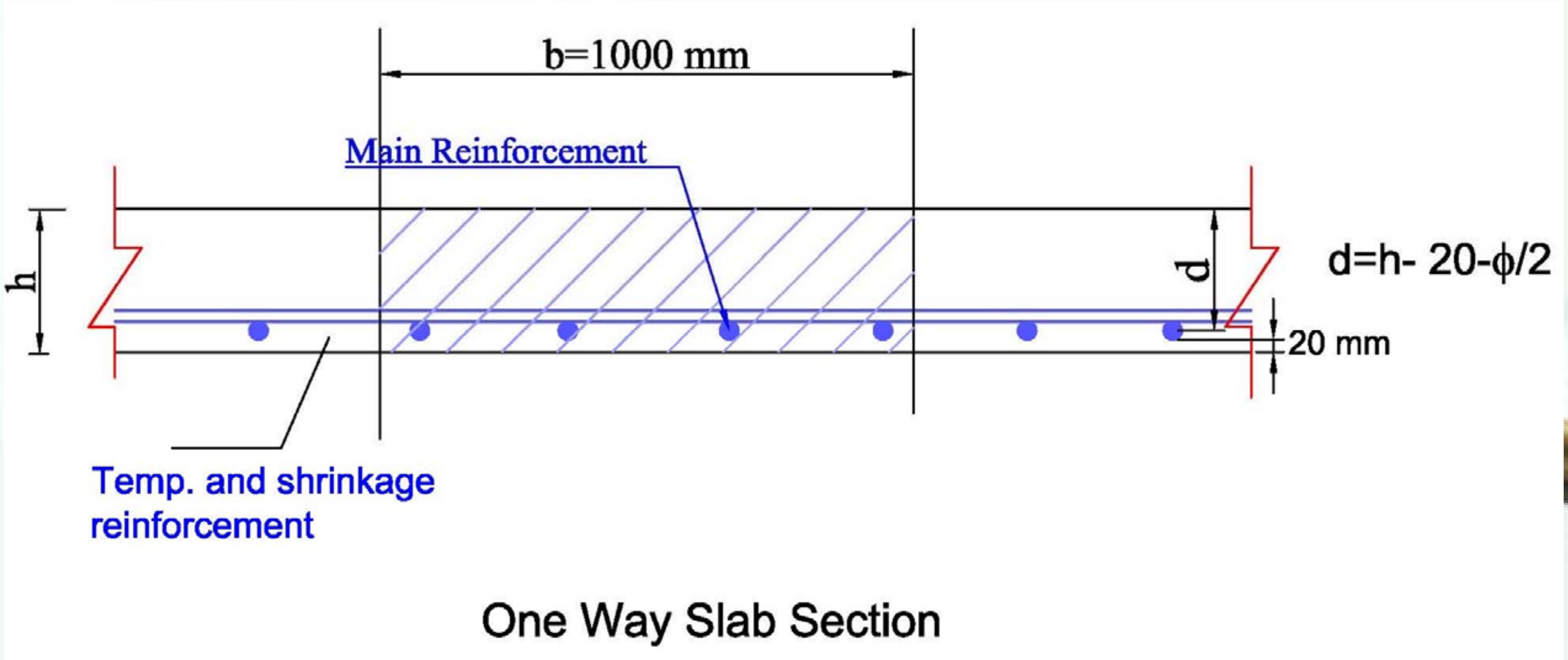
$$\frac{Longer Span}{Short Span} \geq 2$$





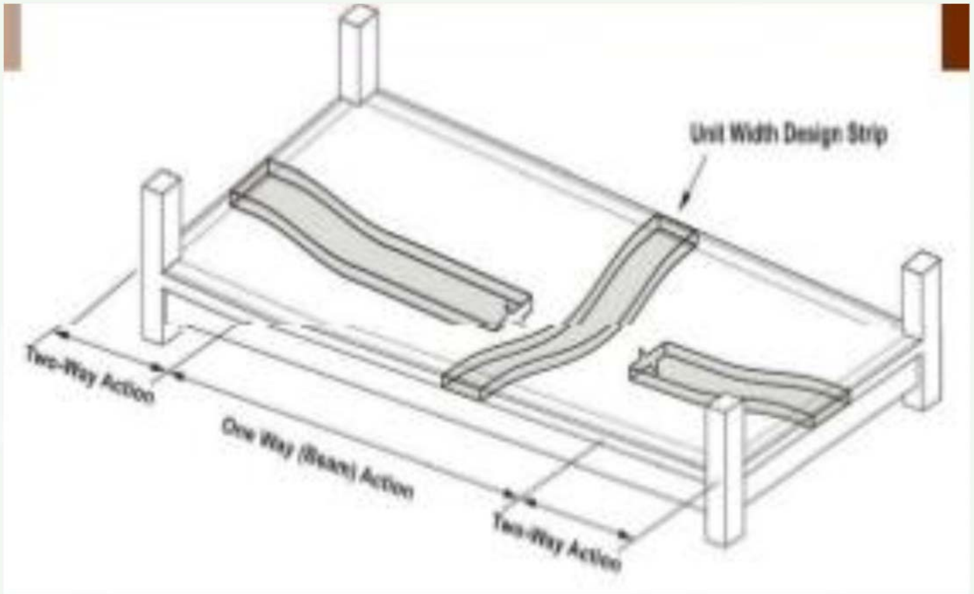


One way Slab Deflection



2. Two Way Slab:

$$\frac{LongerSpan}{ShortSpan} < 2$$



Two way Slab Deflection

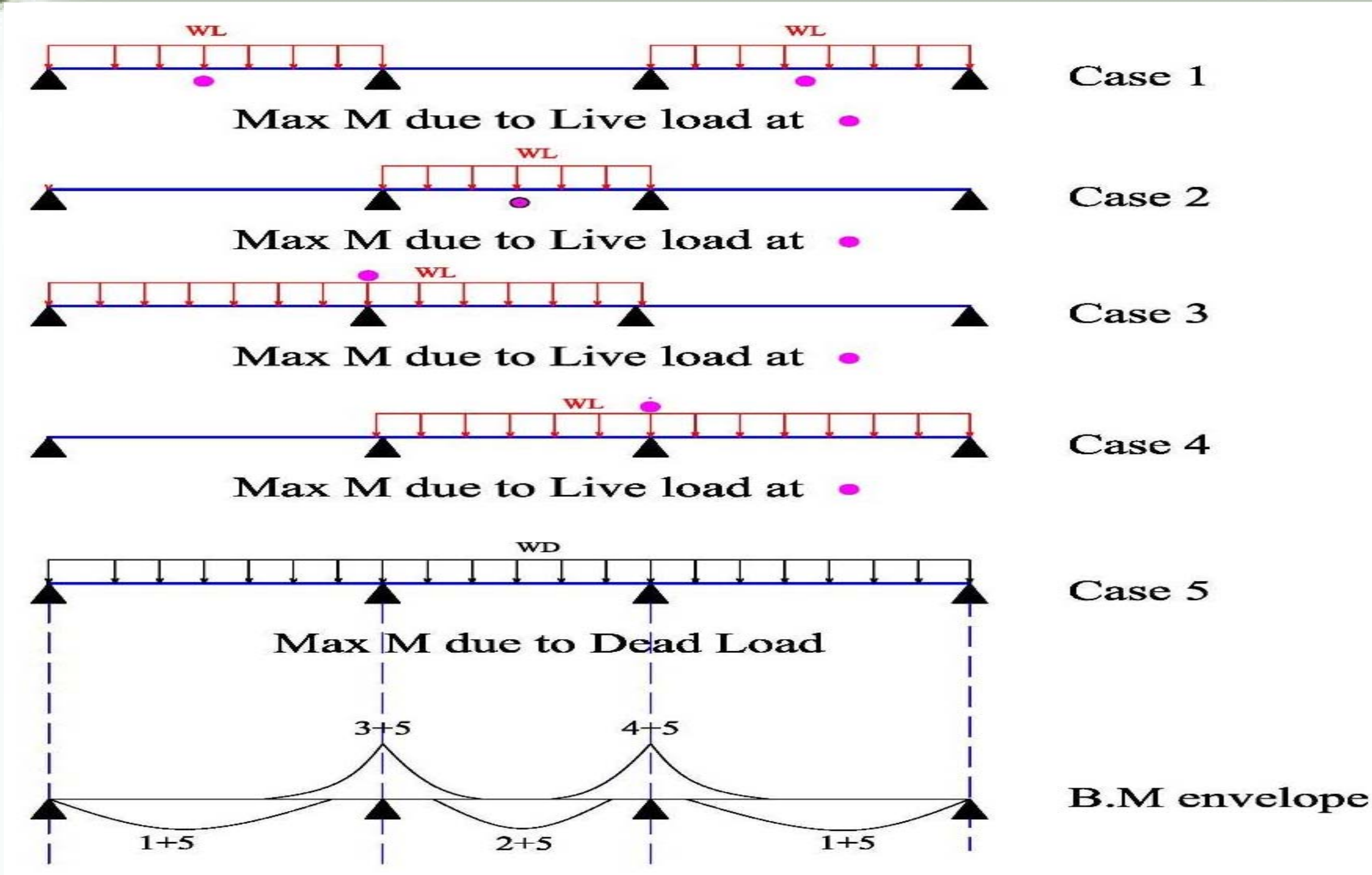
Design of One way slab

For simply supported slab the max. positive bending moment

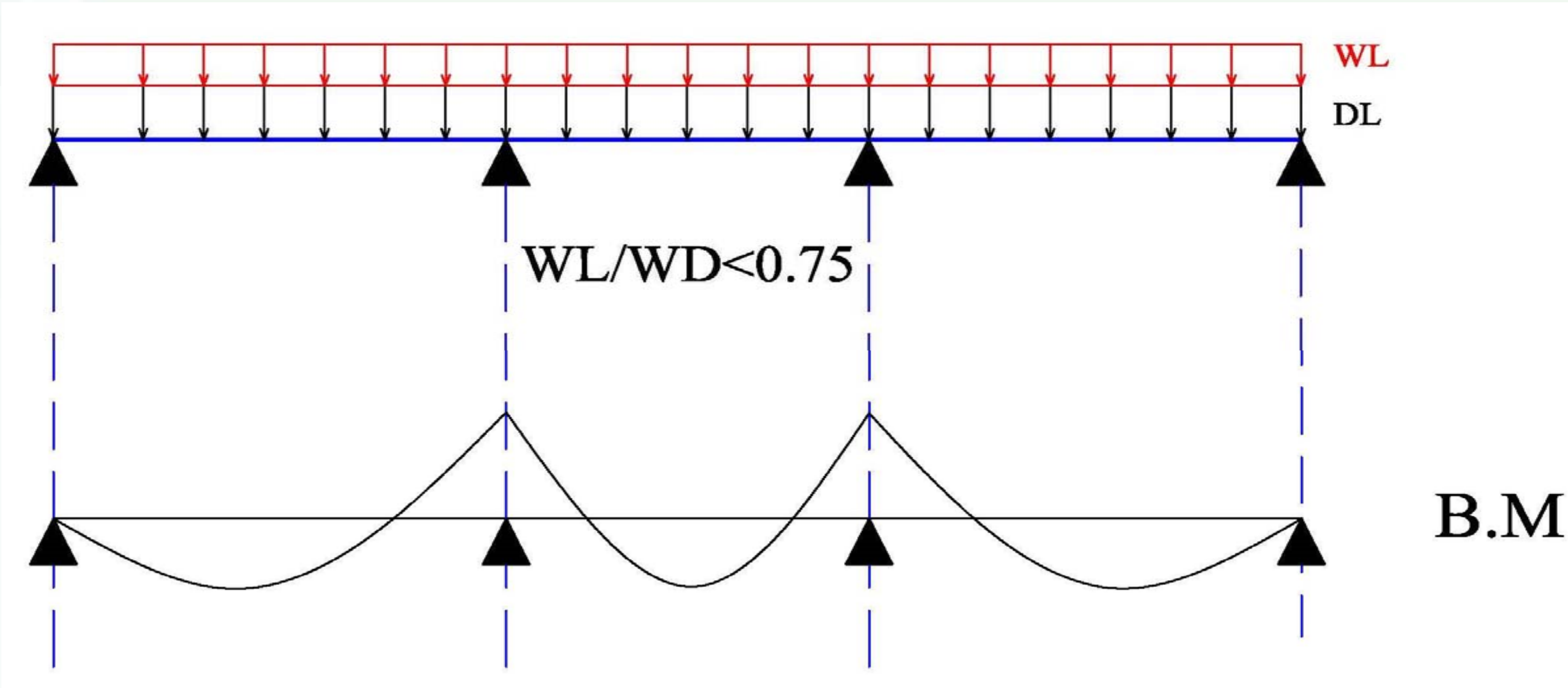
$+M = \frac{Wu\ ln^2}{8}$ where wu = KN/m² · ln = clear span in short direction

While for continuous span the Moment at mid and support should be calculated according to method of structures analysis. To find the Critical section the live should arrange to the spans in different position to get the envelope of bending moment Diagram as shown below:

Loading Cases



Bending moment envelopes for the critical section when the WL/WD> 0.75



Bending moment for the critical section when the WL/WD < 0.75

Thank You

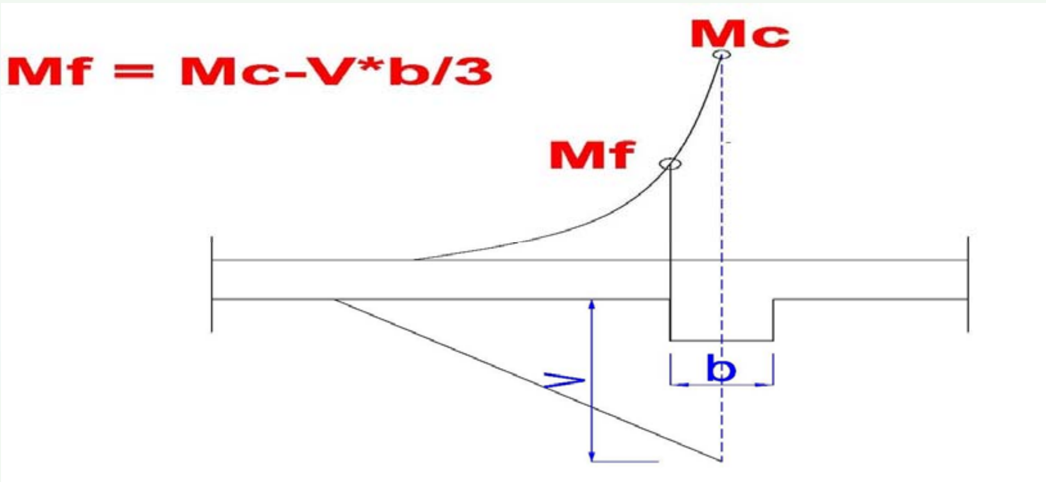
……………To be Continued

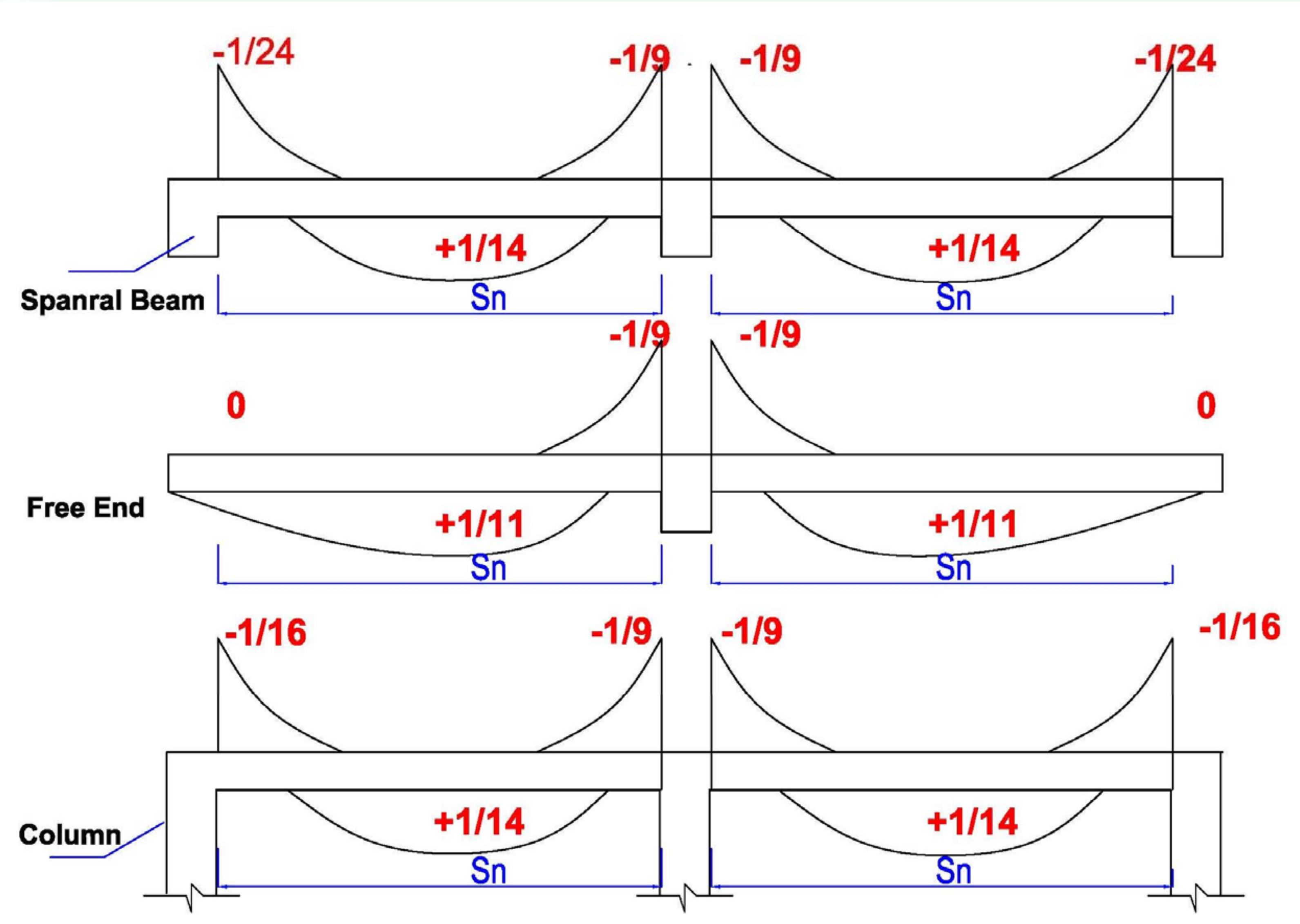
ACI Code Coefficient Methods (ACI Item 6.5)

1. Members are prismatic.
2. Load are uniform ally distributed.
3. Live Load $\leq 3 \times Dead\ Load$
4. There are at least two span.
5. The longer of the two adjacent spans does not exceed the shorter by more than 20 Percent (L ≤ 1.2 S)

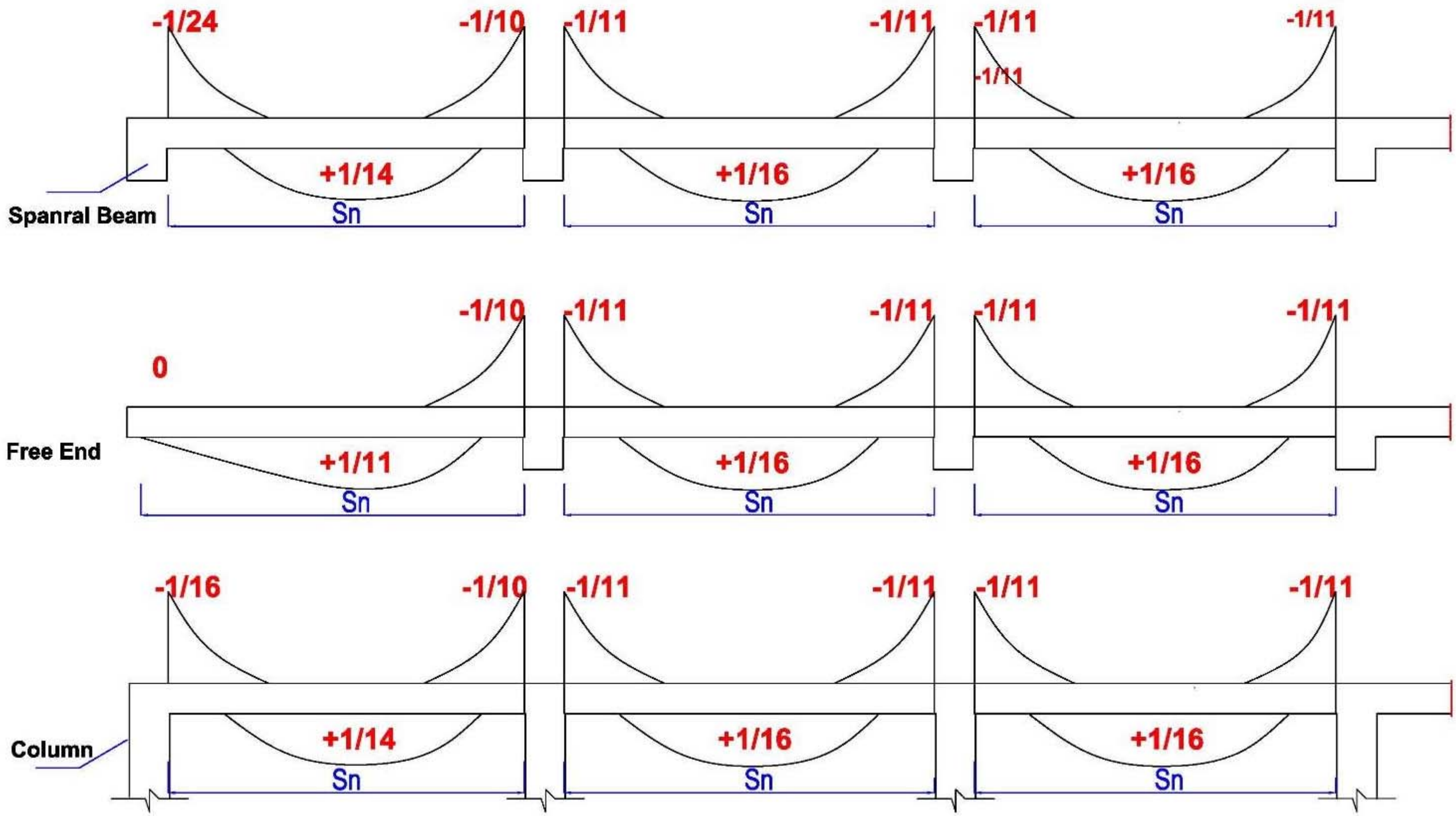
$$Mu = (coefficient)\,(Wu\,Sn^2) = C_f\,Wu\,Sn^2$$

S_n =clear span.

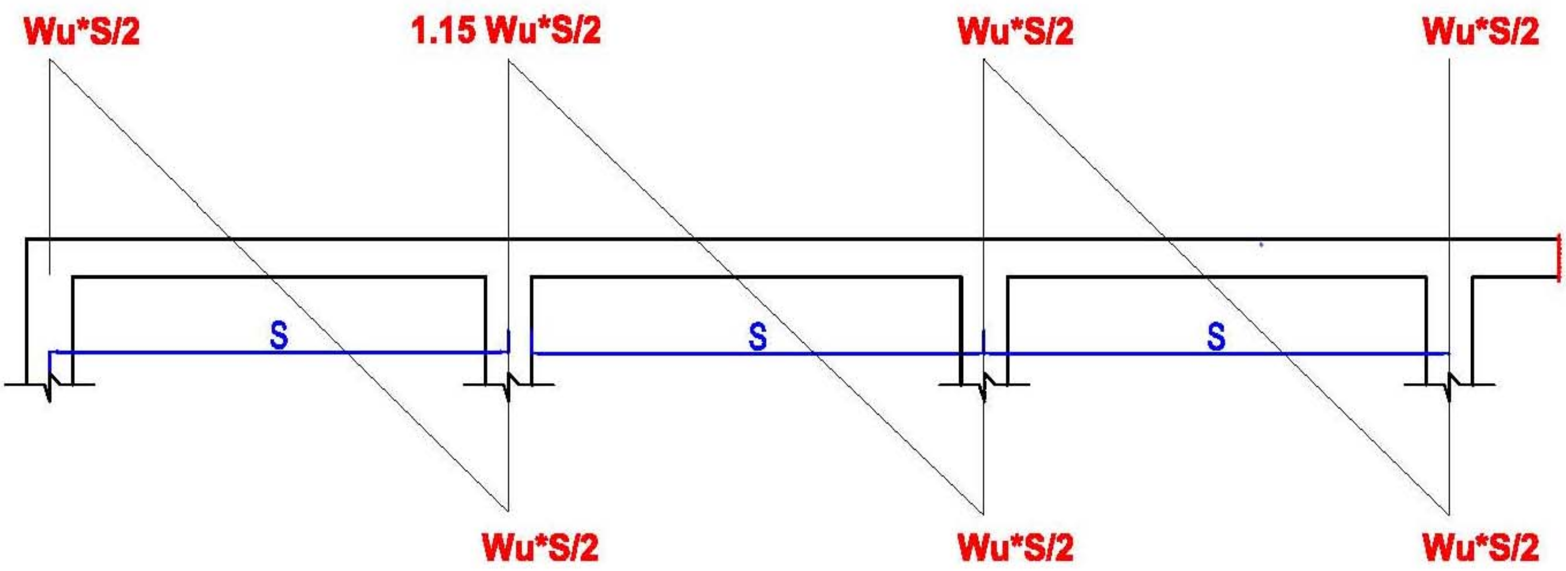



-1/24
-1/9
-1/9
-1/24
+1/14
+1/14
Spanral Beam
Sn
Sn
-1/9
-1/9
0
0
Free End
+1/11
+1/11
Sn
Sn
-1/16
-1/9
-1/9
-1/16
+1/14
+1/14
Column
Sn
Sn

Two Span


-1/24
-1/10
-1/11
-1/11
-1/11
-1/11
-1/11
+1/14
+1/16
+1/16
Spanral Beam
Sn
Sn
Sn
-1/10
-1/11
-1/11
-1/11
-1/11
0
Free End
+1/11
+1/16
+1/16
Sn
Sn
Sn
-1/16
-1/10
-1/11
-1/11
-1/11
-1/11
Column
+1/14
+1/16
+1/16
Sn
Sn
Sn

B.M Factors



Shear Force Factors

Design Limitations of ACI Code.

Design Limitations of ACI Code.

1- minimum depth of Slab (h) when Fy=420 Mpa for solid one way slab should follow the ACI item 7.3.1.1 for normal concrete only

Support condition	Minimum Thickness (h)
Simply Supported	L/20
One End Continuous	L/24
Both End Continuous	L/28
Cantilever	L/10

For f_y other than 420 MPa, these values shall be multiplied by $(0.4 + f_y/700)$

2- Deflection is to be checked when the slab supports are attached to construction likely to be damaged by large deflections. Deflection limits are set by the ACI Code, Table 24.2.2.

3- It is preferable to choose slab depth to the nearest 10 mm.

4. Shear should be checked, although it does not usually control.

5-Concrete cover in slabs shall not be less than (20 mm) at surfaces not exposed to weather

or ground for bar dia. 36 mm and less, while concrete cover not less than 40 mm for bar greater than 36 mm in dia. ACI table 20.6.13.1

6-In structural one way slabs of uniform thickness, the minimum amount of reinforcement (As min. in the direction of the span shall not be less than that required for shrinkage and temperature

reinforcement (ACI Code, Sections 7.6.1 and 24.4.3).

7. The main reinforcement maximum spacing shall be the lesser of three times the slab thickness (3 * h) and 450 mm . (ACI Code, Section 7.7.2.3).

8- In addition to main reinforcement, steel bars at right angles to the main must be provided. this additional steel is called secondary, distribution, shrinkage, or temperature reinforcement.

Temperature and shrinkage reinforcement .

$The\ minimum\ reinforcement\ \ should\ be\ equal\ or\ greater\ than$: ACI 2019 24.4.3.2

$$\rho_{\ min} = 0.0018$$

Max spacing of reinforcement should be greater than

1. $5 * h$ $\quad (h = slab\ thickness)$
2. $450\ mm$

Choose the smaller of above value

The width of slab strip= 1000 mm

$As_{min} = \rho_{\ \mathrm{min}} \times b \times h = 0.0018 \times b \times h$ minimum shrinkage and temperature steel

Reinforcement Details

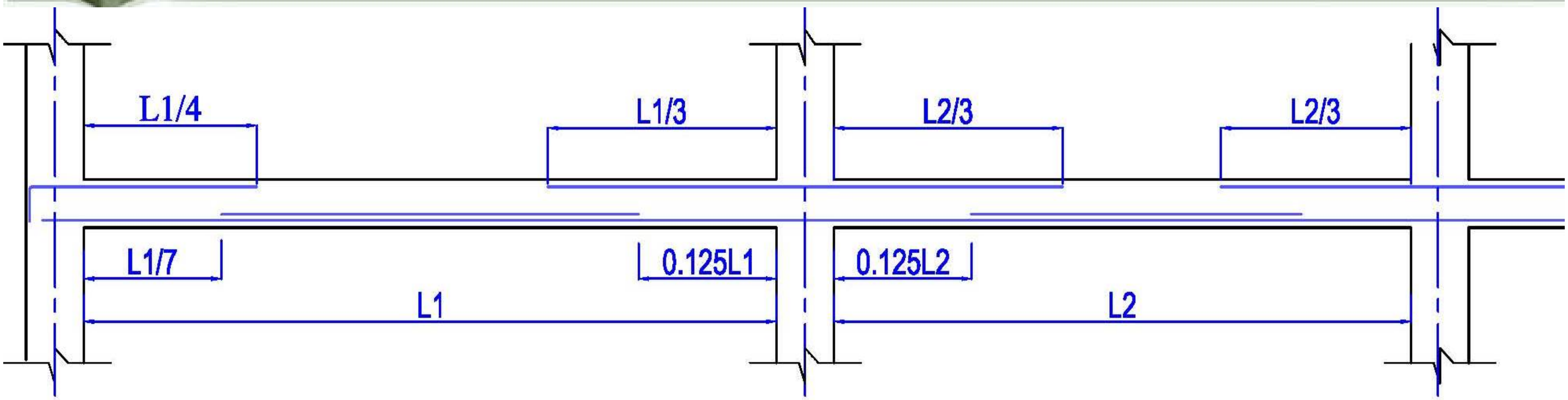
In continuous one-way slabs, the steel area of the main reinforcement is calculated for all critical sections, at midspans, and at supports. There is two reinforcement system can be applied

1- Straight-bar.

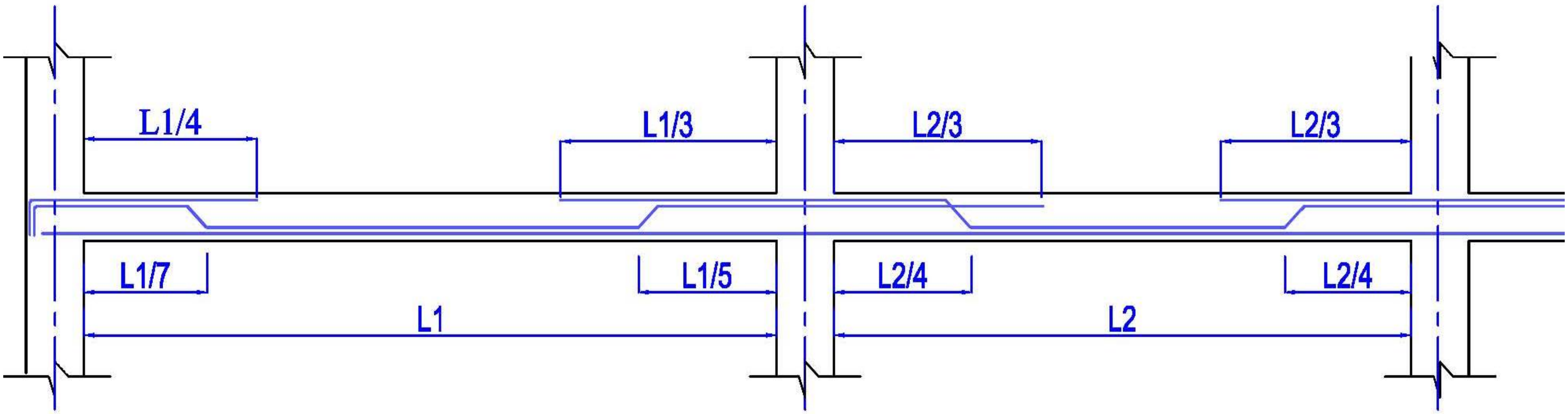
2-Bent-bar, or trussed system.

straight and bent bars are placed alternately in the floor slab.

The location of bent points should be checked for flexural, shear, and development length requirements.

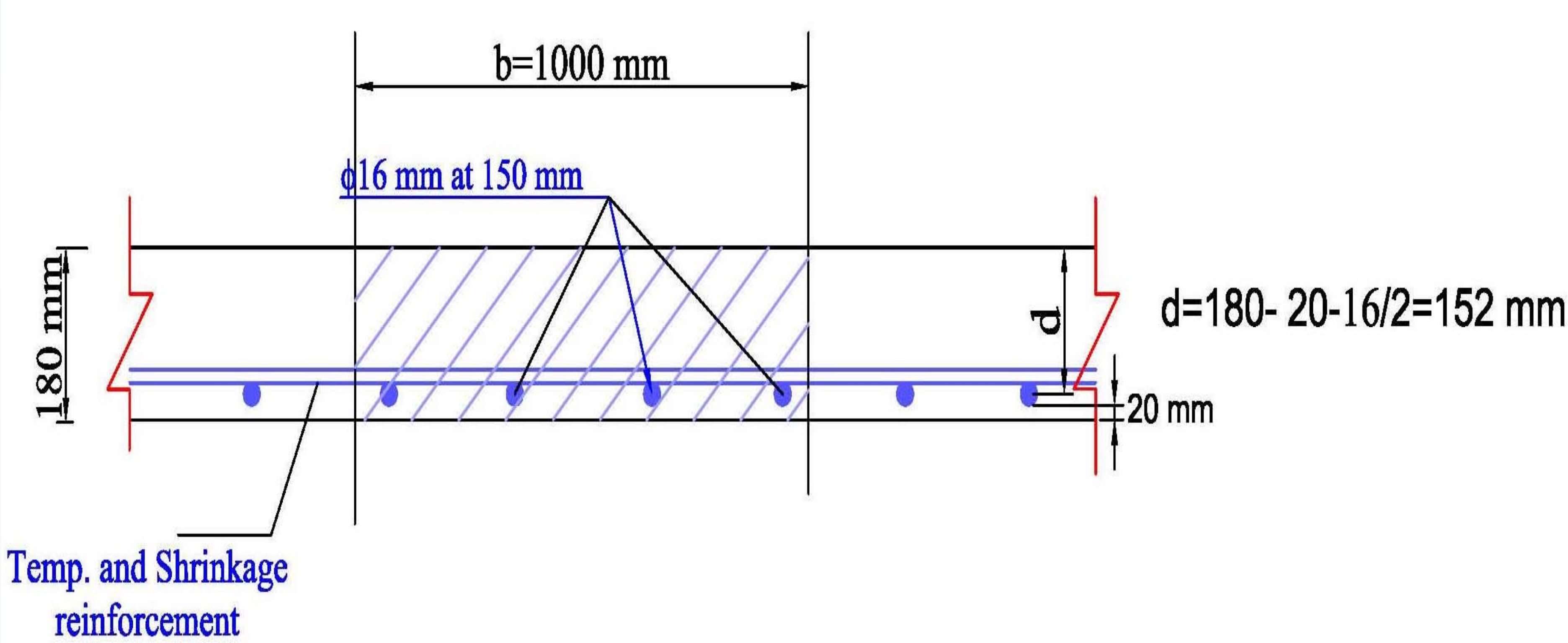


Straight Bar



Bent Bar

Example (1):calculate the design moment of one way solid slab that has a total depth of h=180 mm and is reinforced with 16 mm diameter bar spaced s =150 mm, used fc =21 MPa , fy = 420 MPa

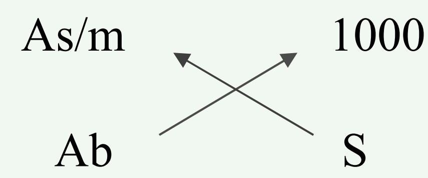


Sol.

d=h−concrte cover (20) $- \frac{\emptyset}{2}$

$$d = 180 - 20 - \frac{16}{2} = 152mm$$

$Taking\ width\ strip\ =\ 1000\ mm$



As/m=1000 $\times \frac{\text{Ab}}{\text{S}}$

A_b= 201 mm^2 (Ab for bar diameter = 16 mm)

As/m=1000 $\times \frac{Ab}{s}$ =1000 × 201/150 =1340 mm^2

Check As $_{min}$= ρ $_{min} \times b \times h = 0.0018 \times 1000 \times 180 = 270\ mm^2$

$$m = \frac{fy}{0.85f'c} = \frac{420}{0.85 \times 21} = 23.53$$

claculate $\rho_b = \frac{\boldsymbol{\beta}_1}{m}\left(\frac{600}{600+fy}\right)\left(\frac{dt}{d}\right) = \frac{0.85}{23.53}\left(\frac{600}{600+420}\right)(1) = 0.02127$

and calculate $\rho_{max} = \left(\frac{0.003+\frac{fy}{Es}}{0.008}\right)\rho_b = \left(\frac{0.003+\frac{420}{200000}}{0.008}\right) \times 0.02127 = 0.01355$

$As_{max} = \rho_{max} \times \text{b} \times \text{d} = 0.01355 \times 1000 \times 152 = 2059\text{mm}^2$

As/m= 270 mm^2 < As /m=1340 mm^2 < Asmax=2059 mm^2

Tension Control $\emptyset = 0.9$

-Section Capacity ϕMu

$C = T$

$0.85 f'c\, a\, b = As\, fy$

$a = \frac{1340 \times 420}{0.85 \times 21 \times 1000} = 31.5\ mm$

$\phi Mu = 0.8\ As \times fy\ \left(d - \frac{a}{2}\right)$

$= 0.9 \times 1370 \times 420 \times \left(152 - \frac{31.5}{2}\right)$

$= 69\ KN.m/m$

Or

$Mu = \phi Rbd^2$

$R = \rho\, fy(1 - 0.5\, \rho\, m)$

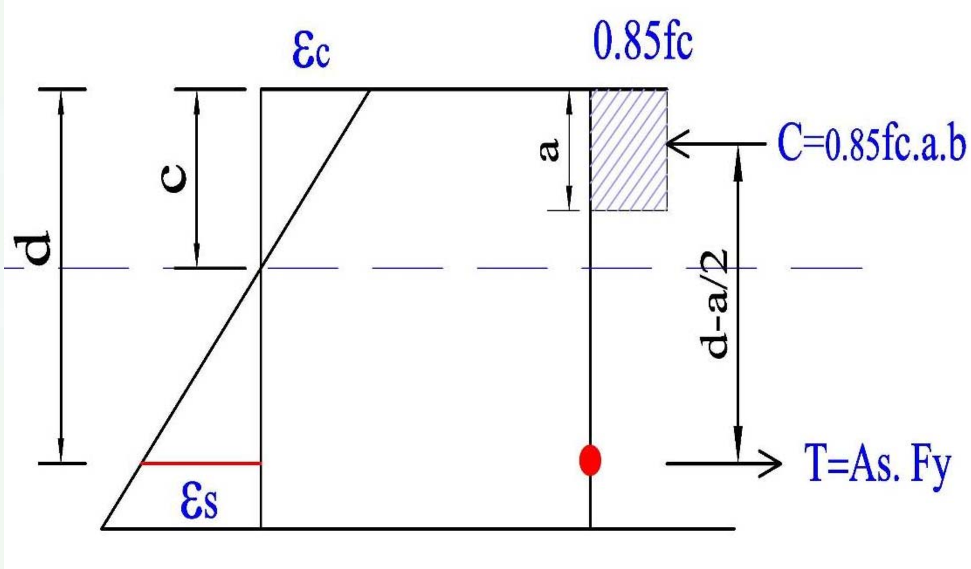
$\rho = \frac{As}{bd} = 0.00882$

$R = 0.00882 \times 420 \times (1 - 0.5 \times 0.00882 \times 23.53)$

$= 3.32$

$Mu = \phi Rbd^2 = 0.9 \times 3.32 \times 1000 \times 152^2$

$Mu = 69\ KN.m\ /m$



Example (2):Determine the allowable uniform live load of example (1) that can be applied on the simply supported slab with span 4.9 m and carries a uniform dead load (excluding self weight) of 4.8 KN/m^2 .

Sol.

$\phi M_n = 69\ KN.m/m$ (*example* (1))

$Mu = \phi\ Mn = \frac{w_u \times L^2}{8}$

$69 = \frac{Wu \times 4.9^2}{8}$ $\therefore W_u = 23\ KN/m^2$

$W_u = 1.2\ D.L + 1.6\ L.L$

Self weight of slab $= h\ .b.1.\gamma c = 0.18 \times 1 \times 1 \times 24 = 4.32\ KN/m^2$

$23 = 1.2 \times (4.32 + 4.8) + 1.6 \times W_L$

$W_L = 7.54\ KN/m^2$

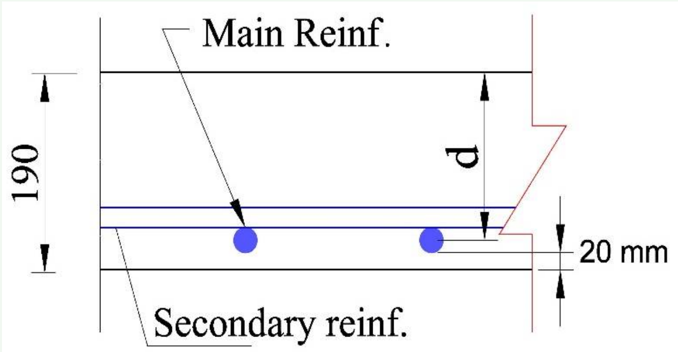
Example (3): Design a 3.65 m simply supported slab to carry a uniform dead load (excluding self weight) of 5.75 KN/m^2 and a uniform Live load of 4.8 KN/m^2 , normal concrete , fc =21 Mpa, fy= 420 Mpa.

Sol.

❖ $Minimum\ Slab\ thickness, fy = 420\ Mpa\ and\ simply\ supported\ slab$

$$h = \frac{L}{20} = \frac{3650}{20} = 182.5\ mm \quad (ACI\ code\ Table\ 7.3.1.1)$$

$Use\ h = 190\ mm$



❖ $Applied\ load$

$$Wu = 1.2\,DL + 1.6\,WL$$

$$Self\ weight\ of\ slab = 0.19 \times 1 \times 1 \times 24 = 4.56\ KN/m^2$$

$$Wu = 1.2 \times (4.56 + 5.75) + 1.6 \times 4.8 = 20.05\ KN/m/m$$

$For\ 1\ m\ strip\ width$

$$Mu = wu\frac{L^2}{8} = 20.05 \times \frac{3.652}{8} = 33.39 KN.m/m$$

$$d = h - 20 - \frac{\phi}{2} = 190 - 20 - \frac{12}{2} = 164\ mm \qquad (use\ \phi = 12\ mm\ Ab = 113\ m2)$$

$$m = \frac{fy}{0.85f'c} = 23.53$$

$$Mu = \phi\, Rbd^2$$

$$R = \frac{Mu}{\phi\, bd^2} \qquad assume\ Tension\ control, use\ \phi = 0.9$$

$$R = \frac{33.39 \times 10^6}{0.9 \times 1000 \times 164^2} = 1.379$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$

$$\rho = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \times 23.53 \times 1.379R}{420}}\right) = 0.00342$$

$$As/m \;=\; \rho.b.d \;=\; 0.00342 \times 1000 \times 164 = 561\; mm^2/m$$

$$As\ \text{min.} = \;0.0018 \times b \times h = \;0.0018 \times 1000 \times 190 \;=\; 342\; mm^2/m$$

$$calculate\quad \rho_b \;=\; \frac{\boldsymbol{\beta}_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right) = \frac{0.85}{23.53}\left(\frac{600}{600 + 420}\right)(1) = 0.02127$$

$$\text{and calculate}\quad \rho_{max} = \left(\frac{0.003 + \frac{fy}{Es}}{0.008}\right)\rho_b = \left(\frac{0.003 + \frac{420}{200000}}{0.008}\right) \times 0.02127 = 0.01355$$

$$As_{\ \text{max.}} = \;0.01355 \times 1000 \times 164 = \;2222\; mm^2/m \quad > \;As/m \;= 561\; mm^2 \qquad (\,OK)$$

$$Ab \;=\; 113\; mm^2 \qquad S = \;1000 \times \frac{113}{561} \;= 201\; mm$$

❖ *Check maximum spacing* $\qquad 3 \times hf \;=\; 3\; \times 190 = \;570\; mm\;\; or\; 450\; mm$

$$use\; S = \;190\;\; mm\; c/c$$

Secondary steel (shrinkage and temperature reinforcement)

$\rho_{min} = 0.0018$

$As\ \min = 0.0018 \times b \times h = 00.0018 \times 190 \times 1000 = 342\ mm^2$

$If\ use\ \phi\ 10mm \quad Ab = 78\ mm^2$

$$S = \frac{1000 \times 78}{342} = 228\ mm^2/m < 5 \times hf = 5 \times 190 = 950\ mm < 450\ mm \qquad (OK)$$

$Use\ \phi 10\ mm\ at\ 220\ mm\ c/c$

$Check\ the\ shear\ requirement$

$$V_u = \frac{Wu \times L}{2} = 20.05 \times 3.65 = 36.59\ KN/m$$

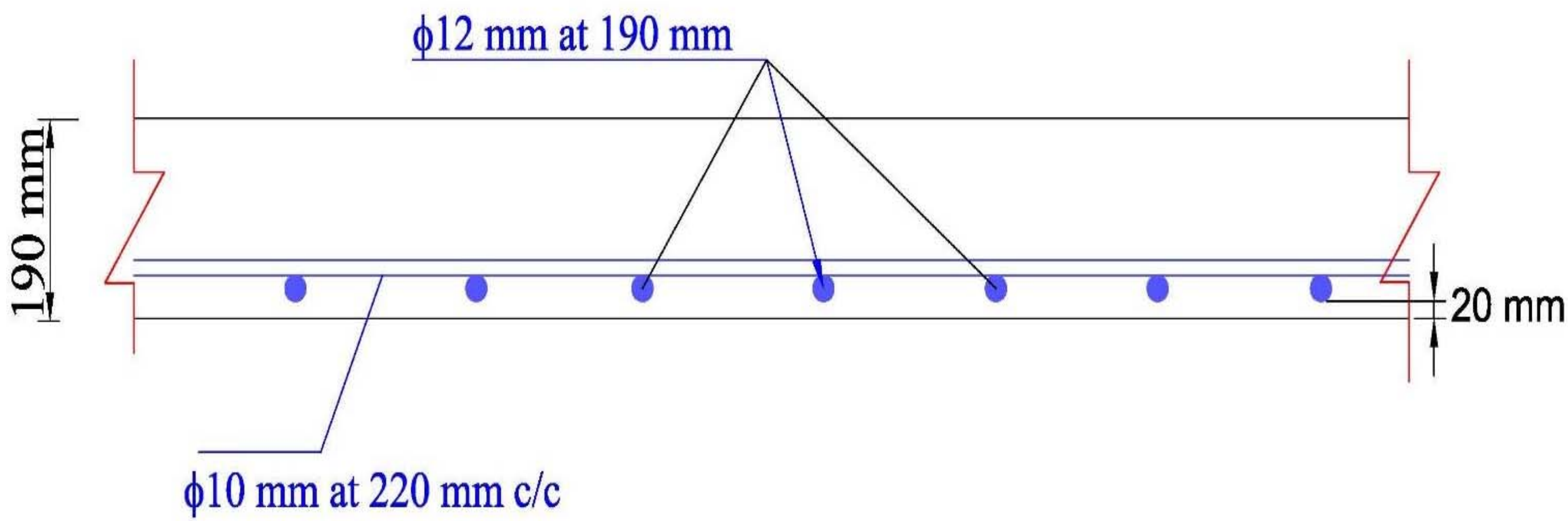
The critical section at d distance from the face of support

$$V_{ud} = Vu - w_u \times d$$

$$= 35.59 - 20.05 \times 0.164 = 33.3\ KN/m$$

$$\phi\, Vc = \phi \times (0.17 \sqrt{f'c}\, b.d) = 0.75 \times 0.17 \times \sqrt{21} \times 1000 \times 164 = 95.82\ KN/m$$

$Vud < \phi\, Vc \qquad (OK)$



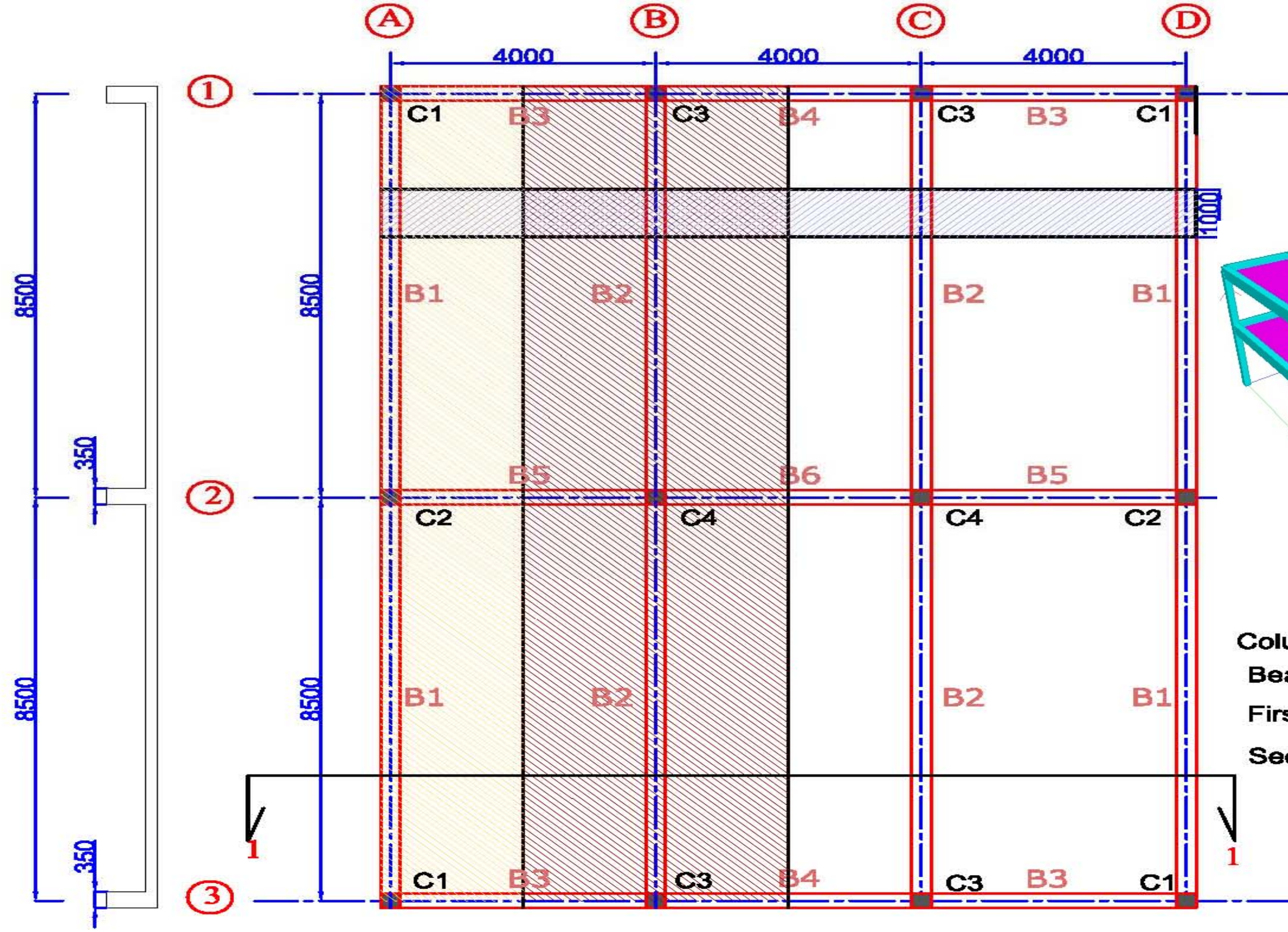
ϕ12 mm at 190 mm
190 mm
20 mm
ϕ10 mm at 220 mm c/c

Thank You

……………….

Example
For Design Floor system One Way Slab

…………………

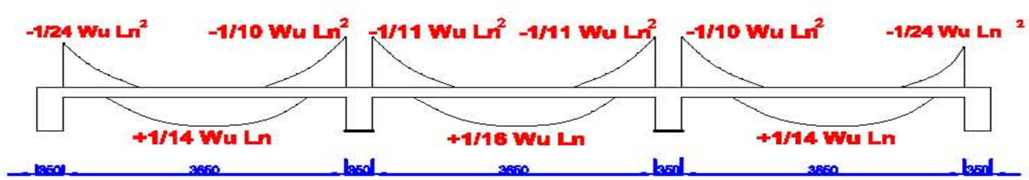


Column Dim. =350 x350 mm

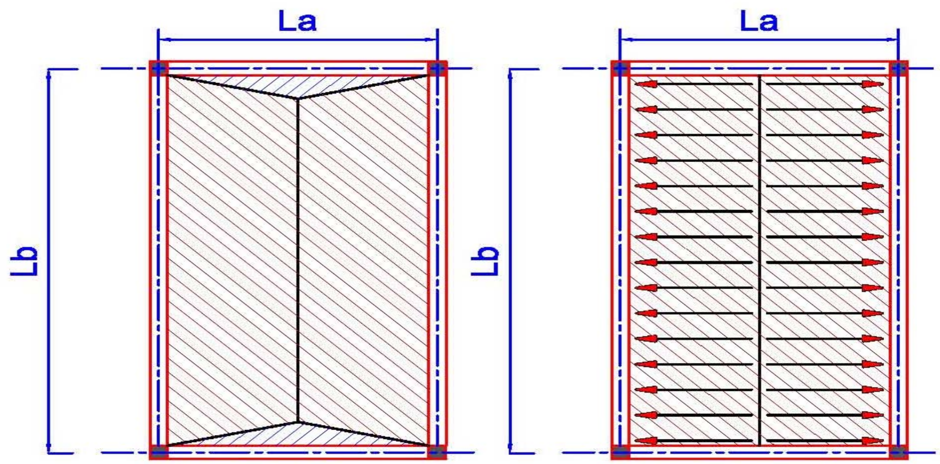
Beam width. =350 mm

First Floor height. =4 m

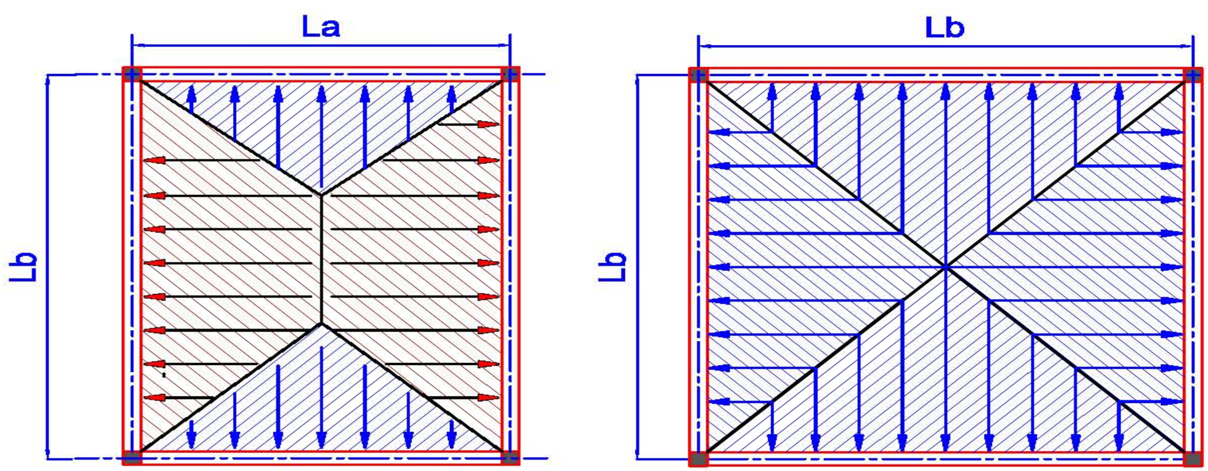
Second Floor height. =3.2 m



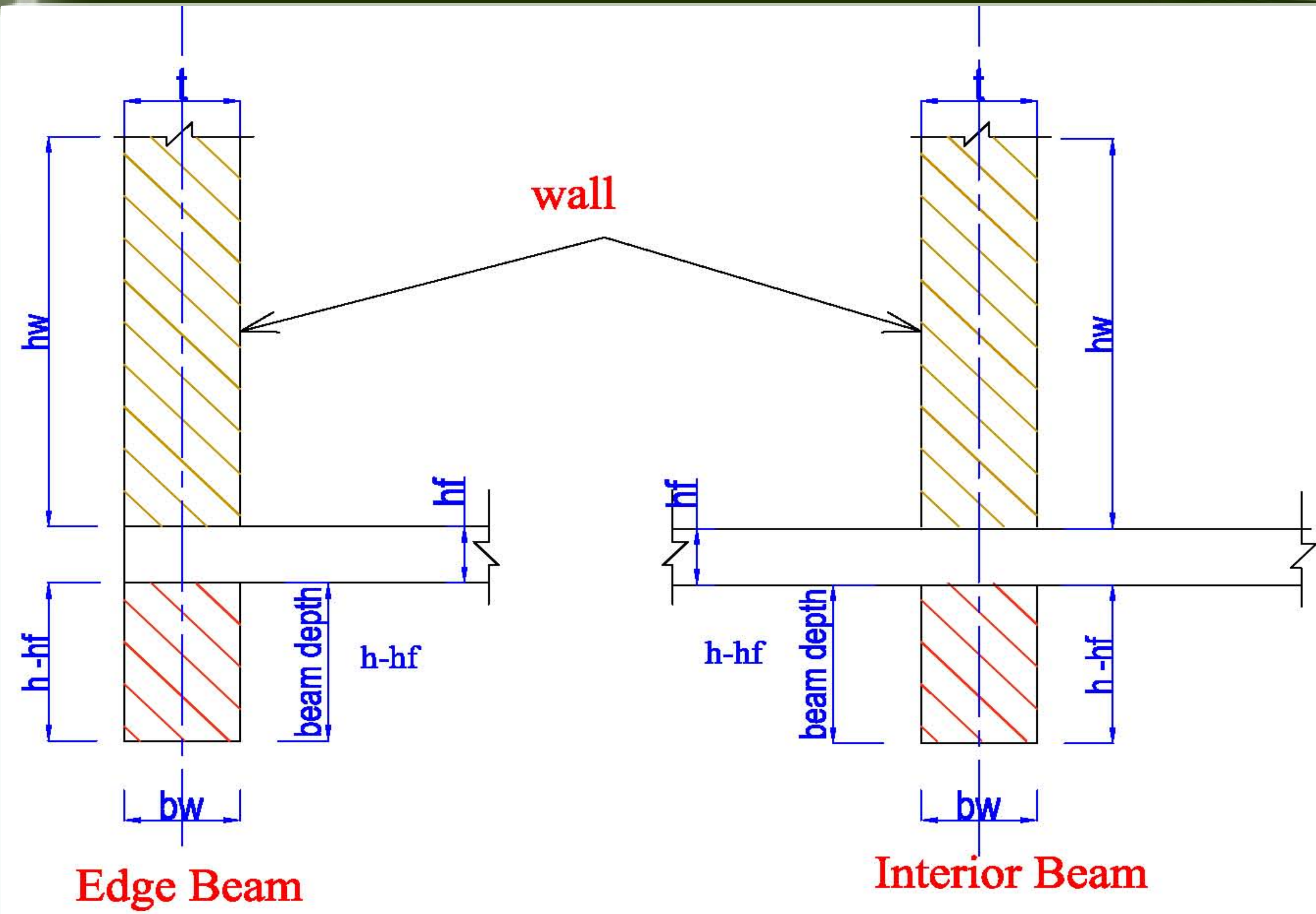
Section 1-1



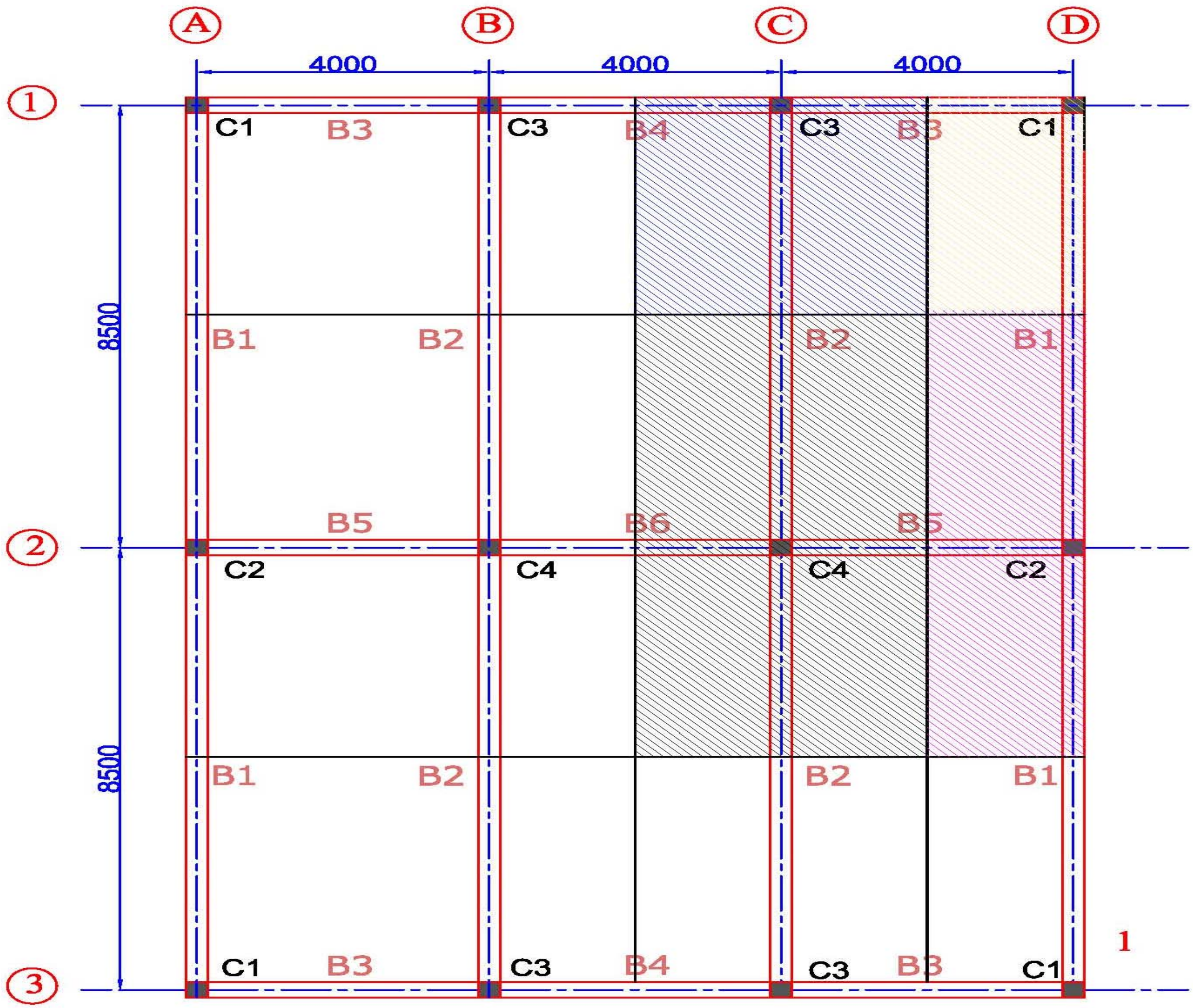
One way slab
Lb/La >= 2



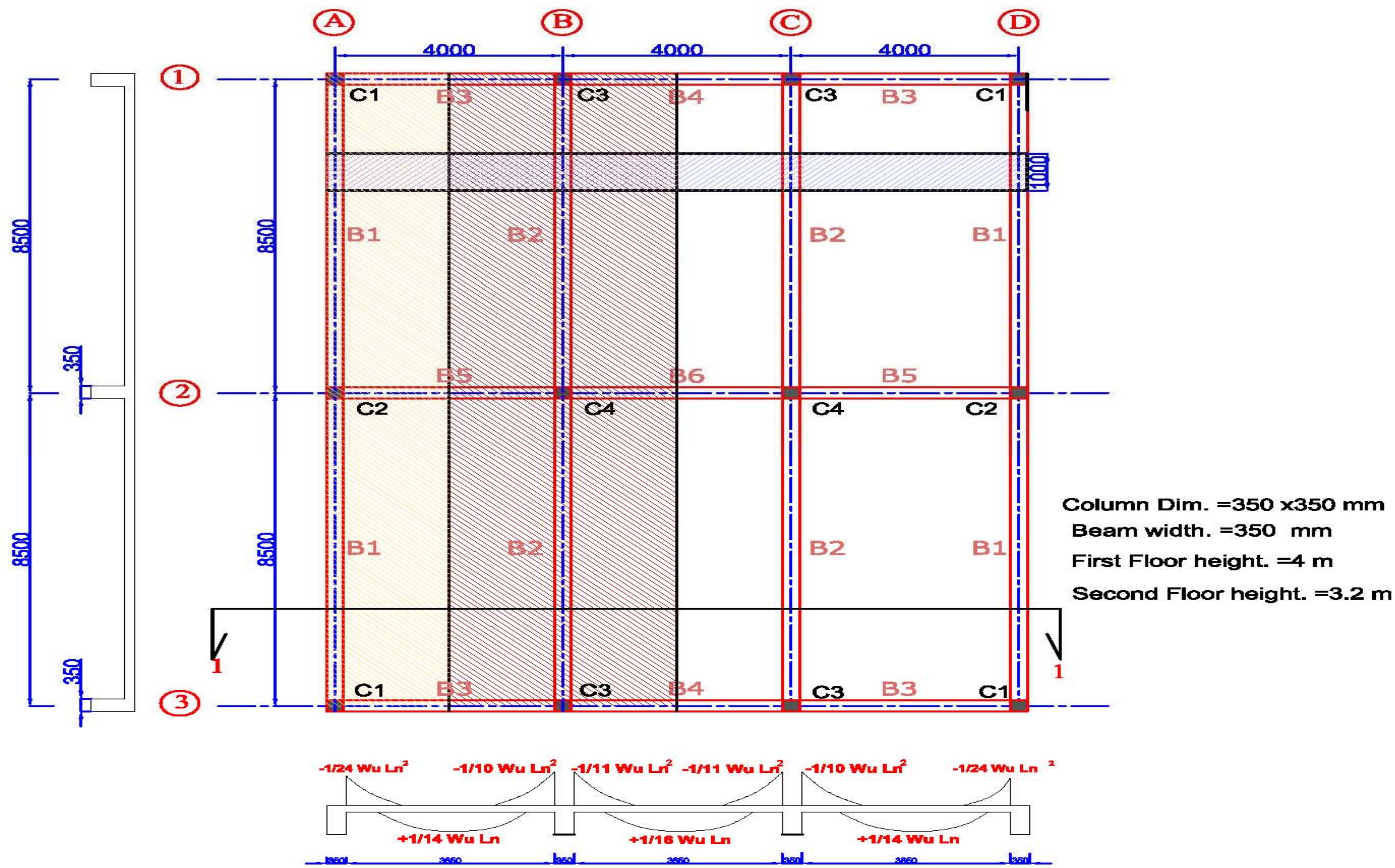
Two way slab
Lb/La < 2



wall
t
hw
hf
h -hf
beam depth
h-hf
bw
Edge Beam
Interior Beam


A
B
C
D
4000
4000
4000
1
2
3
8500
8500
C1
B3
C3
B4
C3
B3
C1
B1
B2
B2
B1
B5
B6
B5
C2
C4
C4
C2
B1
B2
B2
B1
C1
B3
C3
B4
C3
B3
C1

Example (4): Design the one way slab system shown in fig below , subjected to the superimposed dead load 2 KN/m^2 and live load 3 KN / m^2 assume normal concrete , f'c =21 Mpa, and fy= 280 Mpa.



Section 1-1

Column Dim. =350 x350 mm
Beam width. =350 mm
First Floor height. =4 m
Second Floor height. =3.2 m

Sol.

❖ Minimum Slab thickness, (assume one end continuous) and $Fy = 280\ Mpa$

$h = L/24 = 4000/24 = 166.6\ mm$ (*ACI code Table* 7.3.1.1)

$Use\ h = 170\ mm$

$use\ \phi = 12\ mm\ Ab = 113\ m2$

$$d = h - 20 - \frac{\phi}{2} = 170 - 20 - \frac{12}{2} = 144\ mm$$

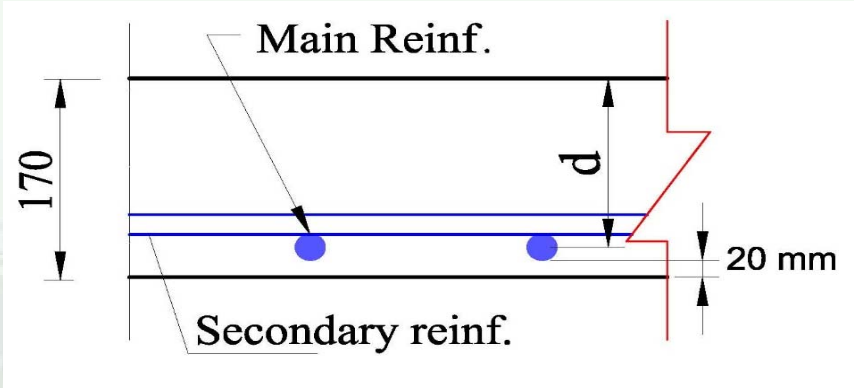
❖ ***Applied load***

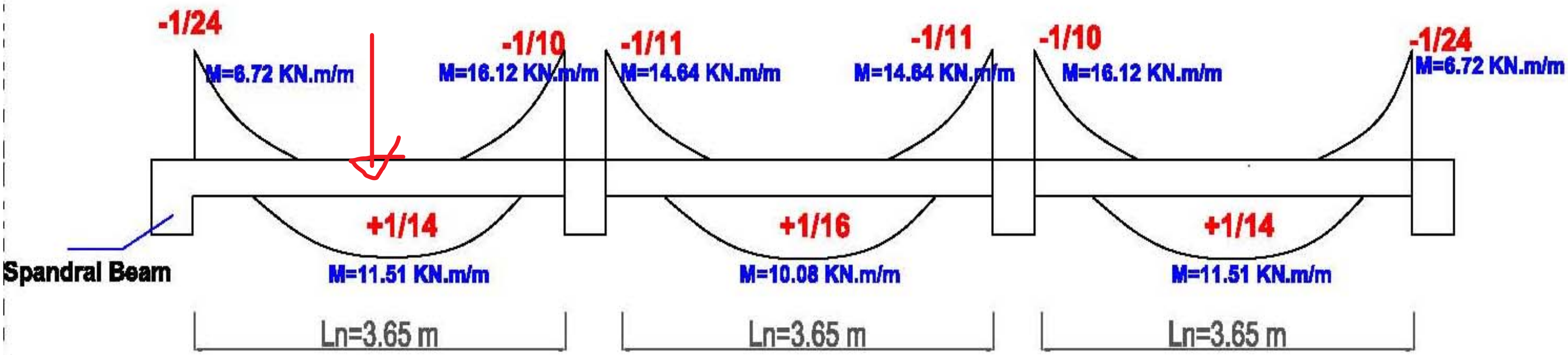
$Wu = 1.2\ DL + 1.6\ WL$

Self weight of slab $= 0.17 \times 1 \times 1 \times 24 = 4.08\ KN/m^2$

$Wu = 1.2 \times (4.08 + 2) + 1.6 \times 3 = 12.1\ KN/m/m$

$$m = \frac{fy}{0.85 f'c} = \frac{280}{0.85 \times 25} = 15.69$$





❖ *Moment Calculation*

−External span

$1 - \text{External Negative Moment} = -M = \left(-\frac{1}{24}\right) Wu\, ln^2 = \left(-\frac{1}{24}\right) \times 12.1 \times 3.652 = 6.72\ KN.m/m$

$2 - \text{Positive Moment} = +M = \left(-\frac{1}{14}\right) Wu\, ln^2 = \left(\frac{1}{14}\right) \times 12.1 \times 3.652 = 11.51\ KN.m/m$

$3 - \text{Internal Negative Moment} = -M = \left(-\frac{1}{10}\right) Wu\, ln^2 = \left(-\frac{1}{10}\right) \times 12.1 \times 3.652 = 16.12\ KN.m/m$

−Interior span

$1 - \text{Internal Negative Moment} = -M = \left(-\frac{1}{11}\right) Wu\, ln^2 = \left(-\frac{1}{11}\right) \times 12.1 \times 3.652 = 14.65\ KN.m/m$

$2 - \text{Positive Moment} = +M = \left(-\frac{1}{16}\right) Wu\, ln^2 = \left(\frac{1}{16}\right) \times 12.1 \times 3.652 = 10.08\ KN.m/m$

$3 - \text{Internal Negative Moment} = -M = \left(-\frac{1}{11}\right) Wu\, ln^2 = \left(-\frac{1}{11}\right) \times 12.1 \times 3.652 = 14.65\ KN.m/m$

No.	Details	External Span			Internal Span	
		-M Exterior supp.	+ M Mid Span	- M Interior supp.	-M Exterior supp.	+M Exterior supp.
1	Mu * 10^6 (N.mm)	6.72	11.51	16.12	14.65	10.08
2	b (mm)	1000	1000	1000	1000	1000
3	d (mm)	144	144	144	144	144
4	$R=Mu/(\phi bd^2)$	0.36	0.617	0.863	0.785	0.54
5	ρ=1/m(1-(1-√1-2mR/fy)	0.001299	0.002243	0.00316		0.001959
6	As =ρ.b.d (mm^2)	187	323	456	413	282
7	As min = ρ.b.h (mm^2) ρmin = 0.0018	306	306	306	306	306
8	As Provided (choosed)	306	306	456	413	306
9	S= 1000*Ab/As (mm)	369	369	247	273	369
10	Smax= 3*h=510 0r 450 mm	450	450	450	450	450
11	S (choosed)	369	369	247	273	369
12	Used Spacing S (use ϕ12 mm)	360	360	240	270	360

Due to $fy = 280\ MPa \quad (from\ ACI\ code\ 24.4.3.2)$

$\rho_{min} = 0.0018$

As shrinkage and Temperature $= \rho_{min} \times b \times h = 0.018 \times 1000 \times 170 = 306\ mm^2/m$

Use ϕ 10 mm ($Ab = 78\ mm^2$)

$$S = \frac{1000 \times 78}{288} = 229\ mm$$

Check Maximum spacing for shrinkage and temperature steel

$S_{max} = 5 \times h = 5 \times 170 = 850\ mm \quad or \quad 450 < s_{max}$

$Use\ \phi\ 10\ mm\ \ 220mm\ c/c$

$S_{\ max} = 5 \times h = 5 \times 170 = 850\ mm\ or\ 450 < s\ max$

Check for Shear:

$$Vu = 1.15\frac{W_u L_n}{2}$$

$$Vu = 1.15 \times 12.1 \times \frac{3.65}{2} = 25.39\ KN/m$$

$$Vu,d = Vu - wu \times d = 25.39 - 12.1 \times 0.144 = 23.65 KN/m$$

$$\phi\,Vc = \phi \times (0.17\sqrt{f'c}\,b.d) = 0.75 \times 0.17 \times \sqrt{21} \times 1000 \times 144 = 84.12\ KN/m$$

$$Vud < \phi\,Vc \qquad (\,OK\ section\ safe\ for\ shear.)$$

Reinforcement Details

1- Bent Bar

A-Additional steel at exterior support (-M)

If we assume that 50% of positive steel will bent then:

As provided will be ϕ12 at 660 mm c/c

$$As/m = \frac{1000 \times 113}{660} = 171\ mm^2/m$$

As (required)/m =340 mm²/m

then As additional = $As_{req.} - As_{provided} = 340 - 171 = 169\ mm^2/m$

$$S = \frac{1000 \times 113}{169} = 668mm \quad use = \quad 12@660\ mm\ c/c$$

B-Additional steel at Interior support (-M)

If we assume that 50% of positive steel will bent then:

As provided will be ϕ12 at 660 mm c/c from left side :

$$As/m = \frac{1000 \times 113}{660} = 171\, mm^2/m$$

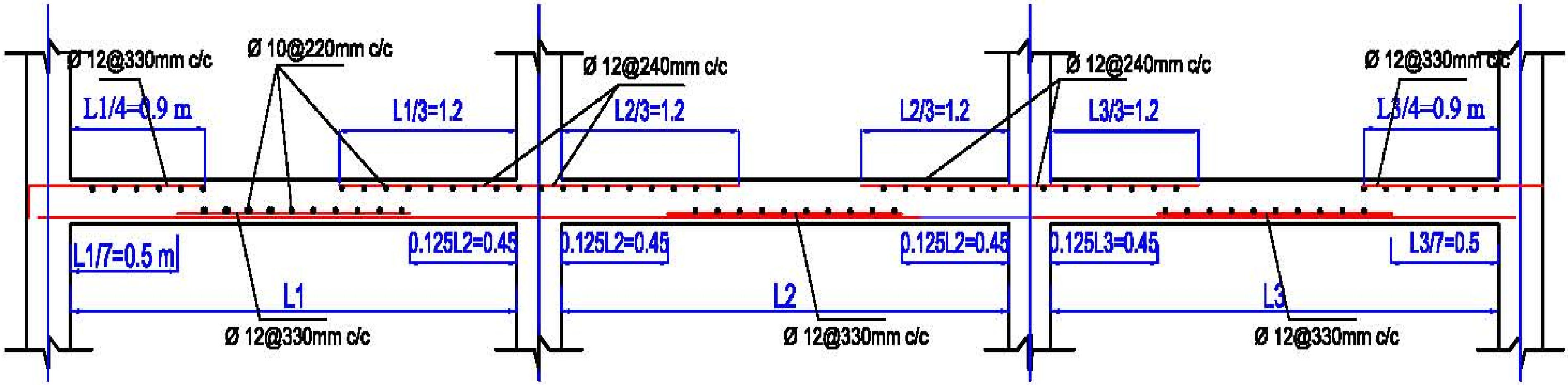
So the Total As provided from both side = 2×171 = 342 mm^2/m

As (required)/m =456 mm^2/m

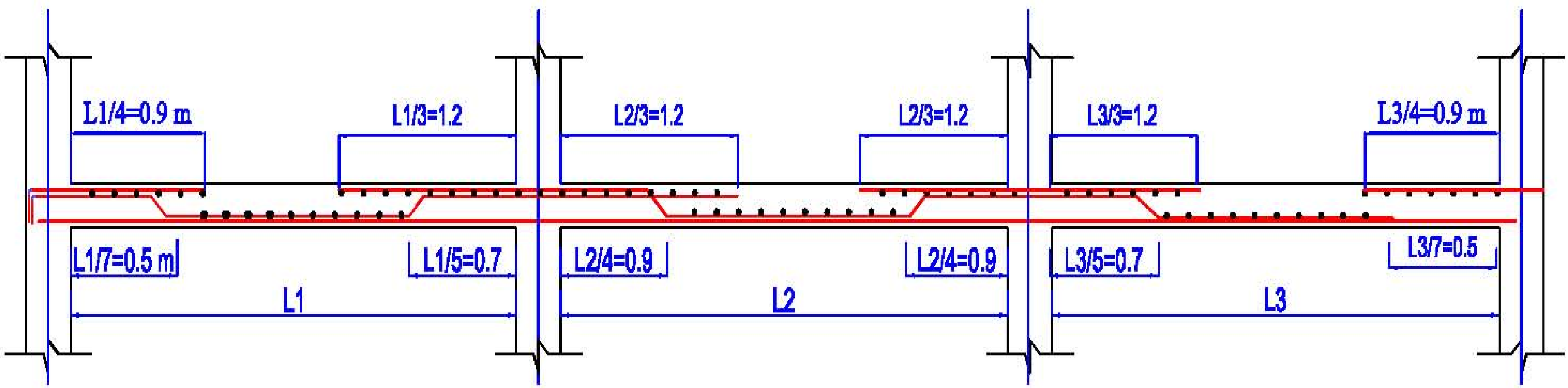
then As additional = As req. – As provided = 456-342=114 mm^2/m

$$S = \frac{1000 \times 113}{114} = 991 mm$$

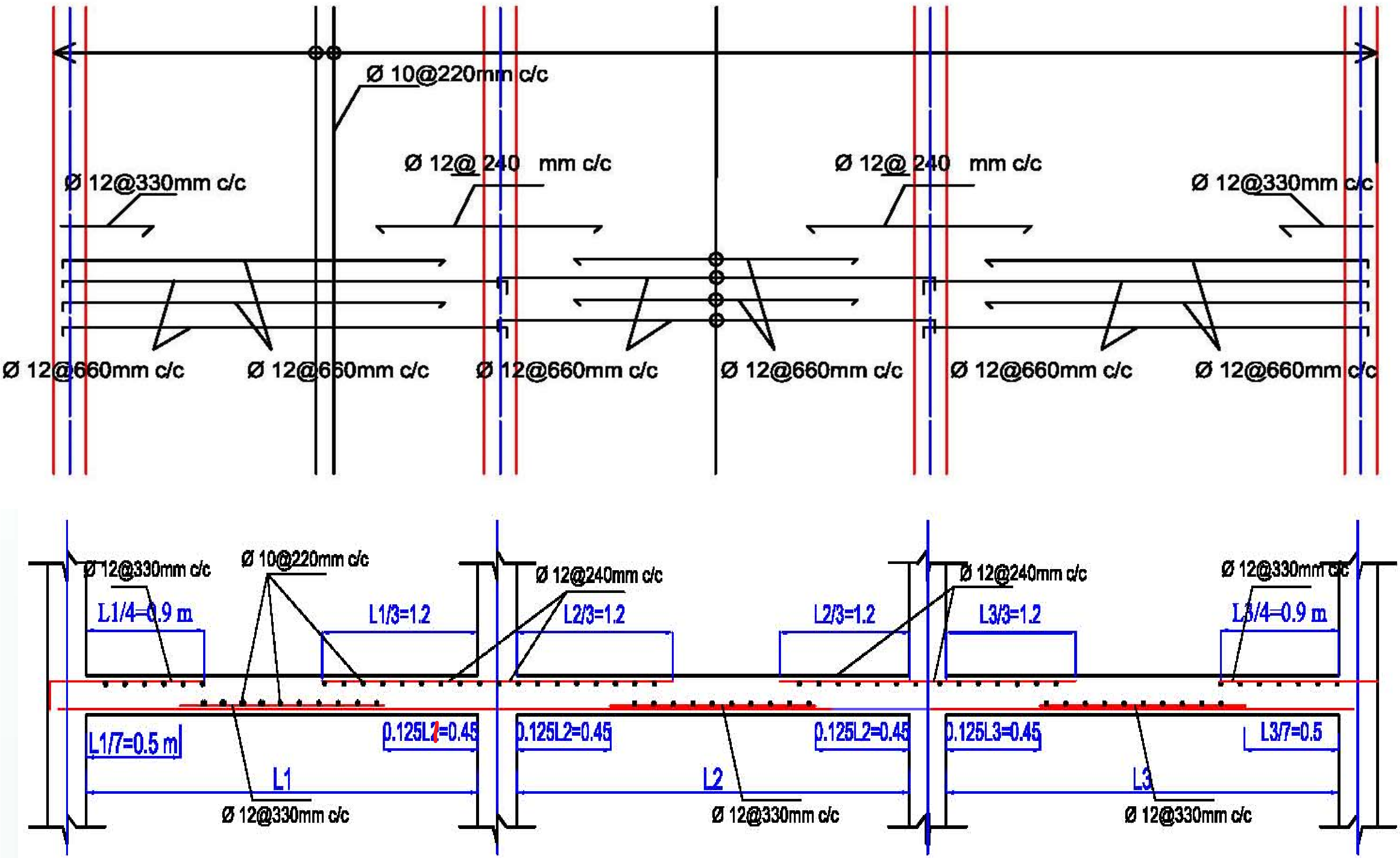
use = ϕ12@990 mm c/c



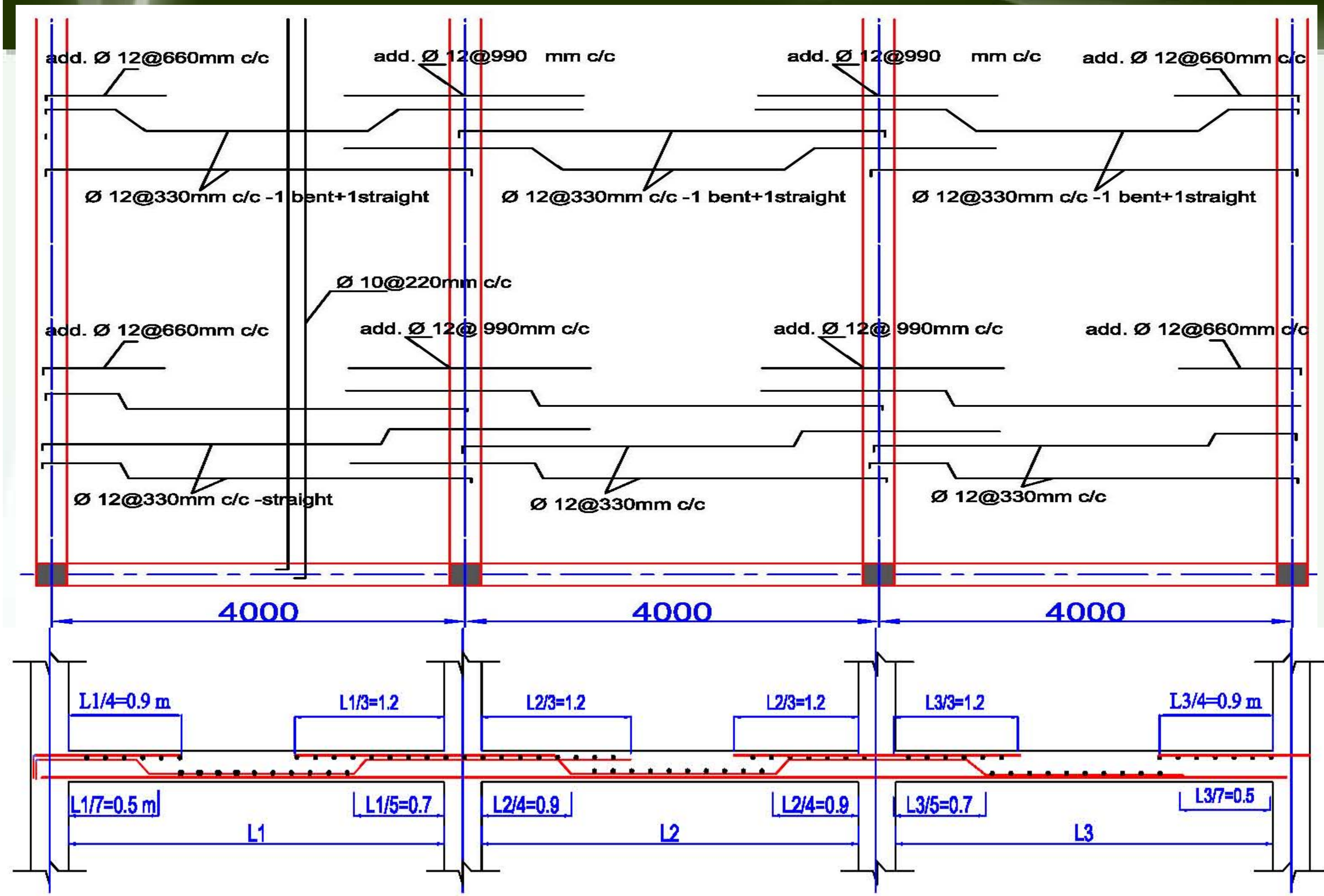
Straight Bar



Bent Bar

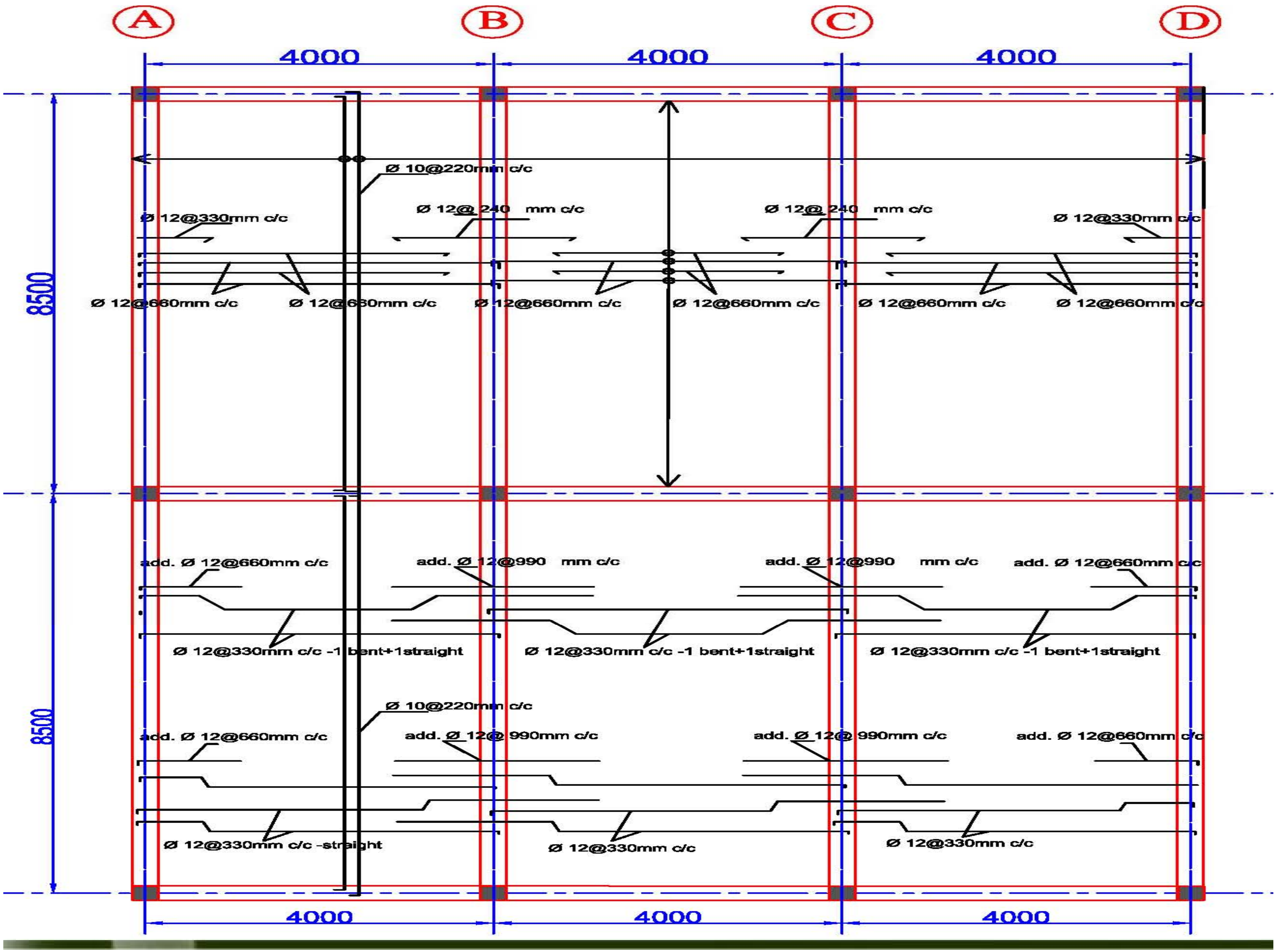


Straight Bar

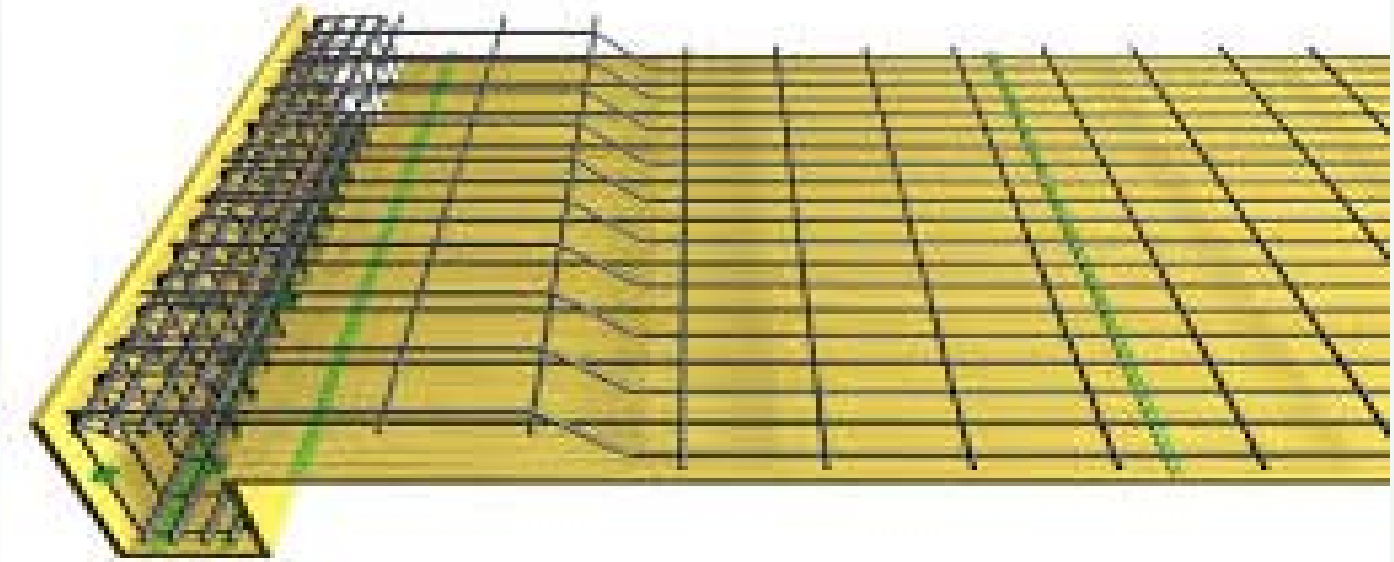


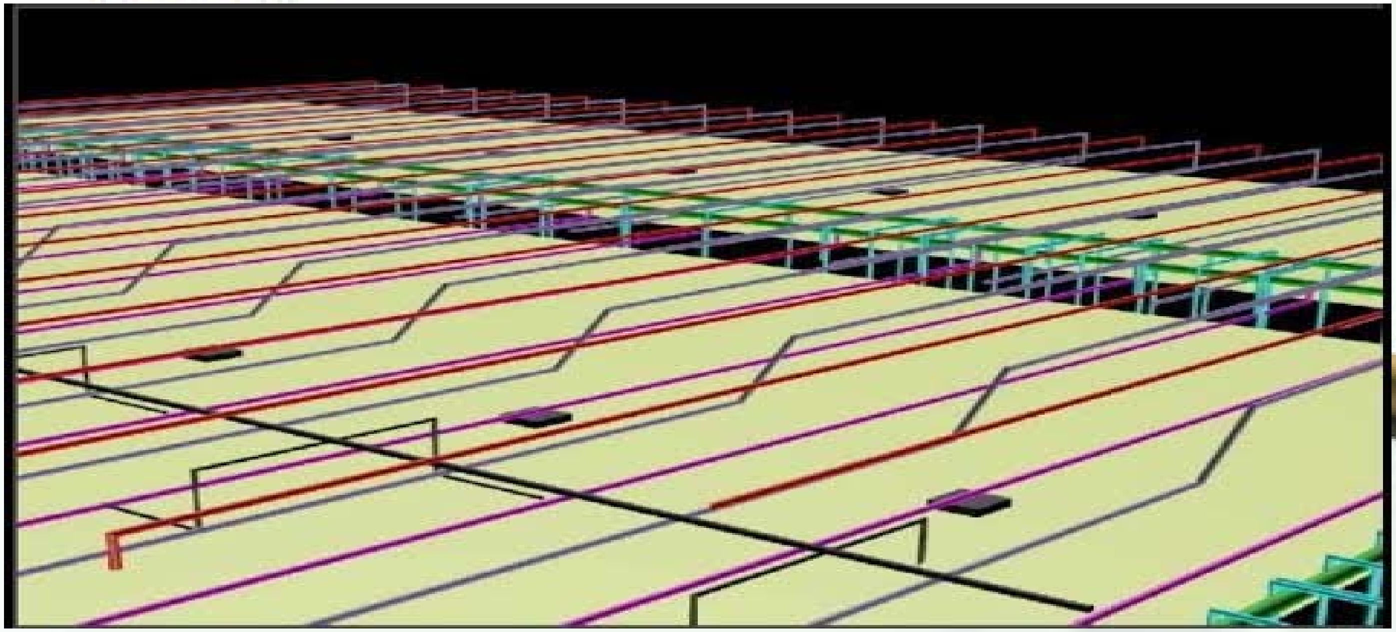
add. Ø 12@660mm c/c
add. Ø 12@990 mm c/c
add. Ø 12@990 mm c/c
add. Ø 12@660mm c/c
Ø 12@330mm c/c -1 bent+1straight
Ø 12@330mm c/c -1 bent+1straight
Ø 12@330mm c/c -1 bent+1straight
Ø 10@220mm c/c
add. Ø 12@660mm c/c
add. Ø 12@ 990mm c/c
add. Ø 12@ 990mm c/c
add. Ø 12@660mm c/c
Ø 12@330mm c/c -straight
Ø 12@330mm c/c
Ø 12@330mm c/c
4000
4000
4000
L1/4=0.9 m
L1/3=1.2
L2/3=1.2
L2/3=1.2
L3/3=1.2
L3/4=0.9 m
L1/7=0.5 m
L1/5=0.7
L2/4=0.9
L2/4=0.9
L3/5=0.7
L3/7=0.5
L1
L2
L3

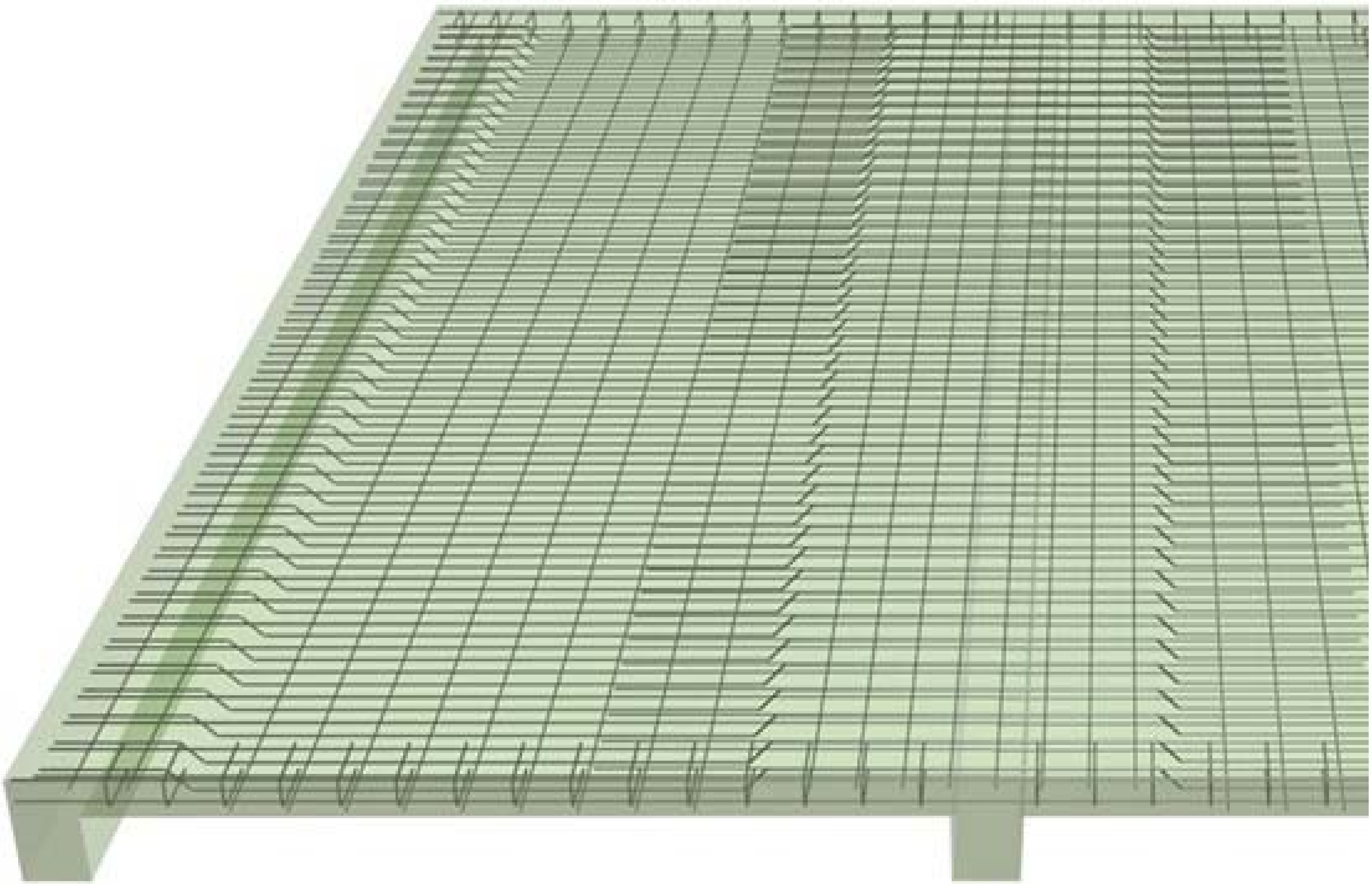
Bent Bar



A
B
C
D
4000
4000
4000
8500
8500
Ø 10@220mm c/c
Ø 12@330mm c/c
Ø 12@ 240 mm c/c
Ø 12@ 240 mm c/c
Ø 12@330mm c/c
Ø 12@660mm c/c
Ø 12@660mm c/c
Ø 12@660mm c/c
Ø 12@660mm c/c
Ø 12@660mm c/c
Ø 12@660mm c/c
add. Ø 12@660mm c/c
add. Ø 12@990 mm c/c
add. Ø 12@990 mm c/c
add. Ø 12@660mm c/c
Ø 12@330mm c/c -1 bent+1straight
Ø 12@330mm c/c -1 bent+1straight
Ø 12@330mm c/c -1 bent+1straight
Ø 10@220mm c/c
add. Ø 12@660mm c/c
add. Ø 12@ 990mm c/c
add. Ø 12@ 990mm c/c
add. Ø 12@660mm c/c
Ø 12@330mm c/c -straight
Ø 12@330mm c/c
Ø 12@330mm c/c
4000
4000
4000



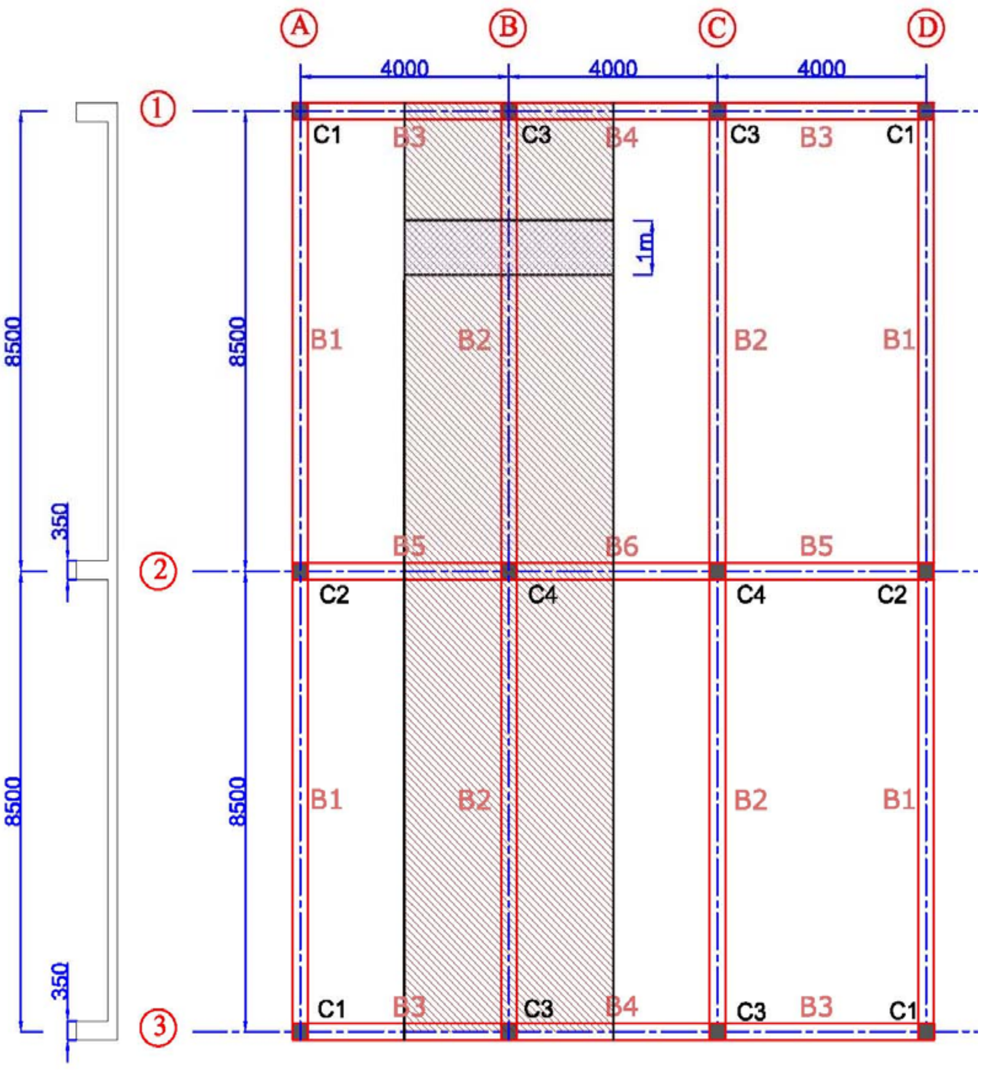




Thank You

……………………

Example (5): From previous Ex. (4) Design the interior beams shown in fig below , where the slabs is subjected to the superimposed dead load 2 KN/m^2 and live load 3 KN / m^2 , normal concrete , f'c =21 mPa, and fy= 280 mPa.



Sol.

1- Interior Beam

-Self weight of drop part of beam =?

Assume Wu (self weight= 5 KN/m) (check Later)

$Wu = 1.2WD + 1.6\,WL$

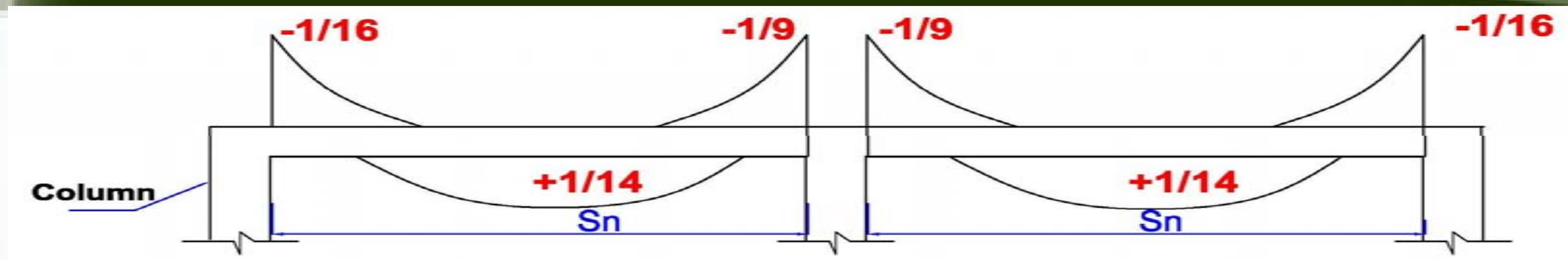
$Wu\ slab = 12.1KN/m^2$

$Wu\ (Load\ on\ beam\ from\ slab/m)$

$= 12.1 \times 4 = 48.4KN/m$

$Wu\ (\ on\ beam\ ) = 1.2 \times 5 + 48.4 = 54.4\ KN/m$

-Calculate Moments (Using ACI code coefficient)



$M = Cf\ Wu\ Sn^2$ $\qquad$ $Sn = 8.5 - 0.35 = 8.15m$

$$Negative\ M\ at\ exterior\ support = \left(-\frac{1}{16}\right) \times 54.4 \times 8.152 = 225.8\ KN.m$$

$$Positive\ M\ at\ mid\ span = \left(\frac{1}{14}\right) \times 54.4 \times 8.152 = 258.1\ KN.m$$

$$Negative\ M\ at\ interior\ support = \left(-\frac{1}{9}\right) \times 54.4 \times 8.152 = 401.49\ KN.m$$

<u>*Flexural Design*</u>

$$m = \frac{fy}{0.85f'c} = \frac{280}{0.85 \times 21} = 15.69$$

$\rho b =$ $\quad$ (*or you can assume* $\rho = 0.5\ \rho max$)

$$\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600 + fy}\right)\left(\frac{dt}{d}\right) \qquad \left(\frac{dt}{d}\right) = 1$$

$$= \frac{0.85}{15.69} \times \left(\frac{600}{600 + 280}\right) = 0.03693$$

$used = 0.5\ \rho\ b = 0.01846$

$R = \rho\ fy(1 - 0.5\ \rho\ m) = 0.01846 \times 280 \times (1 - 0.5 \times 0.01846 \times 15.69) = 4.42$

$Mu = \phi\ Rbd^2$

$$d^2 = \frac{Mu}{\phi\, R\, b} = 401.49 \times 10^6/(0.9 \times 4.42 \times 350)$$

$d = 537\ mm$

$h = 537 + 90 = 627\ mm \quad (two\ layer\ of\ steel)$

$Use\ b \times h = 350 \times 630\ mm$

$Check\ the\ self\ weigh\ of\ beam$

$W_D = 1.2\, Wo\ (self\ Wt.)$

$W_D(self\ wt.) = 1.2 \times 0.35 \times (0.63 - 0.17) = 4.64\ KN/m$

$So\ the\ correct\ Wu = 48.4 + 4.64 = 53.04 KN/m$

Check The ACI code requirement for Minimum Depth of Beam (deflection Control) *ACI Table 9.3.1.1*

Simply supported = L/16

One end Continuous = L/18.5

Both end Continuous =L/21

Cantilever =L/8

If fy not equal 420 MPa then h min. shall be multiplied by factor =(0.4+fy/700) for normal concrete

-And if we use lightweight concrete ((14.4 to 18.4 KN/m^3) the above value of h shall be multiplied with greater of :

1- 1.65-0.0003γc (γc= concrete unit weight)

2- 1.09

In this example the span is one end continuous

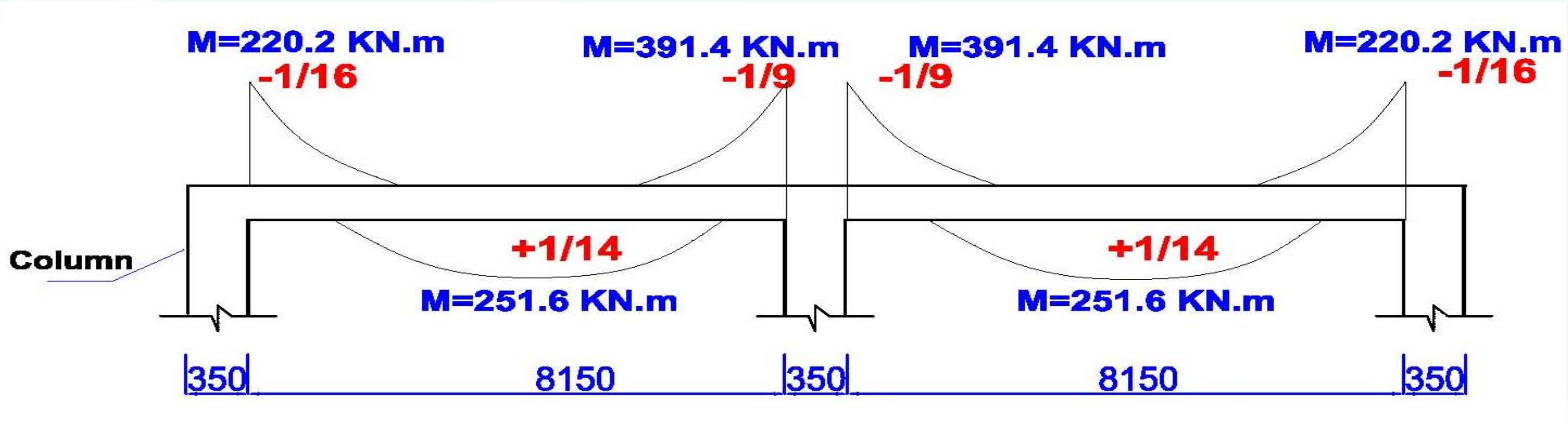
$$h_{min} = \frac{L}{18.5} = \frac{8500}{18.5} = 460\ mm$$

$but\ Fy\ not\ equal\ 420\ Mpa$

$So\ \ f = (0.4 + 280/700) = 0.8$

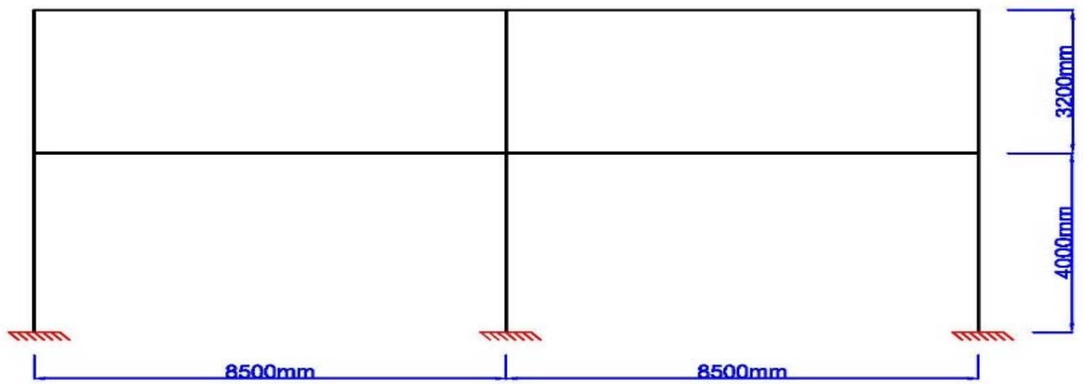
$Corrected\ h_{min} = 0.8 \times 460 = 368\ mm\ <\ h\ used\ =\ 630\ mm.$

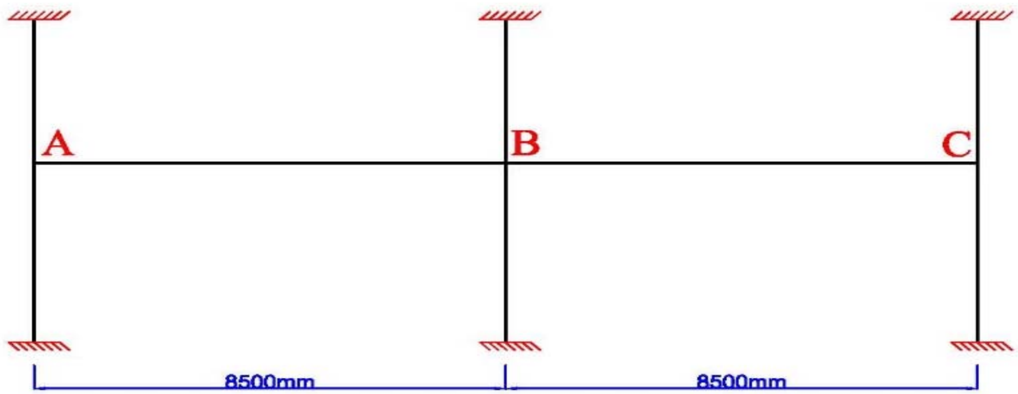
Note: The moment should be corrected according to the modified Wu

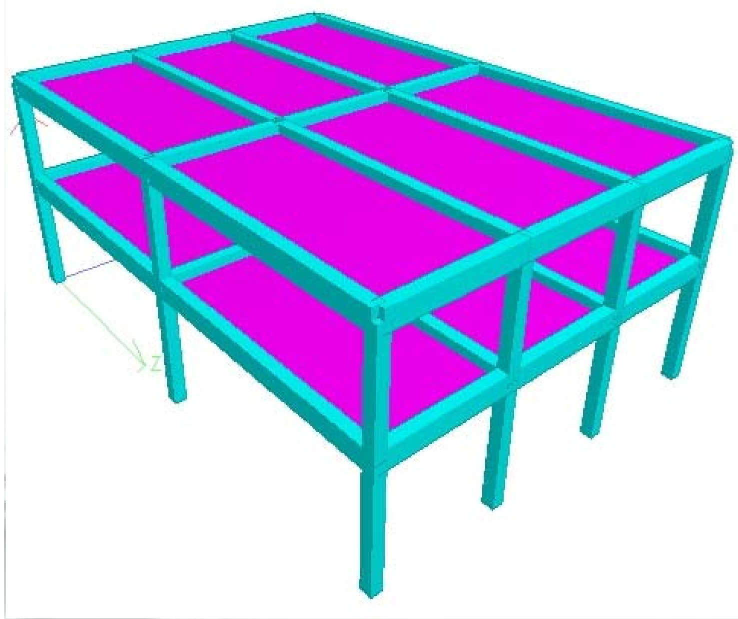


Beams and Column Moment Calculation

- *By using substitute frame method (Moment distribution method)*







Calculate The stiffness of members at each node

$$K = \frac{EI}{L}$$

When use the same concrete properties for whole structure E will be same and constant for all members then :

$$K = \frac{I}{L}$$

K = stiffness of member (mm^3)

I = moment of Inertia (mm^4)

L = length of member (mm)

$$For\ beam\ s\ \ I_b = \frac{bh^3}{12} = \frac{350\ \times 630^3}{12} = 72.93 \times 10^6\ mm^4$$

$$K = \frac{I}{L} = \frac{72.93 \times 10^6}{8500} = 858 \times 10^3$$

$$Stiffness\ of\ \ Upper\ Column\ = 350\ \times \frac{350^3}{12} = 12.505 \times 10^6\ mm^4$$

$$Kc = \frac{Ic}{hc} = \frac{12.505 \times 10^6}{3200} = 390.8 \times 10^3\, mm^3$$

$$Lower\ Column\ \ Kc = \frac{12.505 \times 10^6}{4000} = 312.6 \times 10^3\ mm^3$$

Distributed factor (DF) or **Relative stiffness** $= \left(\frac{\frac{Ib}{Lb}}{\frac{Ic}{ht}+\frac{Ic}{hb}+2*\frac{Ib}{Lb}}\right)$

$$\text{Relative stiffness for beams at B} = \frac{858 \times 10^3}{390.5 \times 10^3 + 312.6 \times 10^3 + 2 \times 858 \times 10^3} = 0.355$$

$$\text{Relative stiffness for beams at A\&C} = \frac{858 \times 10^3}{390.5 \times 10^3 + 312.6 \times 10^3 + 858 \times 10^3} = 0.55$$

$$\text{Fixed end Moment } = \frac{WuL^2}{12} = 53.04 \times \frac{8.52}{12} = 319.35\ KN/m$$

$$Where\ WL/WD < \frac{3 \times 4}{(4.08 + 2) \times 4 + 3.864} = 0.43 < 0.75$$

No need to Use the loading case for Envelope

0.55	0.355	0.355	0.55
+319.35 -175.64	-319.35 0	+319.35 0	-319.34 +175.64
0	-87.82	+87.82	0
+143.71	-407.17	+407.17	-143.71
203.94		*203.94*	

$$M\ positive = \frac{wuL^2}{8} - \left(\frac{M1 + M2}{2}\right) = \frac{53.04 \times 8.52}{8} - \left(\frac{143.71 + 407.17}{2}\right) = 203.94\ KN.m$$

$$Rc = Ra = \frac{wuL}{2} - \frac{M2 - M1}{L}$$

$$= \frac{53.04 \times 8.5}{2} - \frac{407.17 - 143.71}{8.5} = 194.42\ KN$$

$$Rb1 = Rb2 = \frac{WuL}{2} + \frac{M2 - M1}{L}$$

$$= 225.42 + 30.99 = 256.43\ KN$$

$$Vu\ at\ face\ of\ support\ A = Ra - \frac{wu \times x}{2} = 194.42 - 53.04 \times 0.175 = 185.14KN \qquad (x = \frac{350}{2} = 0.175)$$

$$Vu\ at\ face\ of\ support\ B = Rb - \frac{wu \times x}{2} = 256.43 - 53.04 \times 0.175 = 247.15\ KN$$

$$Moment\ at\ face\ of\ support\ A = 194.42 \times 0.175 - 143.71 - \frac{53.04 \times 0.1752}{2} = 110.5\ KN.m$$

$$Moment\ at\ face\ of\ support\ B = 256.43 \times 0.175 - 407.17 - \frac{53.04 \times 0.1752}{2} = 363.11\ KN.m$$

To calculate the positive moment :

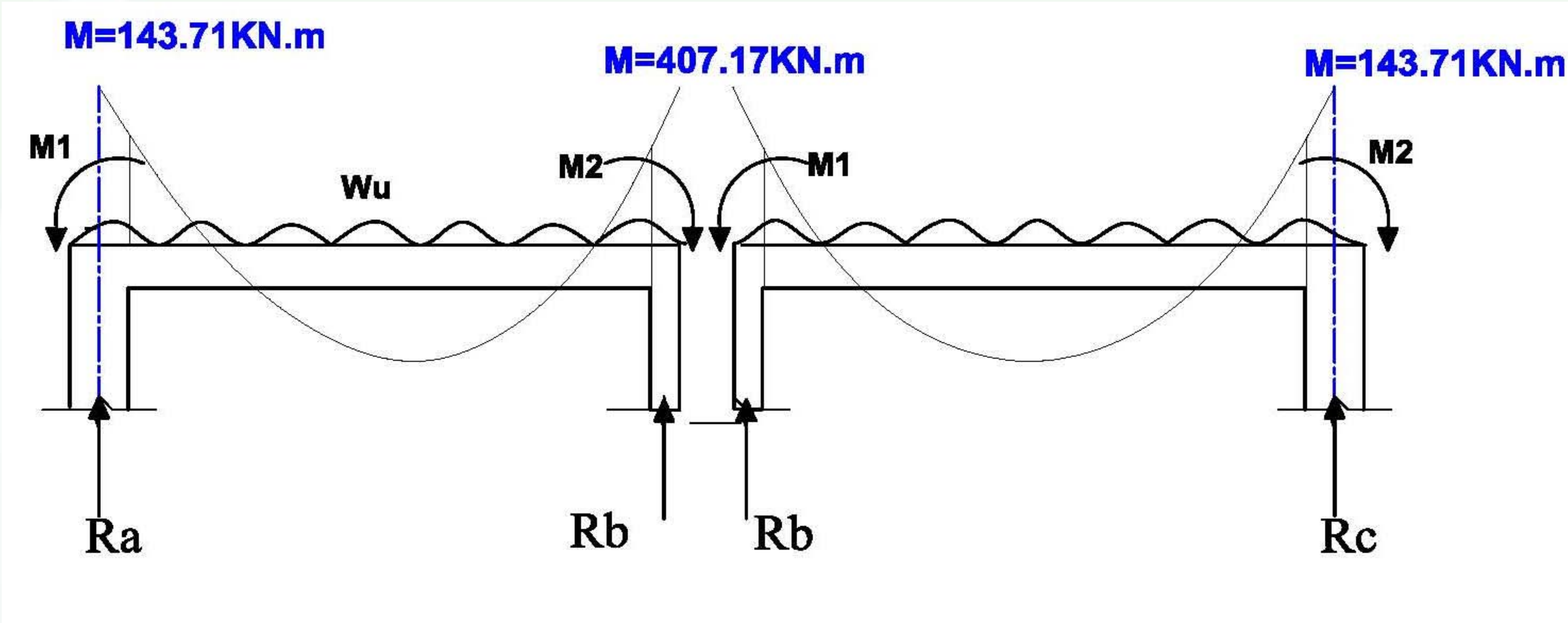
Shear force $= 0$ *at* X

$$X = \frac{194.42 \times 8.5}{194.42 + 256.43} = 3.66\ m$$

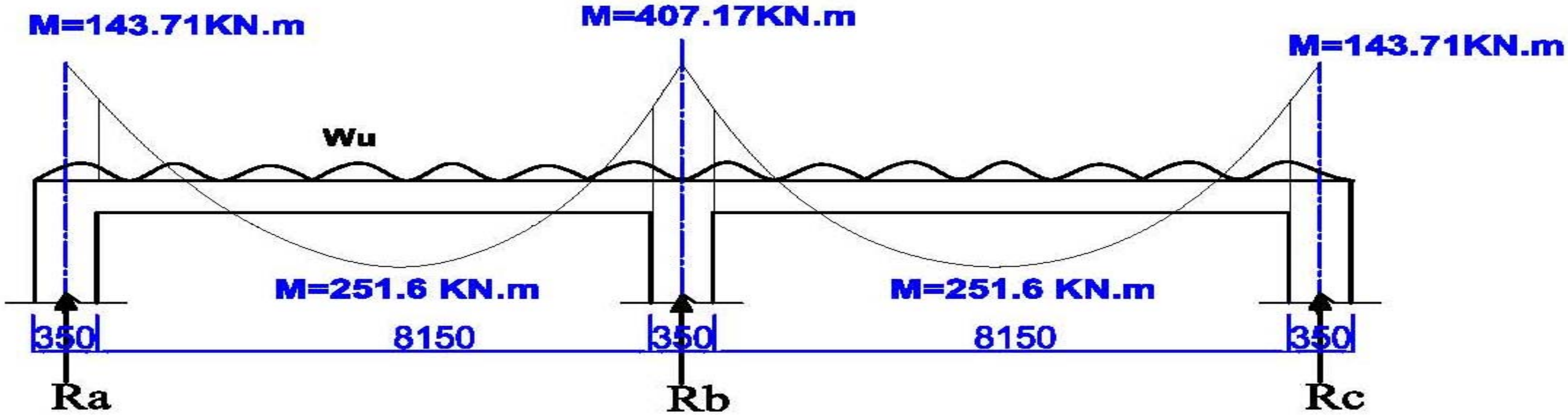
$$Mu = 0 = Ra \times x - 143.71 - \frac{wu \times x^2}{2} = 194.42x - 143.71 - 53.04x^2/2$$

$$x = 0.83\ m$$

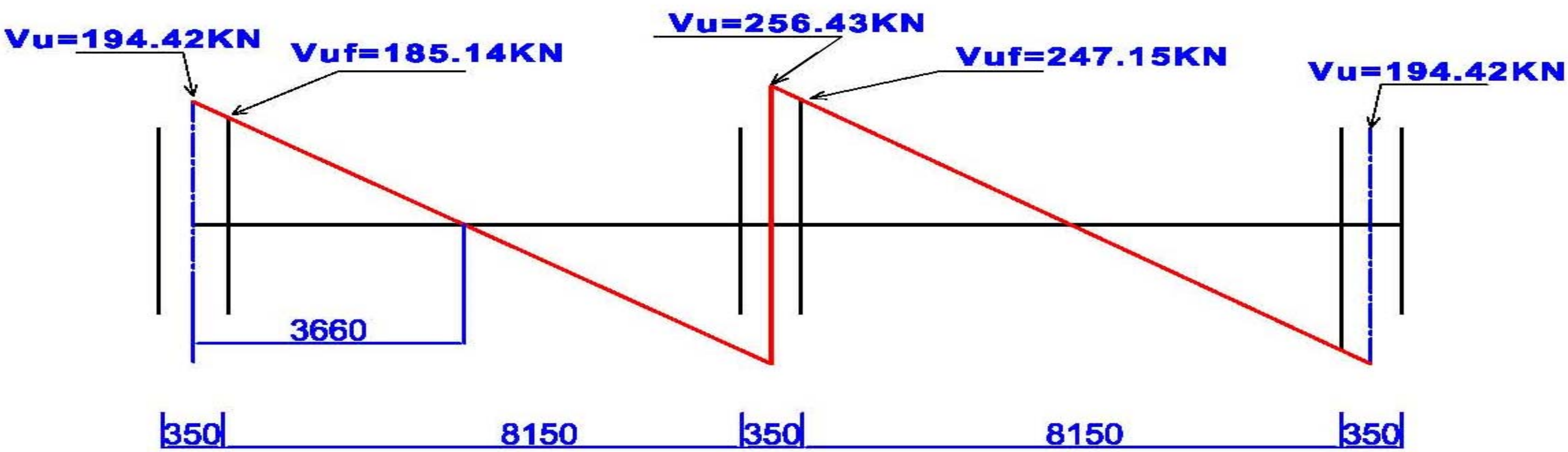
$$Find\ the\ \max positive\ moment = Ra \times x - 143.71 - 53.04 \times \frac{3.662}{2} = 212.62\ KN.m$$



M=143.71KN.m
M=407.17KN.m
M=143.71KN.m
M1
Wu
M2
M1
M2
Ra
Rb
Rb
Rc

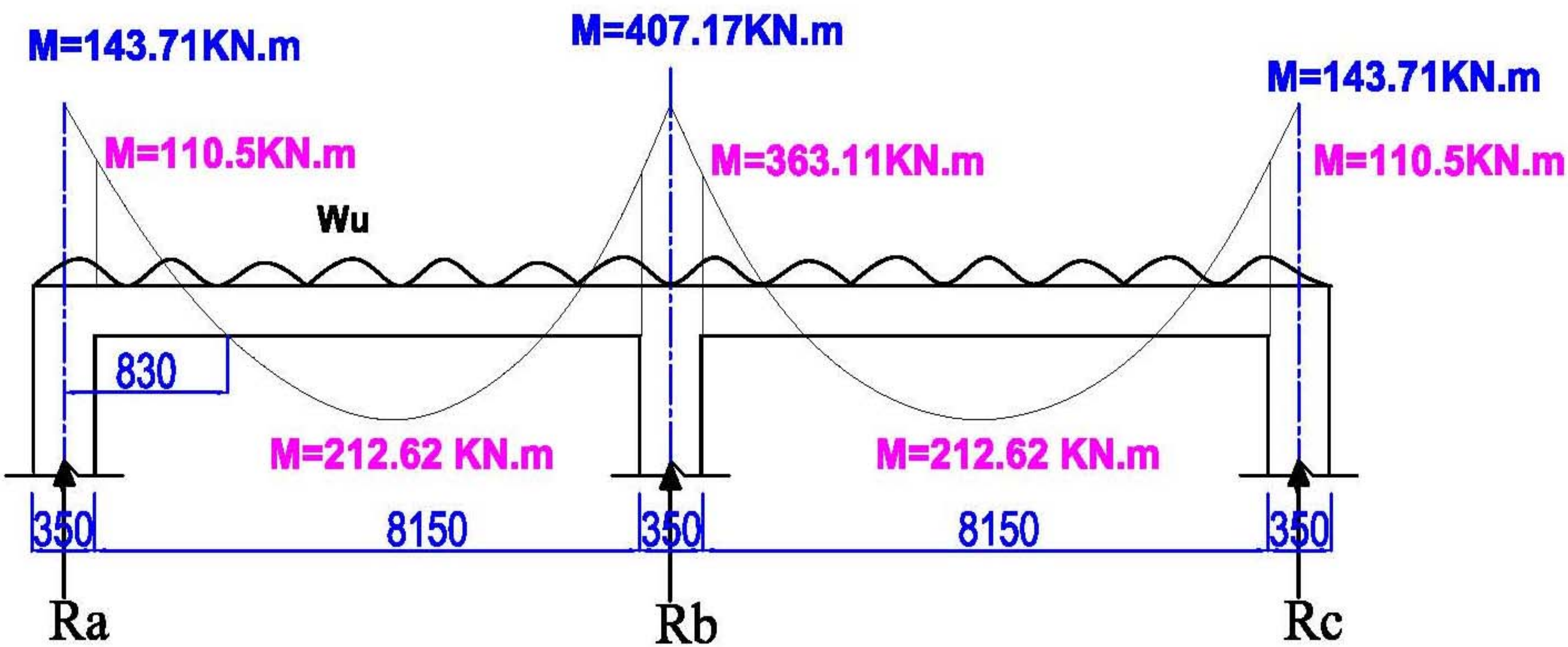


B.M.D



Dimension in mm

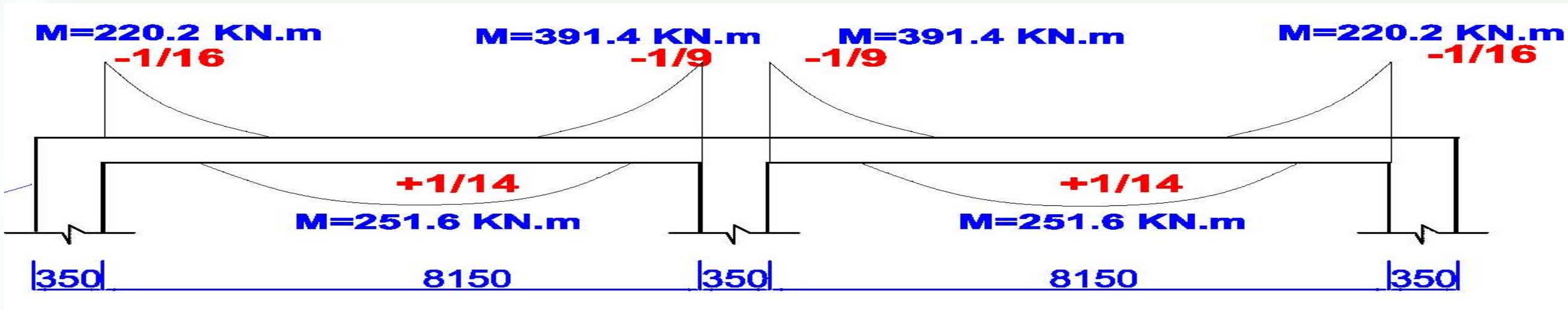
S.F.D



B.M.D

Dimension in mm

Design of interior Beam (B2) *(using ACI Coefficient Methods)*



-Negative Moment

$1 - \ Exterior\ support \quad (-M = 220.2KN.m)$

$R = \ Mu/(\phi\ bd^2)$ → $R = \dfrac{220.2 \times 10^6}{0.9 \times 350 \times 565^2} = \ 2.19$

$m = 15.69$

$\rho = \frac{1}{m} \times (\ 1 - \ \sqrt{1 - \frac{2mR}{fy}})$

$$= \frac{1}{15.69} \times \left(1 - \sqrt{1 - 2 \times 2.19 \times \frac{15.69}{280}} \right) = 0.008371 \ > \ \rho\text{min.} = \frac{1.4}{fy} = 0.005$$

$As = \ \rho bd = \ 0.008371 \times 350 \times 565 = 1655\ mm^2$

$Use\ \ 4\,\phi\, 25\ mm \ = 1964\ mm^2$

$(\ or\ you\ can\ use\ 6\,\phi\,20 = \ 1884\ mm^2\ in\ two\ layer\ then\ we\ have\ to\ corrected\ the\ calculation)$

Check spacing

$$n = \frac{b - 116 - 2ds}{D + S} + 1$$

$$= \frac{350 - 116 - 20}{25 + 25} + 1 = \ 4.3\ Bar \qquad (\ OK)$$

$2 - \ Interior\ support \quad (-M = 391.4 KN.m)$

$$R = \frac{Mu}{\phi \ bd^2}$$

Assume two layer of steel bar

$d = 630 - 90 = 540 \ mm$

$$R = \frac{391.4 \times 10^6}{0.9 \times 350 \times 540^2} = 4.26$$

$m = 15.69$

$$\rho = \frac{1}{m} \times \left(1 - \sqrt{1 - \frac{2mR}{fy}} \right)$$

$$= \frac{1}{15.69} \times \left(1 - \sqrt{1 - 2 \times 4.26 \times \frac{15.69}{280}} \right) = 0.01766 > \rho \text{min.} = \frac{1.4}{fy} = 0.005$$

$As = \rho bd = 0.01766 \times 350 \times 540 = 3337 \ mm^2$

$Use \ \ 8 \phi 25 \ mm = 3928 \ mm^2 \quad (two\ layer)$

$(\ or\ you\ can\ use\ 4 \phi 25 + 4 \phi 22 = 3484 mm^2\ in\ two\ layer\ )$

Check spacing

$$\text{n} = \frac{\text{b} - 116 - 2\text{ds}}{\text{D} + \text{S}} + 1$$

$$= \frac{350 - 116 - 20}{25 + 25} + 1 = 5.25 \ Bar \qquad (OK)$$

Check for maximum steel ratio

calculate $\boldsymbol{\rho_b = \frac{\beta_1}{m}\left(\frac{600}{600+fy}\right)\left(\frac{dt}{d}\right)}$

and calculate $\rho_{max} = \left(\frac{0.003+\frac{fy}{Es}}{0.008}\right)\rho_b$

$d = 540\ mm\ , dt = 565\ mm$

$\rho b = 0.85/15.69 \times (\ 600/(600 + 280)) \times (565/540) = 0.03865$

$$\rho_{max} = \frac{\left(0.003 + \frac{280}{200000}\right)}{0.008} \times \rho b = 0.55 \times \rho b = 0.02126$$

$\rho_{max} = 0.02126 > \rho = 0.01766 > \rho min = 0.005 \qquad (OK)$

$Positive\ \ Moment\ (\ M = 251.6KN.m)$

$We\ have\ T\ section\ ----\ to\ find\ the\ be$

$1 -\ be = 16t\ +\ bw = 16 \times 170 + 350 = 3070mm$

$2 -\ be\ = \frac{L}{4} = \frac{8500}{4} = 2125\ mm$

$3 -\ be = S =\ 4000\ mm$

$Choose\ be\ \ = 2125\ mm$

$Assume\ \ block\ stress\ depth\ \ = a\ \ = h = 170\ mm$

$R = \frac{Mu}{\phi\ \ bd^2}$

$Assume\ two\ layer\ of\ steel\ bar$

$d = 630 - 90 = 540\ mm$

$R = 251.6 \times 10^6/(\ 0.9 \times 2125 \times 540^2) =\ 0.45$

$m = 15.69$

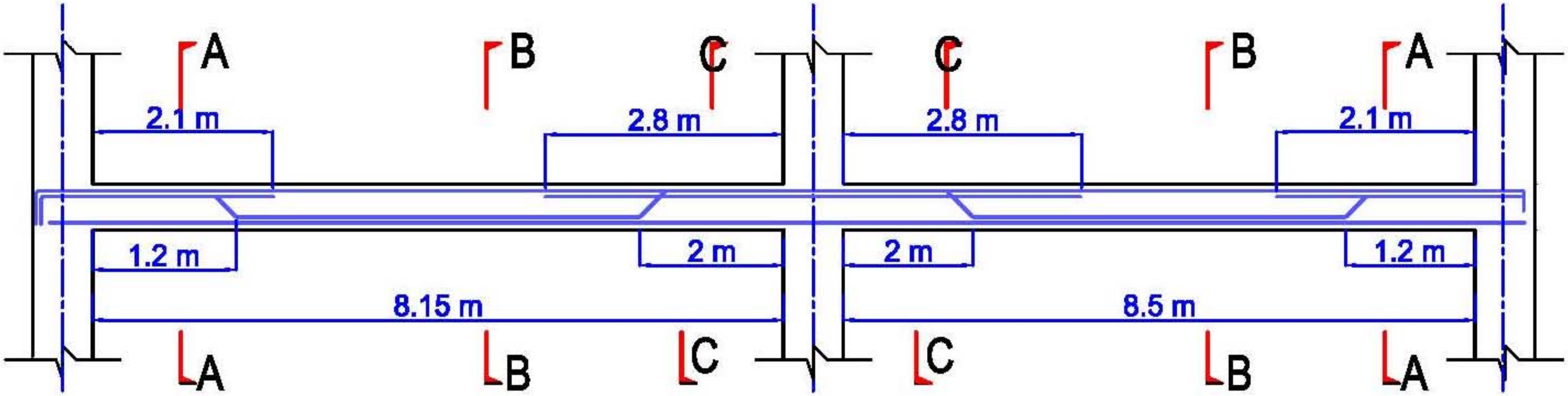
$\rho = \frac{1}{m} \times \left(\ 1 -\ \ \sqrt{1 - \frac{2mR}{fy}}\right)$

$= \frac{1}{15.69} \times \left(\ 1 -\ \ \sqrt{1 - 2 \times 0.45 \times \frac{15.69}{280}}\right) =\ 0.001632\ <\ \rho_{min}. = \frac{1.4}{fy} = 0.005$

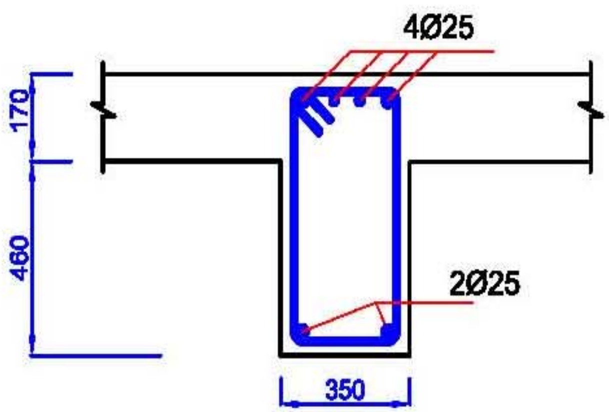
$As =\ \rho_{min}\, bd =\ 0.005 \times 350 \times 540 = 945\ mm^2$

$a =\ \rho.m.d = 0.005 \times 15.69 \times 540 = 42.8\ mm <\ 170\ mm$ $\ \ (\ design\ as\ rectangular\ section\ )$

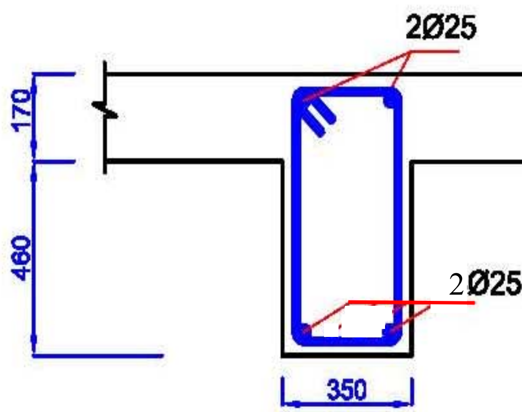
$Use\ \ \ 2\phi\, 25\ mm\ =\ 982mm^2$ $\ (one\ layer)$



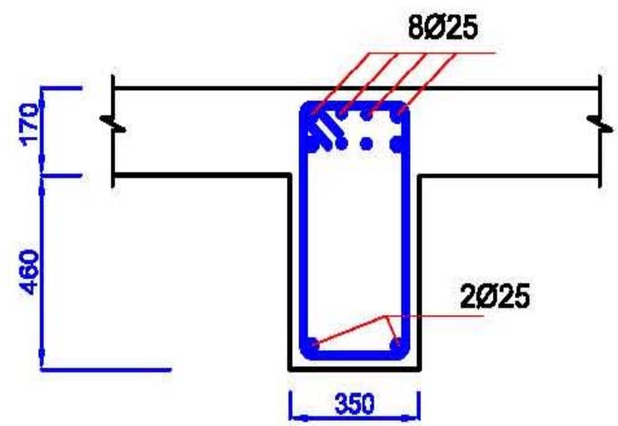
Bent Beams



Beam B2
Section A-A



Beam B2
Section B-B



Beam B3
Section C-C

Thank You

………….To be Continued

Reinforced Concrete Design
Design For Shear

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Introduction

When a simple beam is loaded, as shown in Fig. Below , bending moments and shear forces develop along the beam. To carry the loads safely, the beam must be designed for both types of forces. Flexural design is considered first to establish the dimensions of the beam section and the main reinforcement needed, as explained in the previous chapters.

The beam is then designed for shear. If shear reinforcement is not provided, *shear failure* may occur. Shear failure is characterized by small deflections and lack of ductility, giving little or no warning before failure. On the other hand, flexural failure is characterized by a gradual increase in deflection and cracking, thus giving warning before total failure. This is due to the ACI Code limitation on flexural reinforcement. The design for shear must ensure that shear failure does not occur before flexural failure.

SHEAR STRESSES IN CONCRETE BEAMS

The general formula for the shear stress in a homogeneous beam is

$$\upsilon = \frac{VQ}{I\,b} \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where:

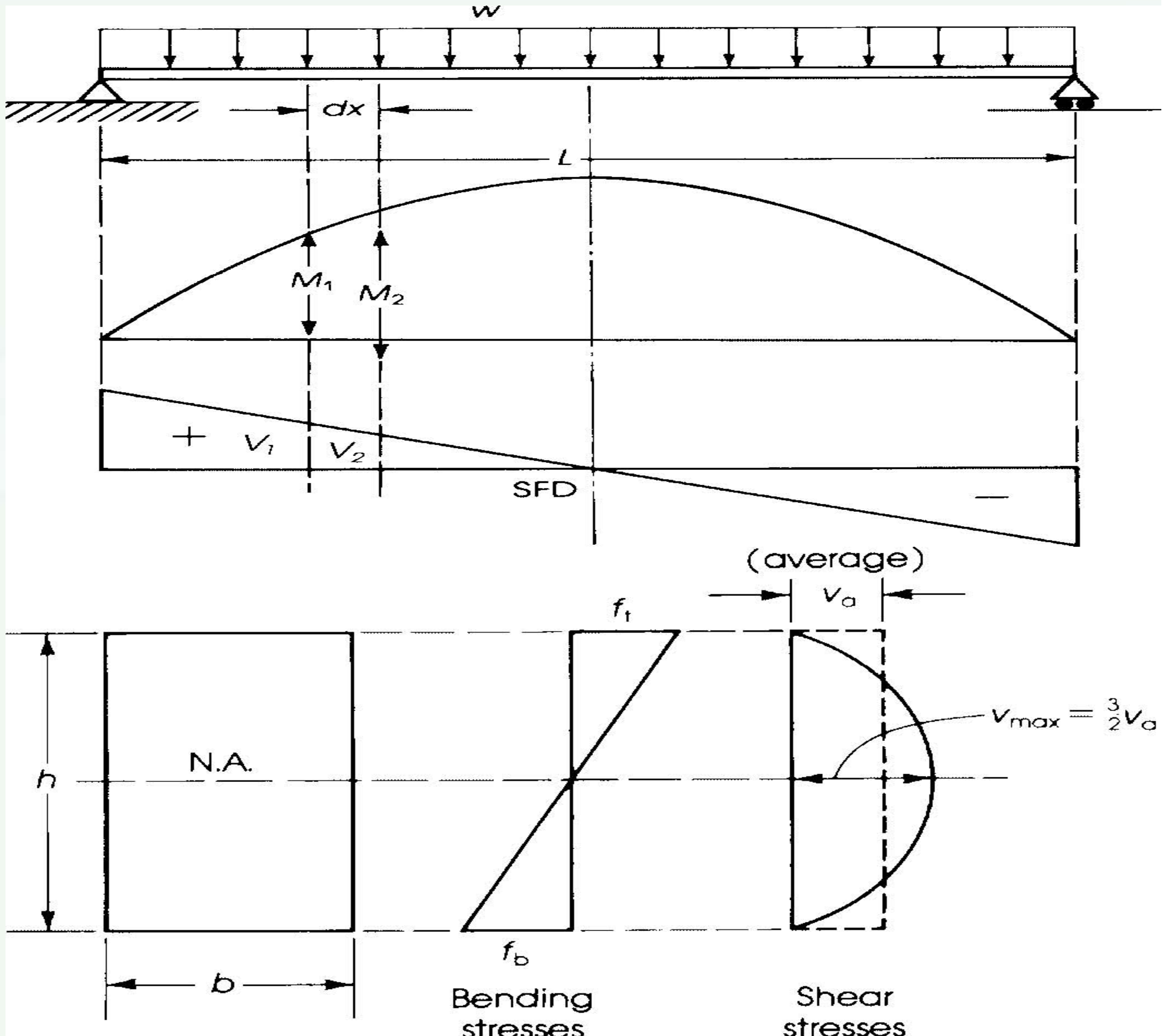
V = total shear at section considered

Q=statical moment about neutral axis of that portion of cross section lying between line through point in question parallel to neutral axis and nearest face, upper or lower, of beam.

I=moment of inertia of cross section about neutral axis.

b=width of beam at given point.

The distribution of bending and shear stresses according to elastic theory for a homogeneous rectangular beam is as shown in Fig. Below. The bending stresses are calculated from the flexural



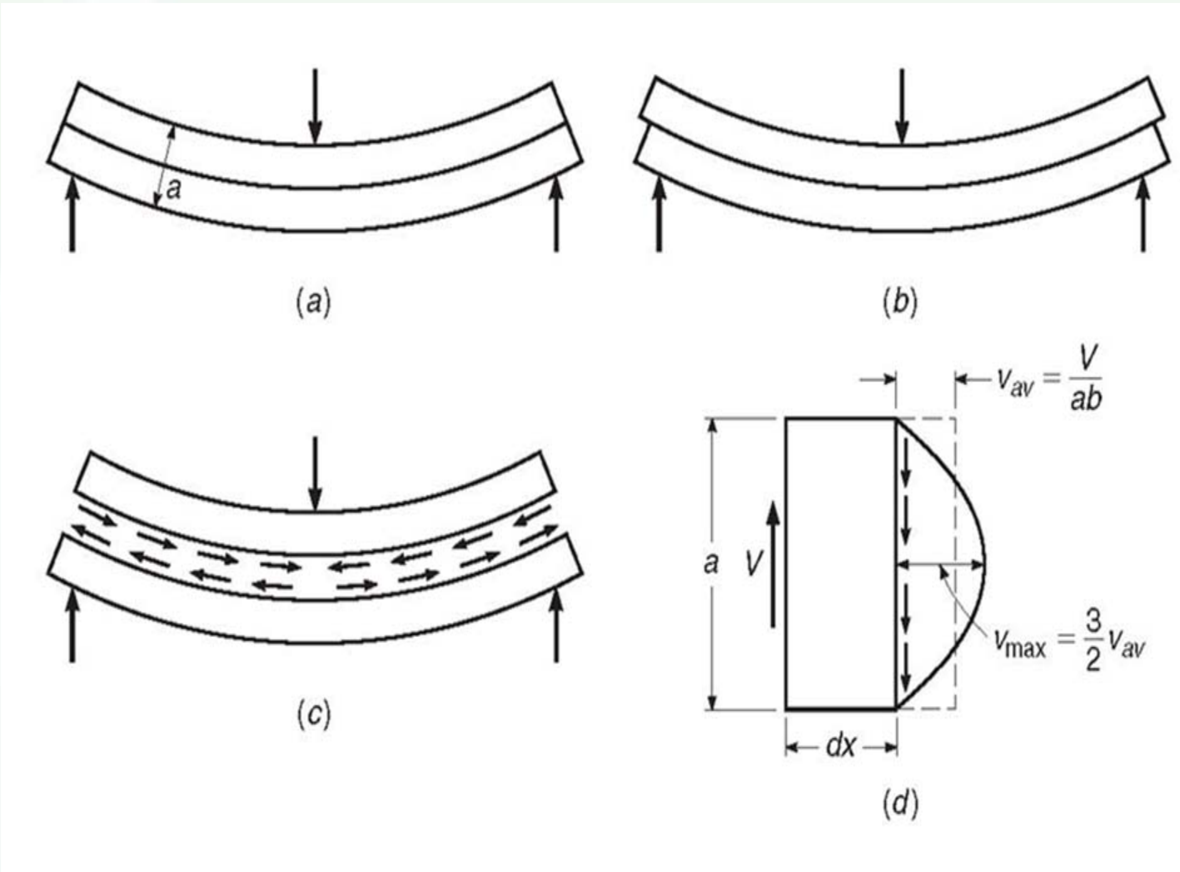
formula $f = \frac{M\,C}{I}$, whereas the shear stress at any point is calculated by the shear formula of Eq.1.

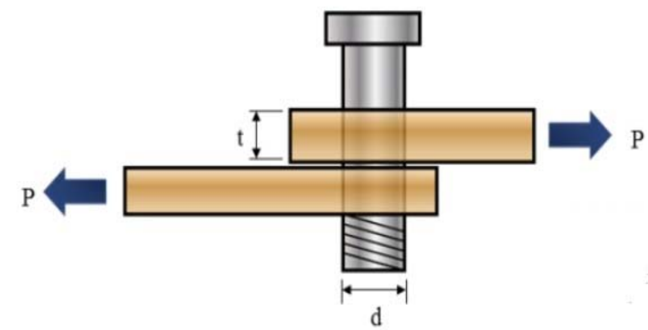
The maximum shear stress is at the neutral axis and is equal to

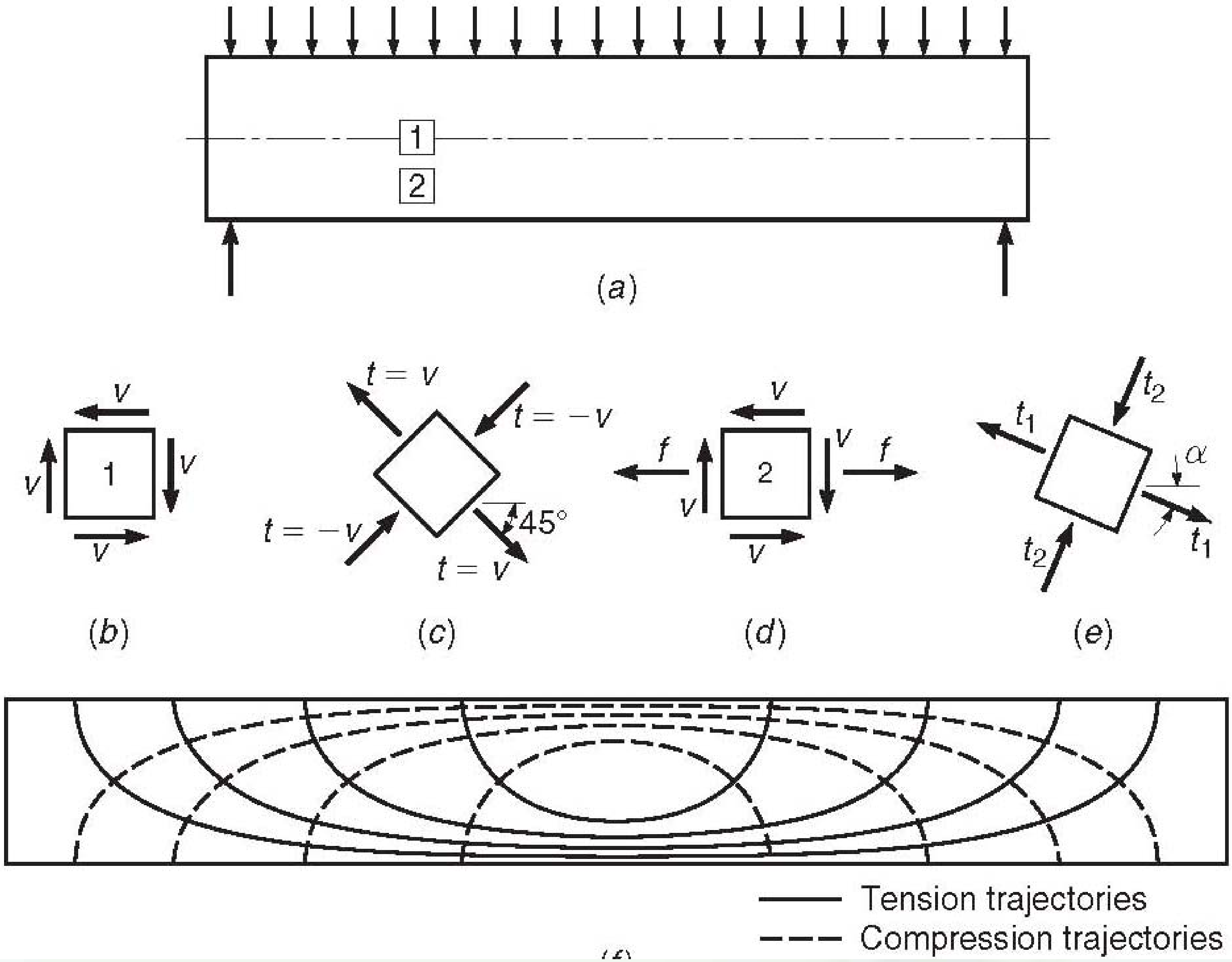
1.5va (average shear),

Where: $v_{max} = \frac{3}{2} v_a = \frac{3\,V}{2\,b\,h}$

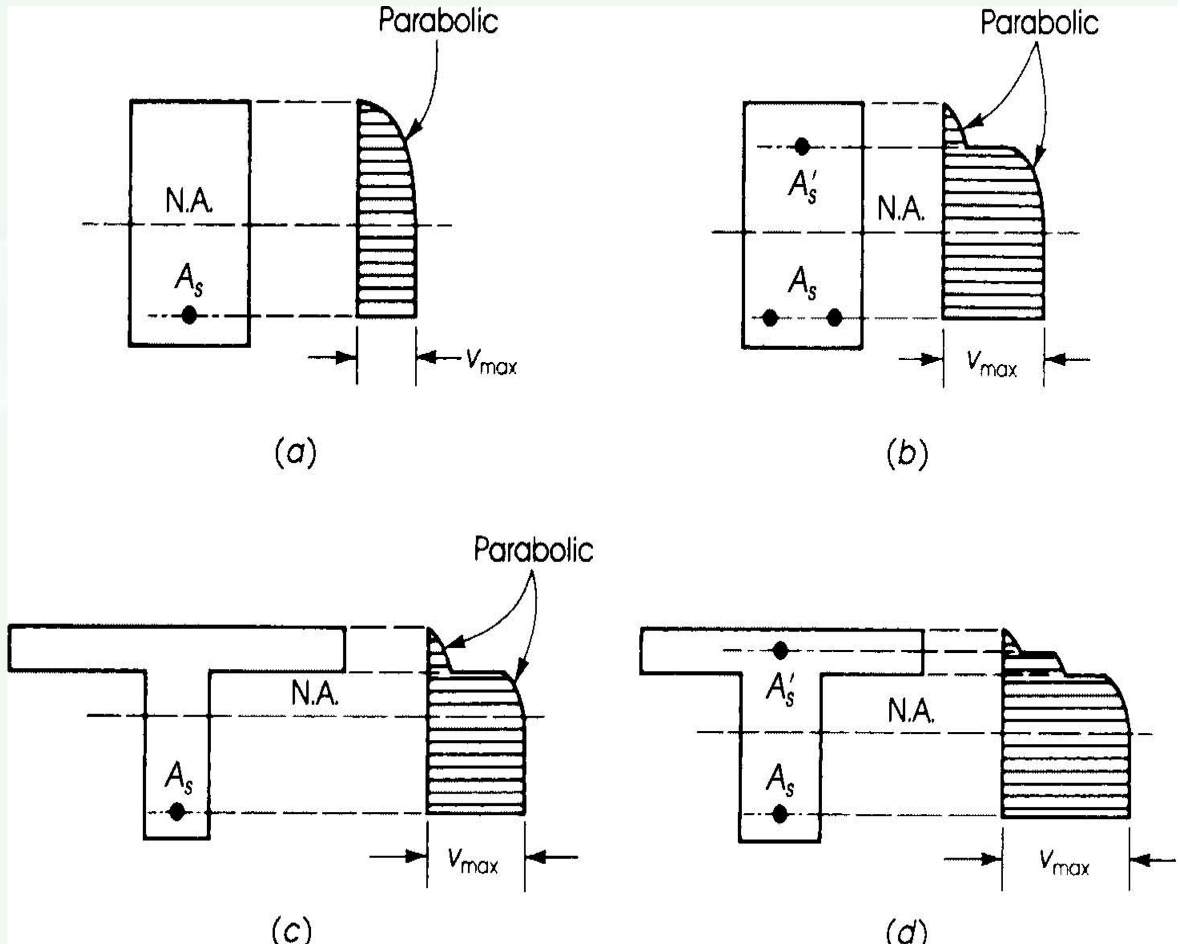
$Fig.$5.1)


a
(a)
(b)
(c)
$v_{av} = \frac{V}{ab}$
a
V
$v_{max} = \frac{3}{2} v_{av}$
dx
(d)


t
P
P
d


1
2
(a)
v
1
(b)
t = v
t = −v
t = −v
t = v
45°
(c)
f
f
2
(d)
t1
t2
α
t2
t1
(e)
Tension trajectories
Compression trajectories

The shear stress curve is parabolic. For a singly reinforced concrete beam, the distribution of shear stress above the neutral axis is a parabolic curve. Below the neutral axis, the maximum shear stress is maintained down to the level of the tension steel, because there is no change in the tensile force down to this point and the concrete in tension is neglected. The shear stress below the tension steel is zero. For doubly reinforced and T-sections, the distribution of shear stresses is as shown in Fig.



It can be observed that almost all the shear force is resisted by the web, whereas the flange resists a very small percentage; in most practical problems, the shear capacity of the flange is neglected.

Referring to Fig. 1 and taking any portion of the beam dx, the bending moments at both ends of the element, M_1 and M_2, are not equal. Because $M_2 > M_1$ and to maintain the equilibrium of the beam portion dx, the compression force C_2 must be greater than C_1 (Fig. 1). Consequently, a shear stress v develops along any horizontal section $a–a_1$ or $b–b_1$ (Fig. 1a). The normal and shear stresses on a small element at levels $a–a_1$ and $b–b_1$ are shown in Fig. 1b. Notice that the normal stress at the level of the neutral axis $b–b_1$ is zero, whereas the shear stress is at maximum.

The horizontal shear stress is equal to the vertical shear stress, as shown in Fig. 1b. When the normal stress f is zero or low, a case of pure shear may occur. In this case, the maximum tensile stress f_t acts at 45∘ (Fig. 1c).

The tensile stresses are equivalent to the principal stresses, as shown in Fig. 5.4d. Such principal stresses are traditionally called diagonal tension stresses. When the diagonal tension stresses reach the tensile strength of concrete, a diagonal crack develops. This brief analysis explains the concept of diagonal tension and diagonal cracking. The actual behavior is more complex, and it is affected by other factors. For the combined action of shear and normal stresses at any point in a beam, the maximum and minimum diagonal tension (principal stresses) f_p are given by the equation

$$f_p = \frac{1}{2}f \mp \sqrt{\left(\frac{1}{2}f\right)^2 + v^2}$$

Where:
f = intensity of normal stress due to bending
v = shear stress

The shear failure in a concrete beam is most likely to occur where shear forces are at maximum, generally near the supports of the member. The first evidence of impending failure is the formation of diagonal cracks.

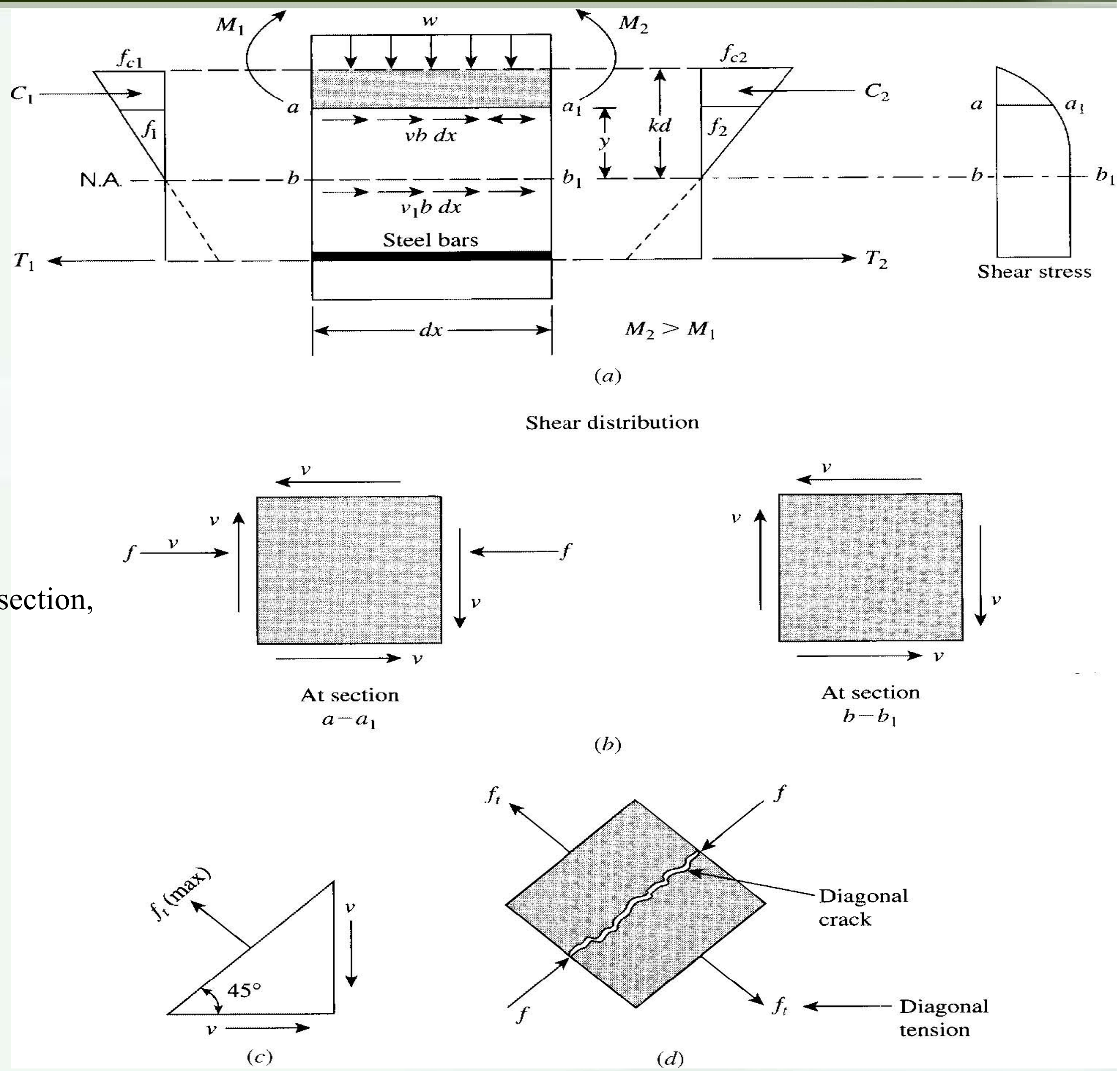


Fig. 1

(a) Forces and stresses along depth of section,

(b) Normal and shear stresses,

(c) Pure shear, and

(d) Diagonal tension.

3. BEHAVIOR OF BEAMS WITHOUT SHEAR REINFORCEMENT

Concrete is weak in tension, and the beam will collapse if proper reinforcement is not provided. The tensile stresses develop in beams due to axial tension, bending, shear, torsion, or a combination of these forces. The location of cracks in the concrete beam depends on the direction of principal stresses. For the combined action of normal stresses and shear stresses, maximum diagonal tension may occur at about a distance d from the face of the support.

The behavior of reinforced concrete beams with and without shear reinforcement tested under increasing load was discussed in chapter of analysis of beam under flexural. In the tested beams, vertical flexural cracks developed at the section of maximum bending moment when the tensile stresses in concrete exceeded the modulus of rupture of concrete, or $f_r = 7.5\ \lambda\sqrt{f_c'}$. Inclined cracks in the web developed at a later stage at a location very close to the support.

An inclined crack occurring in a beam that was previously uncracked is generally referred to as a web-shear crack. If the inclined crack starts at the top of an existing flexural crack and propagates into the beam, the crack is referred to as flexural-shear crack (Fig. 2). Web-shear cracks occur in beams with thin webs in regions with high shear and low moment. They are relatively uncommon cracks and may occur near the inflection points of continuous beams or adjacent to the supports of simple beams.

Flexural-shear cracks are the most common type found in reinforced concrete beams. A flexural crack extends vertically into the beam; then the inclined crack forms, starting from the top of the beam when shear stresses develop in that region. In regions of high shear stresses, beams must be reinforced by stirrups or by bent bars to produce ductile beams that do not rupture at a failure.

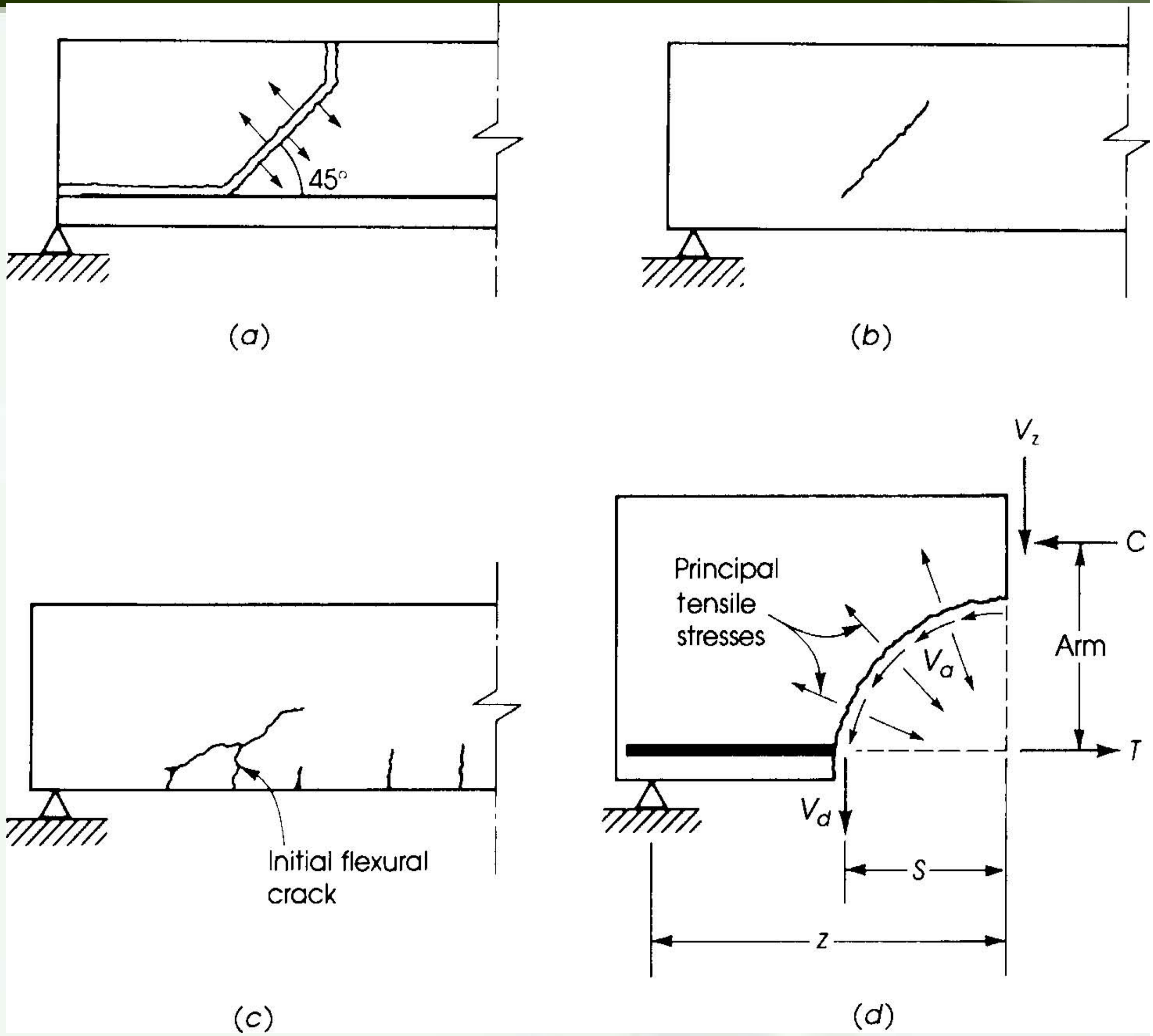


Fig. 2 Shear failure:
(a) General form,
(b) web-shear crack,
(c) Flexural-shear crack,
(d) Analysis of forces involved in shear
V_z = shear resistance,
V_a = interface shear,
V_d = dowel force.

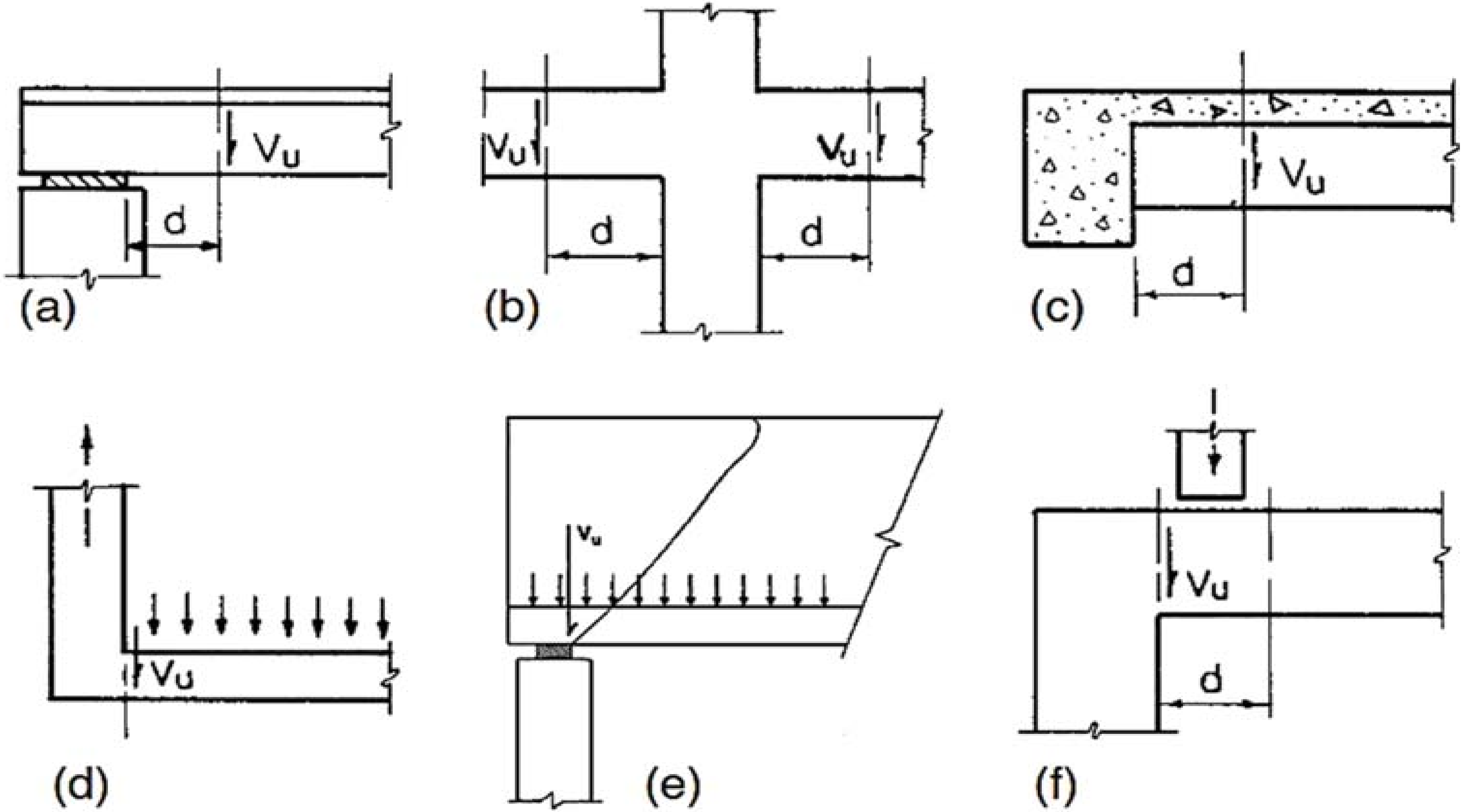


Figure 3 *Typical Support Conditions for Locating Factored Shear Force* V_u

4. MOMENT EFFECT ON SHEAR STRENGTH

In simply supported beams under uniformly distributed load, the midspan section is subjected to a large bending moment and zero or small shear, whereas sections near the ends are subjected to large shear and small bending moments . The shear and moment values are both high near the intermediate supports of a continuous beam. At a location of large shear force and small bending moment, there will be little flexural cracking, and an average stress v is equal to V/bd.

The diagonal tensile stresses are inclined at about 45∘ (Fig. 1c). Diagonal cracks can be expected when the diagonal tensile stress in the vicinity of the neutral axis reaches or exceeds the tensile strength of concrete. In general, the factored shear strength varies between $3.5\sqrt{f_c'}$ and $5\sqrt{f_c'}$. After completing a large number of beam tests on shear and diagonal tension, it was found that in regions with large shear and small moment, diagonal tension cracks were formed at an average shear force of:

$$V_c = 3.5\sqrt{f_c'}\,b_w d$$

where b_w is the width of the web in a T-section or the width of a rectangular section and d is the effective depth of the beam.

In locations where shear forces and bending moments are high, flexural cracks are formed first. At a later stage, some cracks bend in a diagonal direction when the diagonal tension stress at the upper end of such cracks exceeds the tensile strength of concrete. Given the presence of large moments on a beam, for which adequate reinforcement is provided, the nominal shear force at which diagonal tension cracks develop is given by:

$$V_c = 1.9\,\lambda\sqrt{f_c'}\,b_w d$$

This value is a little more than half the value in last Eq. when bending moment is very small. This means that large bending moments reduce the value of shear stress for which cracking occurs. The following equation has been suggested to predict the nominal shear stress at which a diagonal crack is expected:

$$v_c = \frac{V}{b_w d} = (1.9\,\lambda\sqrt{f_c'} + 2500\,\rho\frac{Vd}{M}) \leq 3.5\lambda\sqrt{f_c'}$$

5. BEAMS WITH SHEAR REINFORCEMENT

Different types of shear reinforcement may be used:

1. Stirrups, which can be placed either perpendicular to the longitudinal reinforcement or inclined, usually making a 45∘ angle and welded to the main longitudinal reinforcement. Vertical stirrups, using no. 3 (10 mm) or no. 4 (12 mm) U-shaped bars, are the most commonly used shear reinforcement in beams (Fig. 4a).
2. Bent bars, which are part of the longitudinal reinforcement, bent up (where they are no longer needed) at an angle of 30∘ to 60∘, usually at 45∘.
3. Combinations of stirrups and bent bars.
4. Welded wire fabric with wires perpendicular to the axis of the member.
5. Spirals, circular ties, or hoops in circular sections, as columns.

The shear strength of a reinforced concrete beam is increased by the use of shear reinforcement. Prior to the formation of diagonal tension cracks, shear reinforcement contributes very little to the shear resistance. After diagonal cracks have developed, shear reinforcement augments the shear resistance of a beam, and a redistribution of internal forces occurs at the cracked section. When the amount of shear reinforcement provided is small, failure due to yielding of web steel may be expected, but if the amount of shear reinforcement is too high, a shear–compression failure may be expected, which should be avoided.

Concrete, stirrups, and bent bars act together to resist transverse shear. The concrete, by virtue of its high compressive strength, acts as the diagonal compression member of a lattice girder system, where the stirrups act as vertical tension members. The diagonal compression force is such that its vertical component is equal to the tension force in the stirrup. Bent-up reinforcement acts also as tension members in a truss, as shown in Fig. 4.

In general, the contribution of shear reinforcement to the shear strength of a reinforced concrete beam can be described as follows:

1. It resists part of the shear, V_s.

2. It increases the magnitude of the interface shear, V_a , by resisting the growth of the inclined crack.

3. It increases the dowel force, V_d (Fig. 2), in the longitudinal bars.

4. The confining action of the stirrups on the compression concrete may increase its strength.

5. The confining action of stirrups on the concrete increases the rotation capacity of plastic hinges that develop in indeterminate structures at maximum load and increases the length over which yielding takes place.

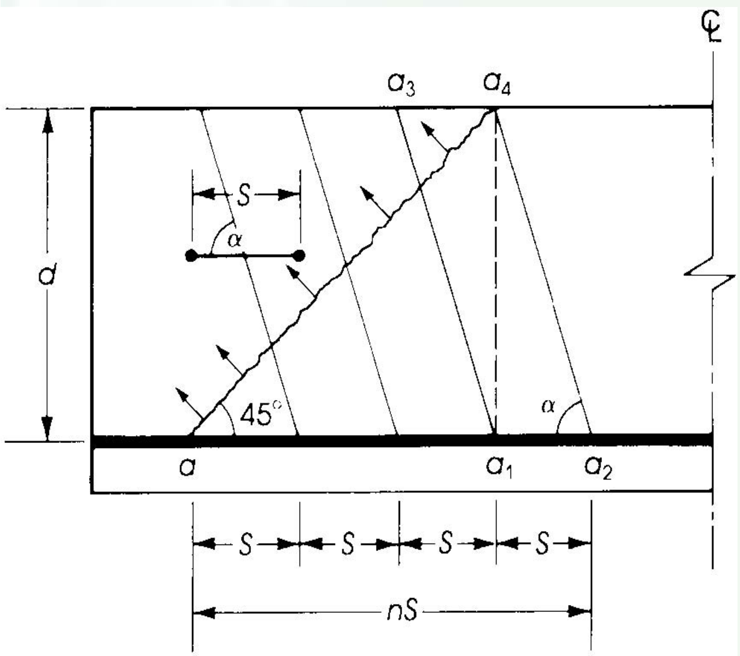
The total nominal shear strength of beams with shear reinforcement V_n is due partly to the shear strength attributed to the concrete, V_c, and partly to the shear strength contributed by the shear reinforcement, V_s:

$$V_n = V_c + V_s$$

The shear force Vu produced by factored loads must be less than or equal to the total nominal shear strength V_n, or

$$V_u \leq \phi V_n = \phi\,(V_c + V_s)$$

where $V_u = 1.2\,V_D + 1.6\,V_L$ and $\phi = 0.75$.



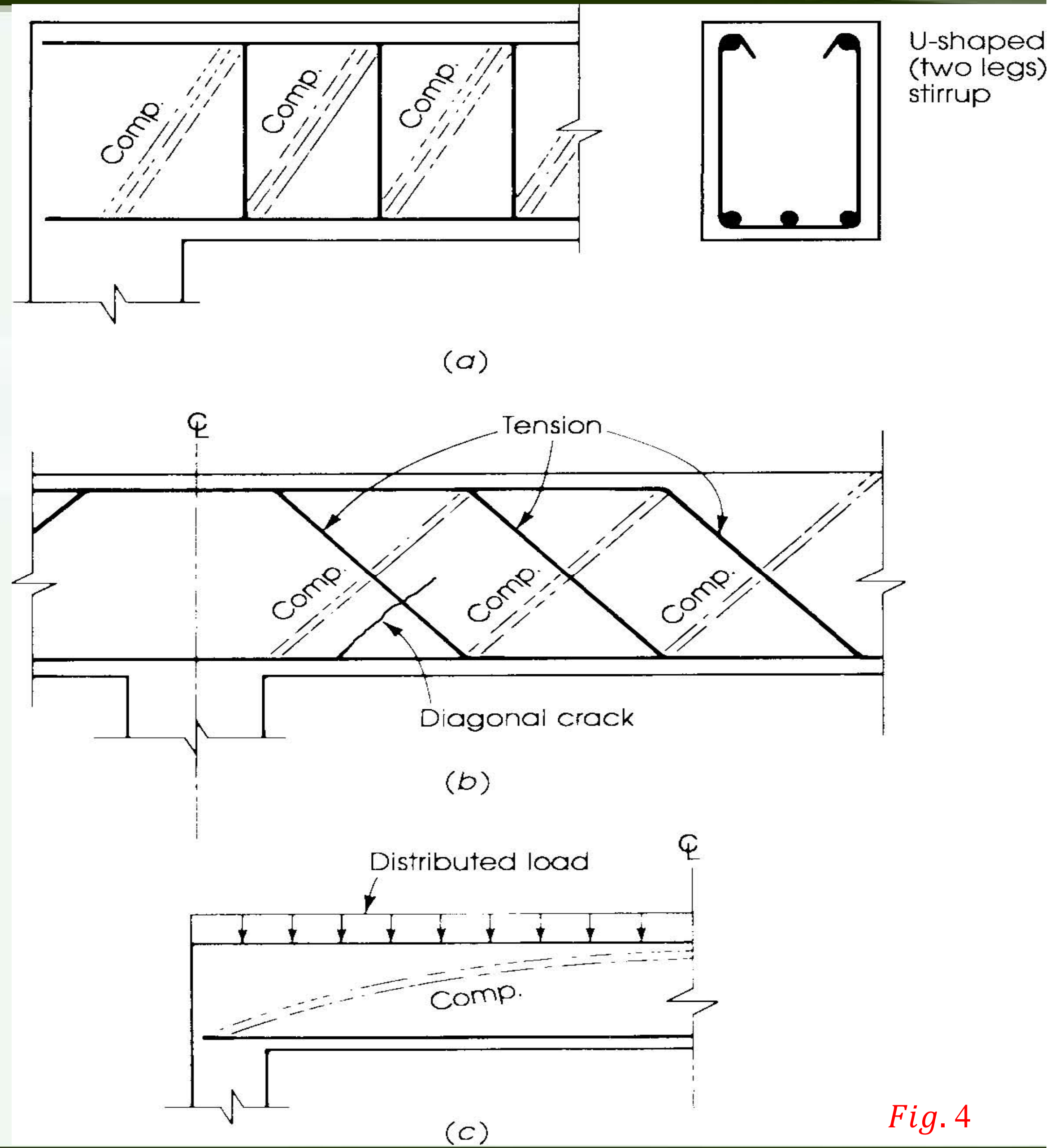


Fig. 4

An expression for V_s may be developed from the truss analogy (Fig. 4). For a 45∘ crack and a series of inclined stirrups or bent bars, the vertical shear force V_s resisted by shear reinforcement is equal to the sum of the vertical components of the tensile forces developed in the inclined bars.

Therefore,

$$V_s = n\, A_v\, f_{yt} \sin\alpha \qquad Eq.2$$

where A_v is the area of shear reinforcement with a spacing s and f_{yt} is the yield strength of shear reinforcement; ns is defined as the distance aa_1a_2:

$$d = \begin{cases} a_1a_4 = aa_1 \tan 45\circ \ (from\ triangle\ a\, a_1a_4) \\ a_1a_4 = a_1a_2 \tan\alpha \ (from\ triangle\ a_1a_2a_4) \end{cases}$$

$$n * s = aa_1 + a_1a_2$$

$$= d(\cot 45^\circ + \cot\alpha) = d(1 + \cot\alpha)$$

$$n = \frac{d}{S}(1 + \cot\alpha)$$

Substituting this value in Eq.2 gives

$$V_s = \frac{A_v\, f_{yt} d}{S} \sin\alpha (1 + \cot\alpha) = \frac{A_v\, f_{yt} d}{S}(\sin\alpha + \cos\alpha)$$

For the case of vertical stirrups, α =90∘ and

$$V_s = \frac{A_v f_{yt} d}{S} \quad or \quad S = \frac{A_v f_{yt} d}{V_s} \quad Eq.3$$

In the case of T-sections, b is replaced by the width of web b*w* in all shear equations. When α =45∘, Eq.3 becomes

$$V_s = 1.4\left(\frac{A_v f_{yt} d}{S}\right) or \ S = 1.4(\frac{A_v f_{yt} d}{V_s})$$

For a single bent bar or group of parallel bars in one position, the shearing force resisted by steel is

$$V_s = A_v \ f_{yt} \sin\alpha \quad or \ Av = \frac{V_s}{f_{yt} \sin\alpha}$$

For α =45°,

$$Av = 1.4\ (\frac{V_s}{f_{yt}})$$

For circular sections, mainly in columns, V_s will be computed from Eq.3 using (d = 0.8 × diameter), and (A_v =two times the area of the bar in a circular tie, hoop, or spiral).

6. ACI CODE SHEAR DESIGN REQUIREMENTS

6.1 Critical Section for Nominal Shear Strength Calculation

The ACI Code, Section 9.4.3.2, permits taking the critical section for nominal shear strength calculation at a distance d from the face of the support. This recommendation is based on the fact that the first inclined crack is likely to form within the shear span of the beam at some distance d away from the support. This critical section is permitted on the condition that the support reaction introduces compression into the end region, loads are applied at or near the top of the member, and no concentrated load occurs between the face of the support and the location of the critical section.

The Code also specifies that shear reinforcement must be provided between the face of the support and the distance d using the same reinforcement adopted for the critical section.

6.2 Minimum Area of Shear Reinforcement

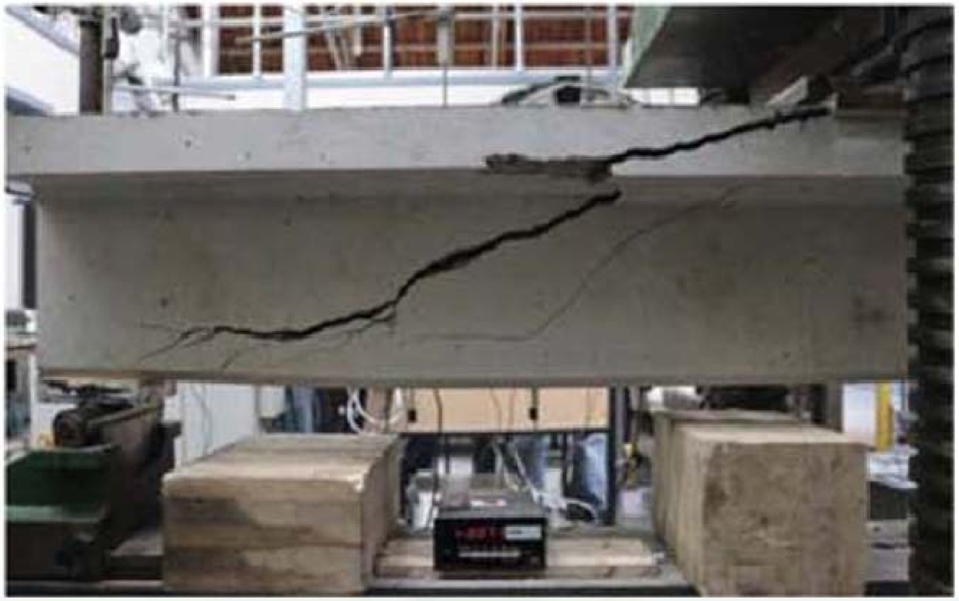
The presence of shear reinforcement in a concrete beam restrains the growth of inclined cracking. Moreover, ductility is increased, and a warning of failure is provided. If shear reinforcement is not provided, brittle failure will occur without warning. Accordingly, a minimum area of shear reinforcement is specified by the Code. The ACI Code, Section 9.6.3.3, requires all stirrups to have a minimum shear reinforcement area, A_v, equal to:

$$A_{v,min} = greater\ of \begin{cases} 0.062\sqrt{f_c'}\,\dfrac{b_w\,s}{f_{yt}} \\ 0.35\,\dfrac{b_w\,s}{f_{yt}} \end{cases}$$

where *bw* is the width of the web and *S* is the spacing of the stirrups. The minimum amount of shear reinforcement is required whenever V_u exceeds $\phi V_c / 2$, except in:

1. Slabs and footings.
2. Concrete floor joist construction.
3. Beams where the total depth (h) does not exceed 10 in.(250 mm), 2.5 times the flange thickness for T-shaped flanged sections, or one-half the web width, whichever is greatest.
4. The beam is integrated with slab, h not greater 24 in.(600 mm) and not greater than the larger of 2.5 times the thickness of the flange and 0.5 times the width of the web.

Shear Failure



6.3 Maximum Shear Carried by Web Reinforcement Vs

To prevent a shear–compression failure, where the concrete may crush due to high shear and compressive stresses in the critical region on top of a diagonal crack, the ACI Code, Section 22.5.1.2, requires that V_s shall not exceed

$(0.66\sqrt{f_c'})\, b_w d$. If V_s exceeds this value, the section should be increased.

6.4 Maximum Spacing of Stirrups

To ensure that a diagonal crack will always be intersected by at least one stirrup. Maximum spacing of legs of shear reinforcement along the length of the member and across the width of the member shall be in accordance with the ACI Code, Table 9.7.6.2.2.

Table 9.7.6.2.2—Maximum spacing of legs of shear reinforcement

Required V_s		Maximum s, mm			
		Nonprestressed beam		Prestressed beam	
		Along length	Across width	Along length	Across width
$\le 0.33\sqrt{f_c'}b_w d$	Lesser of:	$d/2$	d	$3h/4$	$3h/2$
		600			
$> 0.33\sqrt{f_c'}b_w d$	Lesser of:	$d/4$	$d/2$	$3h/8$	$3h/4$
		300			

This is based on the assumption that a diagonal crack develops at 45° and extends a horizontal distance of about d. In regions of high shear, where Vs exceeds $(0.33\sqrt{f_c'})b_w d$, the maximum spacing between stirrups must not exceed d/4. This limitation is necessary to ensure that the diagonal crack will be intersected by at least three stirrups. When V_s exceeds the maximum value of $(0.66\sqrt{f_c'})b_w d$, this limitation of maximum stirrup spacing does not apply, and the dimensions of the concrete cross section should be increased.

A second limitation for the maximum spacing of stirrups may also be obtained from the condition of minimum area of shear reinforcement. A minimum A_v is obtained when the spacing s is maximum.

A third limitation for maximum spacing is 600 mm. when $Vs \leq (0.33\sqrt{f_c'})\, b_w d$ and 300mm. when V_s is greater than $(0.33\sqrt{f_c'})b_w d$ but less than or equal to $(0.66\sqrt{f_c'})b_w d$. The least value of all maximum spacing must be adopted. The ACI Code maximum spacing requirement ensures closely spaced stirrups that hold the longitudinal tension steel in place within the beam, thereby increasing their dowel capacity, V_d (Fig. 5.5).

6.5 DESIGN OF VERTICAL STIRRUPS

Stirrups are needed when $Vu \geq \phi Vc$. Minimum stirrups are used when Vu is greater than 0.5 ϕVc but less than ϕV. This is achieved by using no.3 (10 mm) stirrups placed at maximum spacing. When Vu is greater than ϕV, stirrups must be provided. The spacing of stirrups may be less than the maximum spacing and can be calculated using

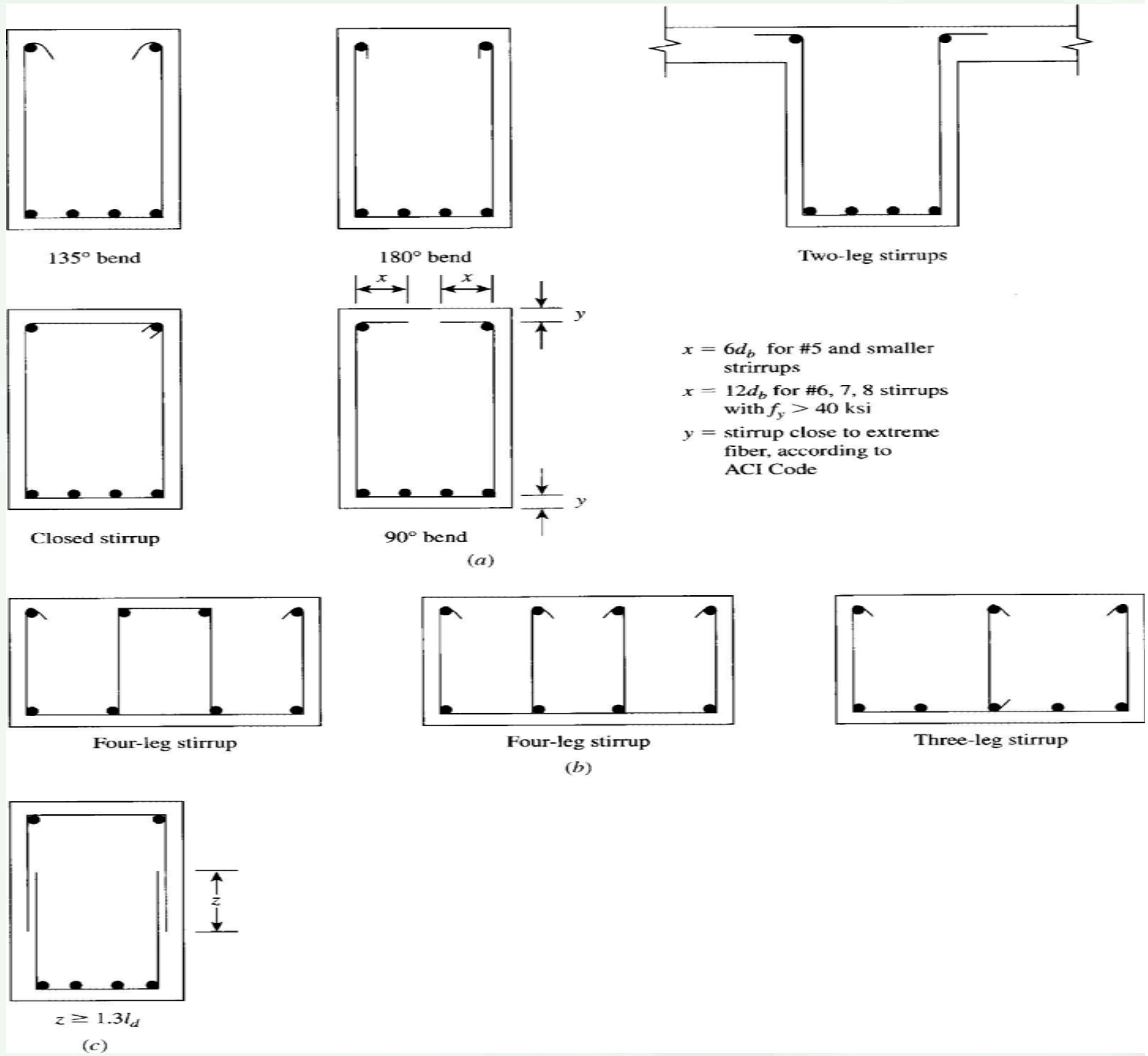
$$S = \frac{A_v\, f_{yt} d}{V_s}$$

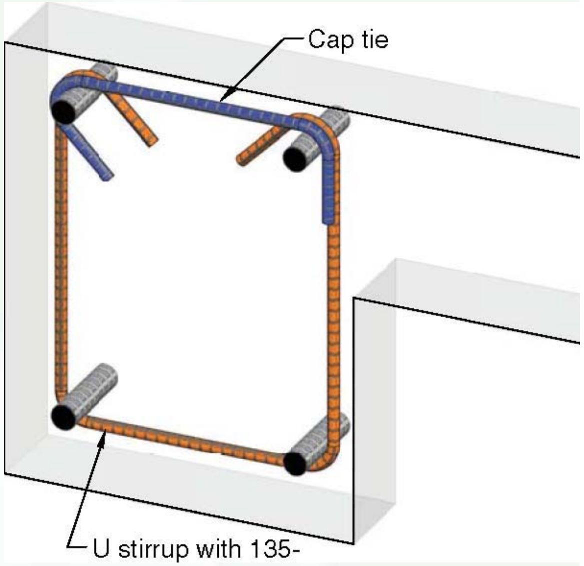
Figure Stirrup types:

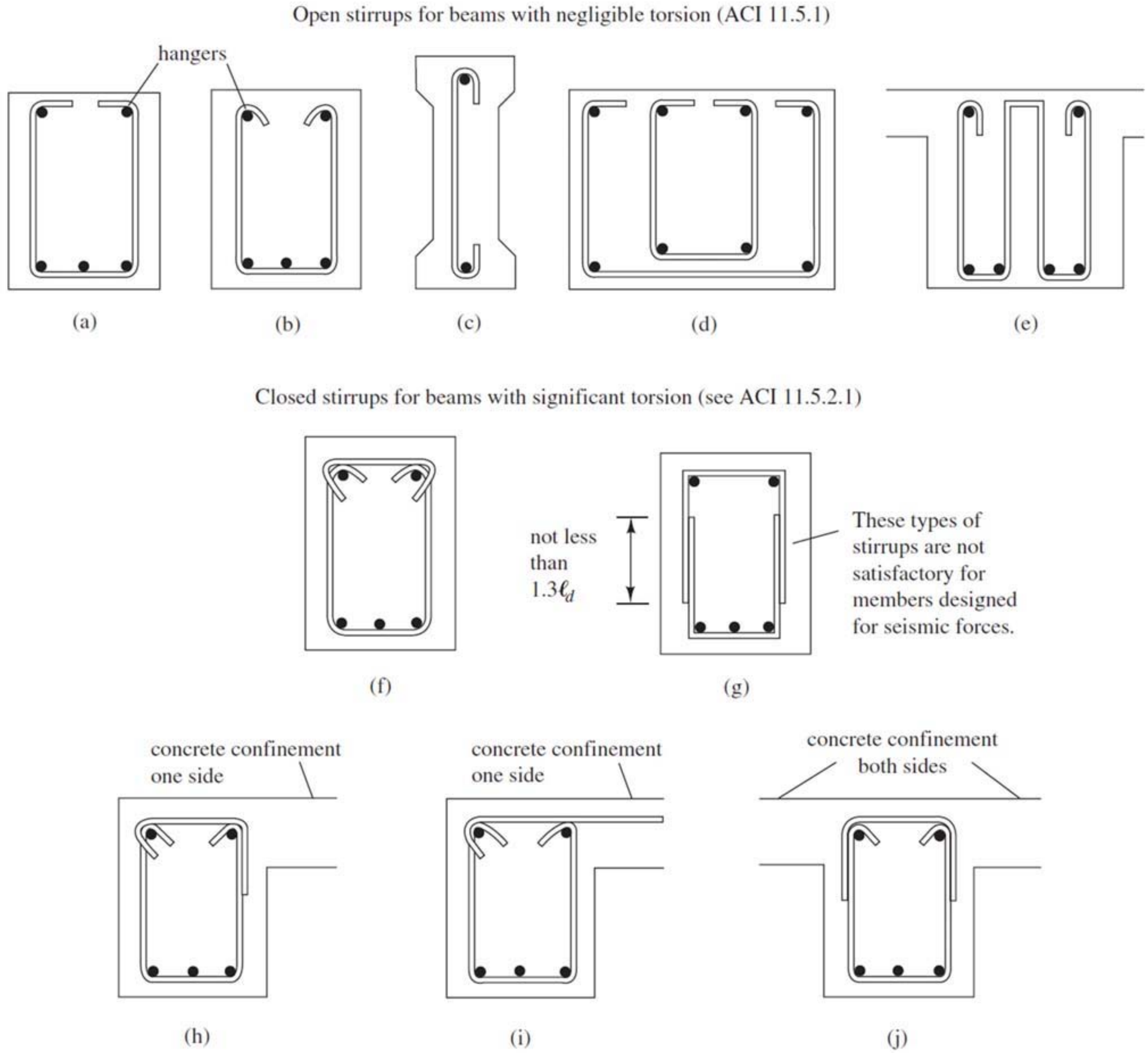
(a)U-stirrups enclosing longitudinal bars, anchorage lengths, and closed stirrups;

(b) Multi leg stirrups; and

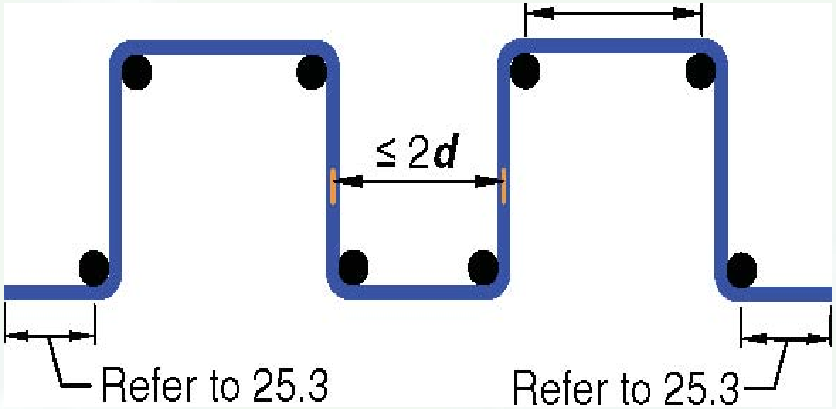
(c) Spliced stirrups.

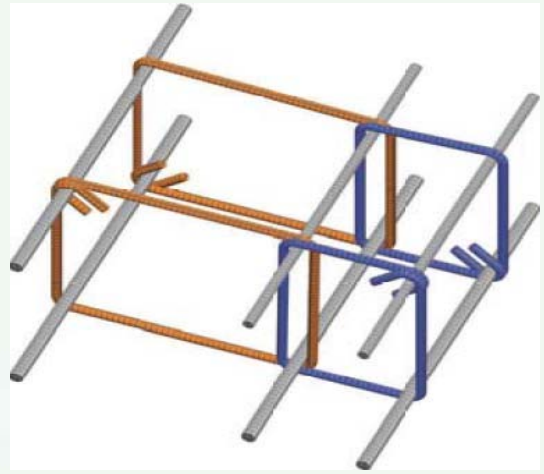


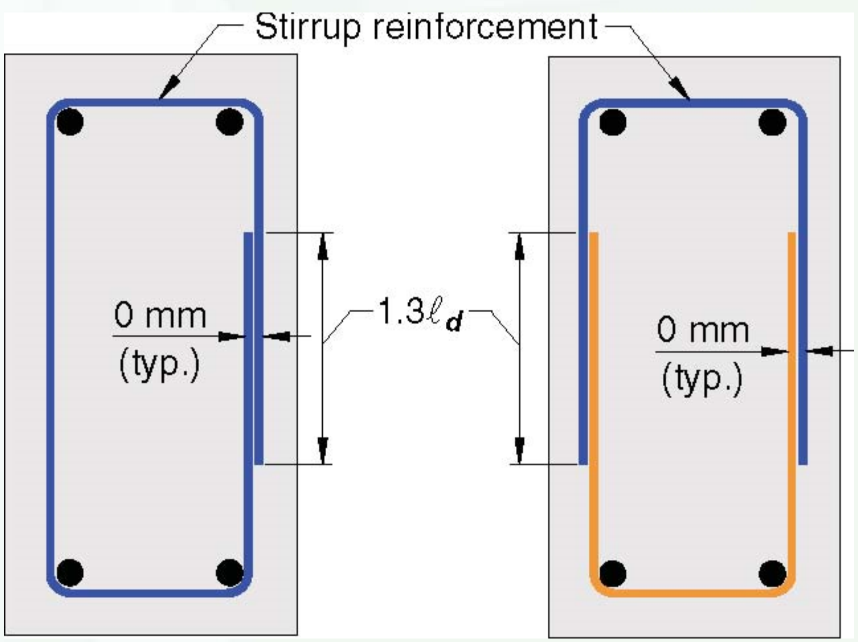


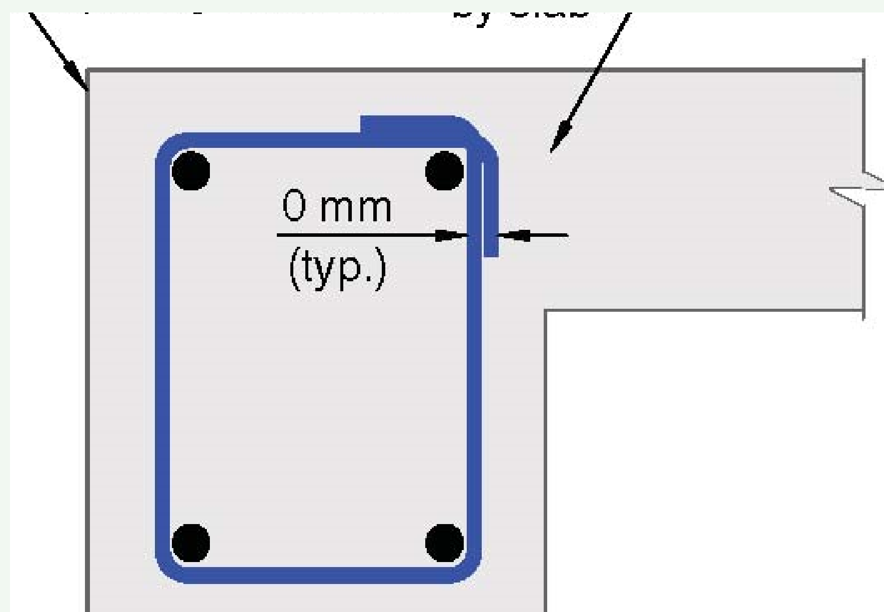


Types of stirrups.


≤ 2d
Refer to 25.3
Refer to 25.3




Stirrup reinforcement
0 mm
(typ.)
1.3ℓd
0 mm
(typ.)


0 mm
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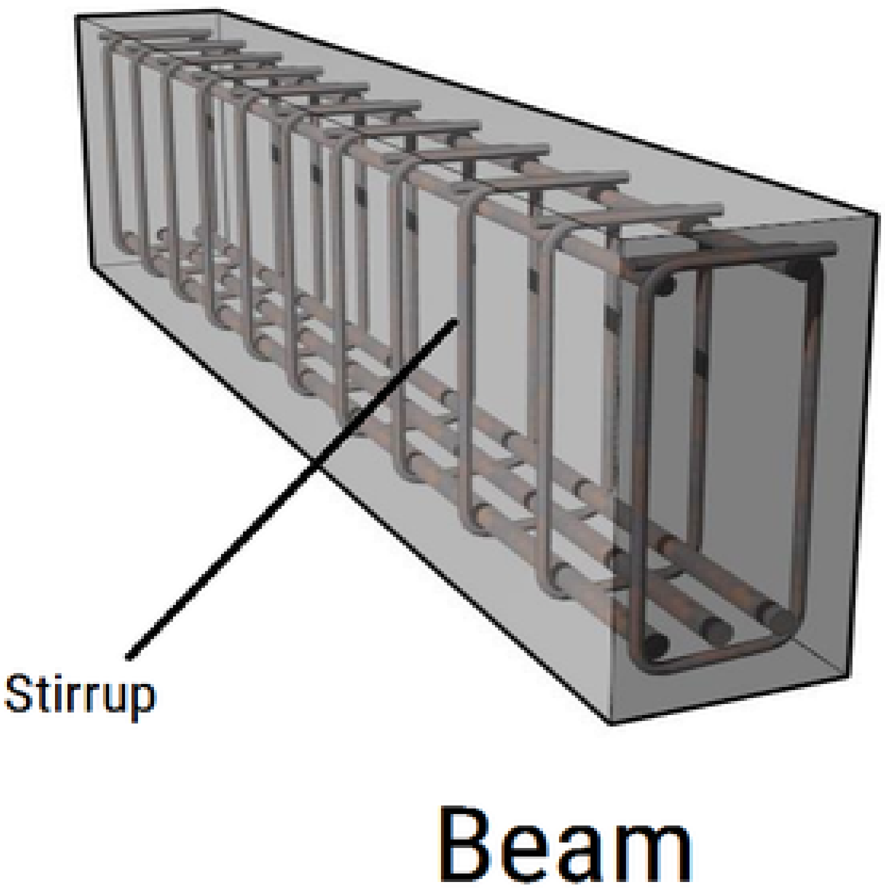
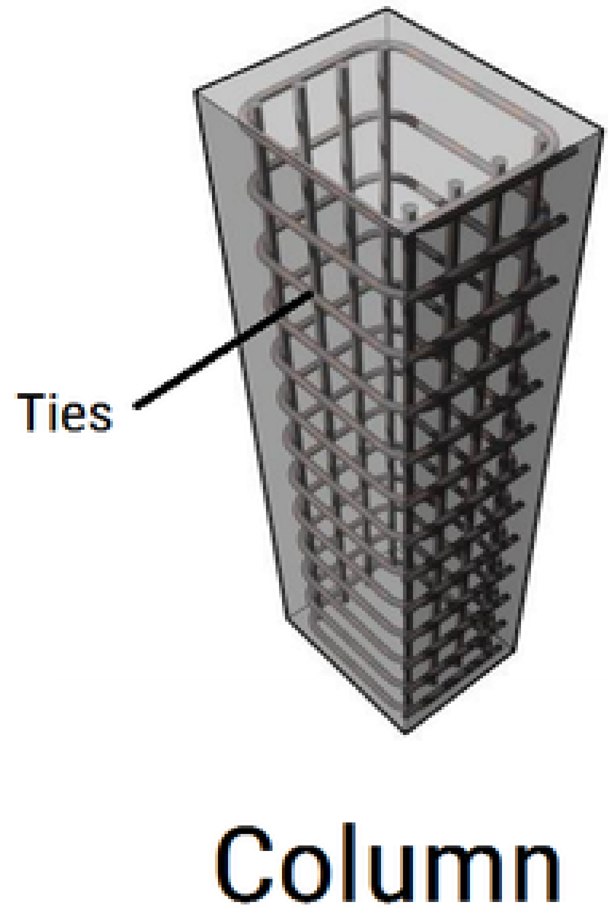
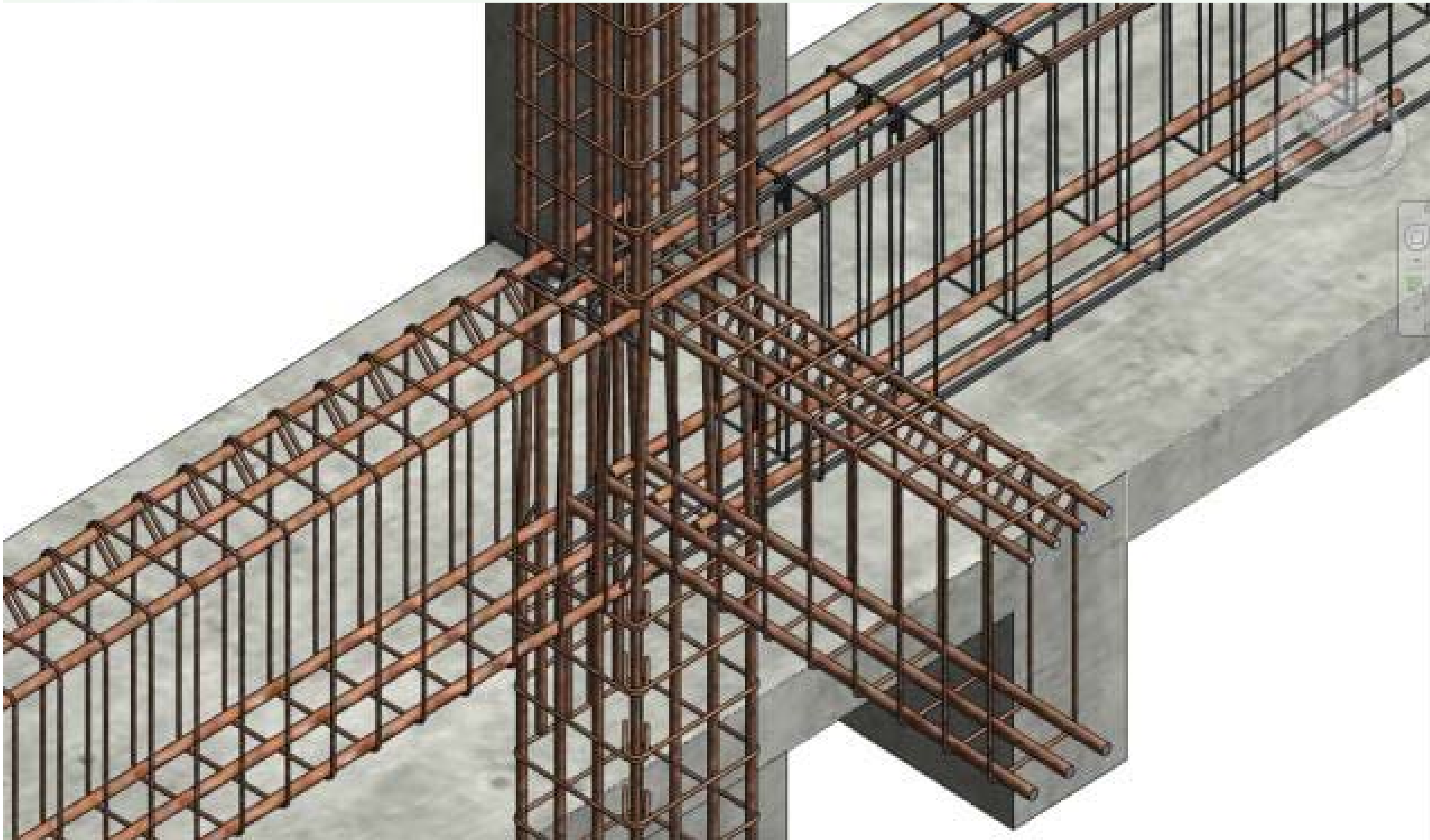
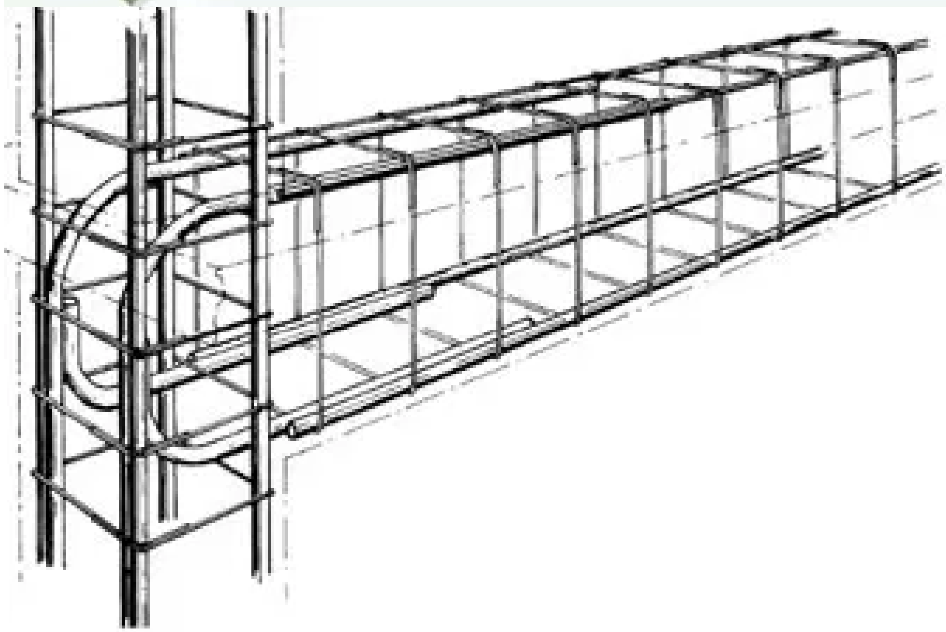
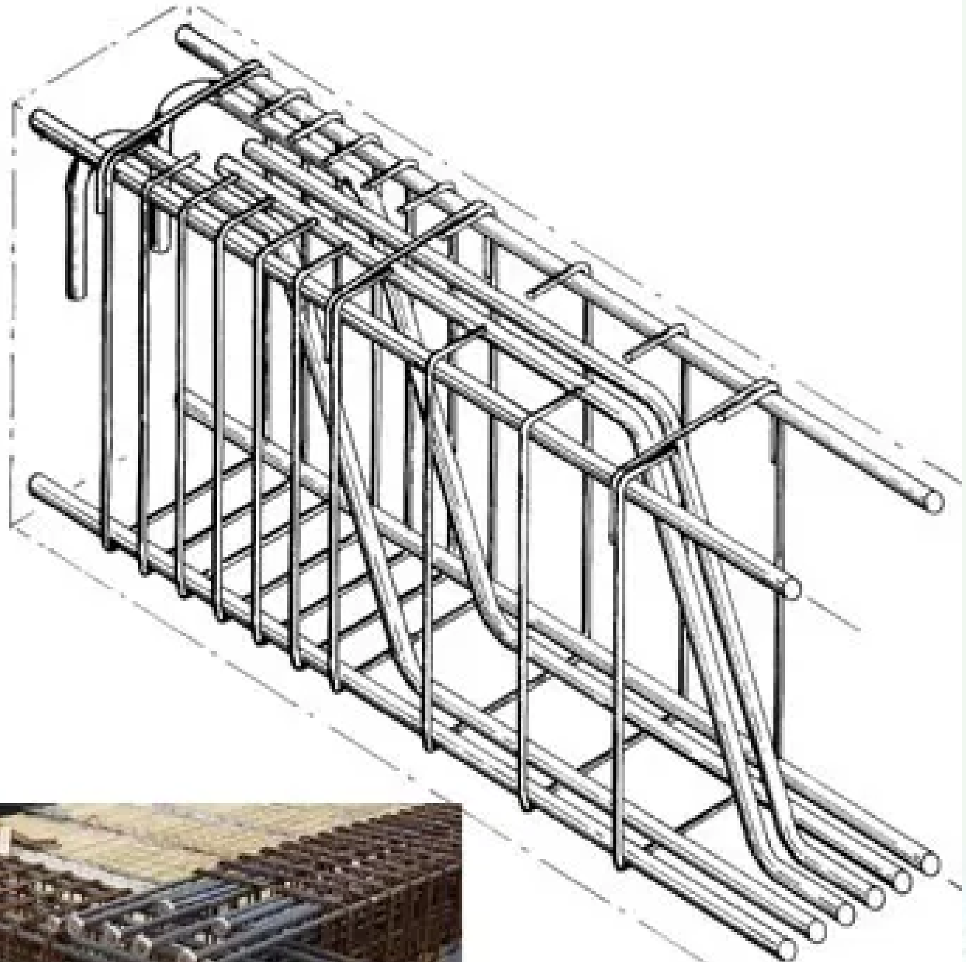
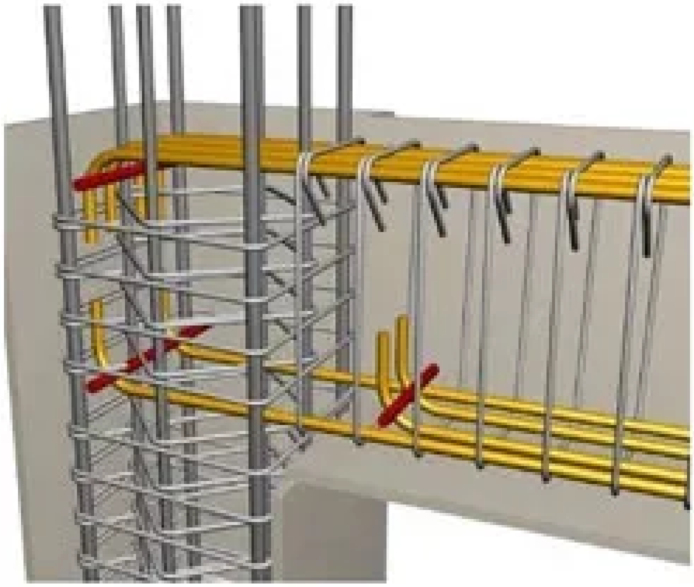

Stirrup
Beam


Ties
Column









7. DESIGN PROCEDURE ACCORDING ACI-2019

The design procedure for shear using vertical stirrups according to the ACI Code can be summarized as follows:

1. Calculate the factored shearing force, V_u, from the applied factored forces acting on the structural member. The critical design shear value is at a section located at a distance d from the face of the support.

Let $V_n = \frac{V_u}{\phi}$

2. Calculate V_c by:

$$for\ A_v \geq A_{v,min} \quad V_c = either\ of \begin{Bmatrix} 0.17\,\lambda\sqrt{f_c'}\,b_w\,d & Eq.a \\ 0.66\,\lambda\,(\rho_w)^{1/3}\sqrt{f_c'}\,b_w\,d & Eq.b \end{Bmatrix}$$

$$for\ A_v < A_{v,min} \quad V_c = 0.66\,\lambda_s\,\lambda\,(\rho_w)^{1/3}\sqrt{f_c'}\,b_w\,d \qquad Eq.c$$

And shall consider the following :

$$V_c \leq 0.42\,\lambda\sqrt{f_c'}\,b_w\,d.$$

$$\lambda s = \sqrt{\frac{2}{1 + 0.004\,d}} \leq 1$$

3. Calculate $0.083\,\lambda\sqrt{f_c'}\;b_w\,d = 0.5\,V_c \;\ldots\ldots\ldots, Eq.a$

4. ***a***. $If\ V_n < 0.5\,V_{c,Eq.a}$, $no\ shear\ reinforcement\ is\ needed$.

b. $If\ 0.5\,V_{c,Eq.a} < V_n \le V_c$ $minimum\ shear\ reinforcement\ is\ required$.

Can use no.3 (dia.10 mm) U-stirrups spaced at maximum spacing, as explained in step 8.

c. $If\ V_n > V_c$, $shear\ reinforcement\ must\ be\ provided\ according\ to\ steps$ 5 $through$ 8.

5. $If\ V_n > V_c$, calculate the shear to be carried by shear reinforcement:

$$V_n = V_c + V_S \quad or \quad V_S = V_n - V_c$$

6. Calculate:

$V_{C1} = 0.33\sqrt{f_c'}\, b_w\, d$ and $V_{C2} = 0.66\sqrt{f_c'}\, b_w\, d = 2\, V_{C1}$ $then$:

$If \quad V_S > V_{C2}$ increase the dimensions of the section.

$If \quad V_S < V_{C2}$ proceed in the design

7. Calculate the stirrups spacing based on

$$S_1 = \frac{A_v\, f_{yt}\, d}{V_s}$$

8. Determine the maximum spacing allowed by the ACI Code. The maximum spacing is the least of S_1, S_2 and S_3: where

$S_2 = \frac{d}{2} \leq 600\, mm$, if $V_S \leq V_{C1}$ or $S_2 = \frac{d}{4} \leq 300\, mm$, if $V_S > V_{C1}$

$$S_3 = smaller\ of \begin{Bmatrix} \dfrac{A_v\, f_{yt}}{0.062\sqrt{f_c'}\, b_w} \\ \dfrac{A_v\, f_{yt}}{0.35\, b_w} \end{Bmatrix}$$

$then, \quad S_{max} = Min\ (S_1, S_2\ and\ S_3)\ (Practical\ value)$.

9. The ACI Code did not specify a minimum spacing. Under normal conditions, a practical minimum S may be assumed to be equal to 75 mm. for d ≤ 500 mm. and 100 mm. for deeper beams. If S is considered small, either increase the stirrup bar number or use multiple-leg stirrups.

10. For circular sections, the area used to compute V_c is the diameter times the effective depth d, where d=0.8 times the diameter, ACI Code, Section 22.5.2.2.

Where:

V_c	Shear resistance of the concrete
λ_s	Factor for considering the component height
λ	Factor for normal or lightweight concrete
ρ_w	Longitudinal reinforcement ratio of the tension reinforcement
f'_c	Concrete compressive strength
N_u	Design axial force
A_g	cross-sectional area
b_w	Width of the cross-section
d	Effective depth

Reinforced Concrete Design

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LOGO

Example (1): A simply supported beam has a rectangular section with b=300mm., d=540 mm, and h=600 mm. and is reinforced with 4 φ 25 mm bars. Check if the section is adequate for each of the following factored shear forces. If it is not adequate, design the necessary shear reinforcement in the form of U-stirrups. Use $f_c' = 28$ MPa and f_{yt} =420 MPa. Assume normal-weight concrete. When :

(a) Vu = 52 kN, (b) Vu = 104 kN, (c) Vu = 243 kN, (d) Vu = 337 kN, (e) Vu = 560 kN

Solution

$Calculate\ V_c$:

$$V_c = 0.17\,\lambda\sqrt{f_c'}\;\; b_w\, d = 0.17 \times 1 \times \sqrt{28} \times 300 \times 540 = 145728\, N \approx 146\, kN$$

$Calculate\;\; 0.5\, V_c$

$$0.5\, V_c = \frac{146}{2} = 73\, KN$$

$$V_{C1} = 0.33\sqrt{f_c'}\, b_w\, d = 0.33 \times \sqrt{28} \times 300 \times 450 = 236\, kN$$ and

$$V_{C2} = 0.66\sqrt{f_c'}\, b_w\, d = 2\, V_{C1} = 472\, kN$$

(a) $V_u = 52\, kN$

$$V_n = \frac{V_u}{\emptyset} = \frac{52}{0.75} = 69.33\, kN$$

$\therefore\; V_n\ (69.3\, kN) < 0.5\, V_c\ (73\, kN)$

$\therefore\; no\ shear\ reinforcement\ is\ needed.$

(b) $V_u = 104\ kN$

$$V_n = \frac{V_u}{\emptyset} = \frac{104}{0.75} = 139\ kN$$

$\therefore\ 0.5\,V_c\ (73\ kN) < V_n\ (139\ kN) < V_c\ (146\ kN)$

$\therefore$ *minimum shear reinforcement is required.*

$$\therefore S_2 = \frac{d}{2} \leq 600\ mm$$

$$\therefore S_2 = \frac{540}{2} = 270\ mm \leq 600\ mm \quad \therefore S_2 = 270\ mm$$

Use ϕ 10 mm therefore $A_v = 2\ leg = 2\ \times (10^2 \times \frac{\pi}{4}) = 157\ mm^2$

$$S_3 = smaller\ of \begin{Bmatrix} \dfrac{A_v\, f_{yt}}{0.062\sqrt{f_c'}\, b_w} \\ \dfrac{A_v\, f_{yt}}{0.35\, b_w} \end{Bmatrix} = min \begin{Bmatrix} \dfrac{157 \times 420}{0.062\sqrt{28} \times 300} \\ \dfrac{157 \times 420}{0.35 \times 300} \end{Bmatrix}$$

$$S_3 = min \begin{Bmatrix} 670\ mm \\ 628\ mm \end{Bmatrix} \quad \therefore S_3 = 628\ mm$$

$\therefore S_{max} = \min(S_2\ and\ S_3) = 270\ mm$

$\therefore$ Use ϕ 10 mm @ 260 mm c/c , U-stirrups

$V_S = \boldsymbol{Vn} - \boldsymbol{Vc}$	$\boldsymbol{Vc} = 0.17\,\lambda\sqrt{f_c'}\ \ b_w\, d$
	$S_1 = \frac{A_v\, f_{yt} d}{V_s}$ (calculated)
$V_S < V_{C\mathbf{1}}$	$\therefore S_2 = \frac{d}{2}, \qquad or\ S_2 = 600\ mm$
$V_{C\mathbf{1}} < V_S < V_{C\mathbf{2}}$	$S_2 = \frac{d}{4}, \qquad or\ S_2 = 300\ mm$
$V_S > V_{C\mathbf{2}}$	Change Section Dimension
	$S_3 = smaller\ of \left\{ \begin{array}{c} \frac{A_v\, f_{yt}}{0.062\sqrt{f_c'}\, b_w} \\ \frac{A_v\, f_{yt}}{0.35\, b_w} \end{array} \right\}$ (calculated)

(c) $V_u = 243\ kN$

$$V_n = \frac{V_u}{\emptyset} = \frac{243}{0.75} = 324\ kN$$

$\therefore\ V_c\ (146\ kN) < V_n\ (324\ kN)$

$\therefore\ shear\ reinforcement\ must\ be\ provided\ and\ calculate\ V_s$

$V_S = V_n - V_c$

$V_S = 324 - 146 = 178\ kN$

$V_S\ (178\ kN) < Vc_1(236kN) < V_{C2}(472\ kN)$ $\quad \therefore\ the\ dimensions\ of\ the\ sec.is\ OK$

Calculate the stirrups spacing, Use ϕ 10 mm therefore $A_v = 157\ mm^2$

$$S_1 = \frac{A_v\, f_{yt} d}{V_s} = \frac{157 \times 420 \times 540}{178 \times 10^3} = 200\ mm$$

$For\ V_S\ (178\ kN) < V_{C1}(236\ kN)$

$$\therefore S_2 = \frac{d}{2} \le 600\ mm$$

$$\therefore S_2\ \frac{540}{2} = 270\ mm \le 600\ mm \quad \therefore S_2 = 270\ mm \quad and$$

$$S_3 = smaller\ of \begin{Bmatrix} \dfrac{A_v\, f_{yt}}{0.062\sqrt{f_c'}\ b_w} \\ \dfrac{A_v\, f_{yt}}{0.35\ b_w} \end{Bmatrix} = min \begin{Bmatrix} \dfrac{157 \times 420}{0.062\sqrt{28} \times 300} \\ \dfrac{157 \times 420}{0.35 \times 300} \end{Bmatrix}$$

$$S_3 = min \begin{Bmatrix} 670\ mm \\ 628\ mm \end{Bmatrix} \quad \therefore S_3 = 628\ mm$$

$$S_{max} = \min(S_1, S_2\ and\ S_3) = 200\ mm$$

∴ Use ϕ 10 mm @ 200 mm c/c , U-stirrups

(d) $V_u = 337\ kN$

$$V_n = \frac{V_u}{\emptyset} = \frac{337}{0.75} = 449\ kN$$

$\therefore\ V_c\ (146\ kN) < V_n\ (449\ kN)$

$\therefore\ shear\ reinforcement\ must\ be\ provided\ and\ calculate\ V_s$

$V_S = V_n - V_c$

$V_S = 449 - 146 = 303\ kN$

$V_S\ (303\ kN) < V_{C2}(472\ kN) \qquad \therefore\ the\ dimensions\ of\ the\ sec. is\ OK$

Calculate the stirrups spacing, Use ϕ 10 mm therefore $A_v = 157\ mm^2$

$$S_1 = \frac{A_v\ f_{yt} d}{V_s} = \frac{157 \times\ 420 \times 540}{303 \times 10^3} = 117\ mm$$

$For\ V_S\ (303\ \ kN) >\ V_{C1}(236\ kN)$

$$\therefore S_{\ 2} = \frac{d}{4} \leq 300\ mm$$

$$\therefore S_{\ 2} = \frac{540}{4} = 135\ mm \leq 300\ mm \quad \therefore S_{\ 2} = \ 135\ mm$$

$$and \qquad S_3 = smaller\ of \begin{Bmatrix} \dfrac{A_v\, f_{yt}}{0.062\sqrt{f_c'}\, b_w} \\ \dfrac{A_v\, f_{yt}}{0.35\, b_w} \end{Bmatrix} = min \begin{Bmatrix} \dfrac{157 \times 420}{0.062\sqrt{28} \times 300} \\ \dfrac{157 \times 420}{0.35 \times 300} \end{Bmatrix}$$

$$S_3 = min \begin{Bmatrix} 670\ mm \\ 628\ mm \end{Bmatrix} \qquad \therefore S_3 = 628\ mm$$

$S_{max} = \min(S_1, S_2\ and\ S_3) = 117\ mm$

∴ Use ϕ 10 mm @ 110 mm c/c , U-stirrups

(e) $V_u = 560\ kN$

$$V_n = \frac{V_u}{\emptyset} = \frac{560}{0.75} = 747\ kN$$

$\therefore\ V_c\ (146\ kN) < V_n\ (747\ kN)$

$\therefore\ shear\ reinforcement\ must\ be\ provided\ and\ calculate\ V_s$

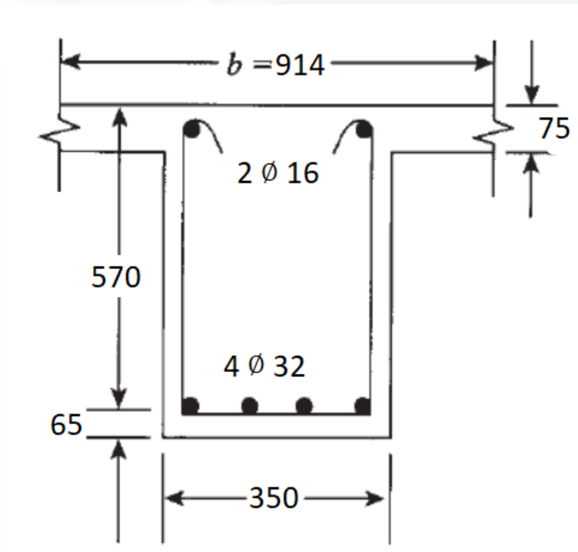
$V_S = V_n - V_c$

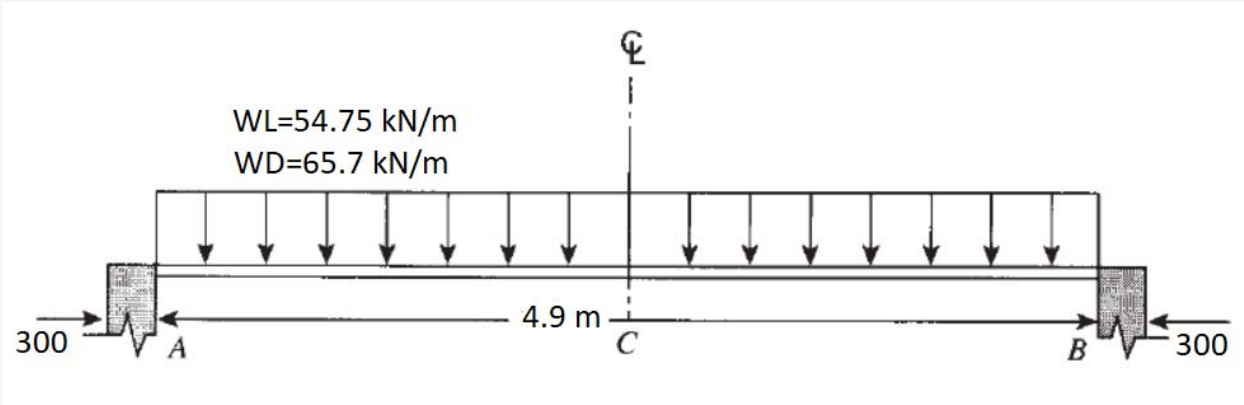
$V_S = 747 - 146 = 601\ kN$

$V_S\ (601\ kN) > V_{C2}(472\ kN) \qquad \therefore Not\ OK\ and\ change\ the\ dimensions\ of\ the\ section.$

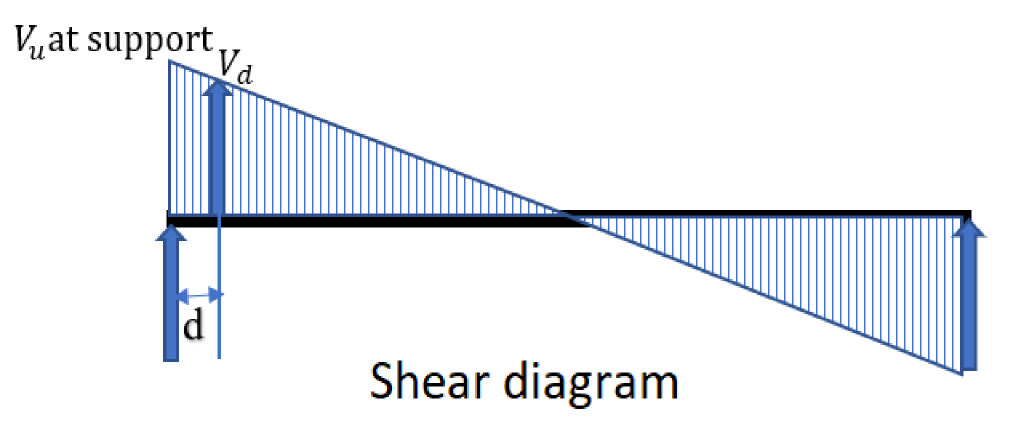
Example (2)

A 5.2 m, span simply supported beam has a clear span of 4.9 m and carries uniformly distributed dead and live loads of 65.7 kN/m and 54.75 kN/m, respectively. The dimensions of the beam section and steel reinforcement are shown in Fig. below. Check the section for shear and design the necessary shear reinforcement. Given $f_c' = 21\ MPa$ normal-weight concrete and $f_{yt} = 420\ MPa$.





Solution



Calculate W_u

$W_u = 1.2\, WD + 1.6\, WL$

$W_u = 1.2\,(65.7) + 1.6\,(54.75) = 166.44\; kN/m$

Calculate V_u (at face of support)

$$V_{u,f} = \frac{W_u\, l}{2} = \frac{166.44\ \times 4.9}{2} = 407.8\; kN$$

Design V_u (at distance d from the face of the support) = $V_{u,d} = V_{u,f} - W_u\, d$

$V_{u,d} = V_{u,f} - W_u\, d = 407.8 - 166.44 \times 0.57 = 313\; kN$

$\therefore V_u = 313\; kN$

$$V_n = \frac{V_u}{\emptyset} = \frac{313}{0.75} = 417\; kN$$

Calculate V_c

$V_c = 0.17\, \lambda \sqrt{f_c'}\ b_w\, d = 0.17 \times 1 \times \sqrt{21} \times 350 \times 570$

$V_c = 155\; kN$

Calculate $0.5\, V_c$

$$0.5\, V_c = 0.5Vc == \frac{155}{2} = 77.5\; kN$$

$V_{C1} = 0.33\sqrt{f_c'}\, b_w\, d = 0.33 \times \sqrt{21} \times 350 \times 570 = 302\, kN$

and $V_{C2} = 0.66\sqrt{f_c'}\, b_w\, d = 2\, V_{C1} = 604\, kN$

$\therefore\ V_c\ (155\, kN) < V_n\ (417\, kN)$

$\therefore\ shear\ reinforcement\ must\ be\ provided\ and\ calculate\ V_S$

$V_S = V_n - V_c$

$V_S = 417 - 155 = 262\, kN$

$V_S\ (262\, kN) < V_{C2}(604\, kN)$ $\therefore\ the\ dimensions\ of\ the\ sec.\ is\ OK$

Calculate the stirrups spacing, Use ϕ 10 mm, therefore $A_v = 157\, mm^2$

$$S_1 = \frac{A_v\, f_{yt} d}{V_s} = \frac{157 \times 420 \times 570}{262 \times 10^3} = 143\, mm$$

$For\ V_S\ (262\ kN) < V_{C1}(302\, kN)$

$$\therefore S_2 = \frac{d}{2} \leq 600\, mm$$

$$\therefore S_2 = \frac{570}{2} = 285\, mm \leq 600\, mm \quad \therefore S_2 = 285\, mm$$

$$S_3 = smaller\ of \begin{Bmatrix} \dfrac{A_v\, f_{yt}}{0.062\sqrt{f_c'}\, b_w} \\ \dfrac{A_v\, f_{yt}}{0.35\, b_w} \end{Bmatrix} = min \begin{Bmatrix} \dfrac{157 \times\ 420}{0.062\sqrt{21}\ \times 350} \\ \dfrac{157 \times\ 420}{0.35\ \times 350} \end{Bmatrix}$$

$$S_3 =\ min \begin{Bmatrix} 663\ mm \\ 538\ mm \end{Bmatrix} \quad \therefore S_3 =\ 538\ mm$$

$$S_{max} = \min(S_1\ , S_2\ and\ S_3) = 143\ mm \approx 130\ mm$$

∴ Use ϕ 10 mm @ 130 mm c/c

From shear diagram, the shear force on beam not constant and decrease to zero in center of beam, therefore using the spacing (S= 130 mm) for all beam is not economic, because this value (S= 130 mm) determined according to maximum shear force at distance d from support. So, for such cases when shear force not constant, the beam can divide to 3 or 2 zones according to the following.

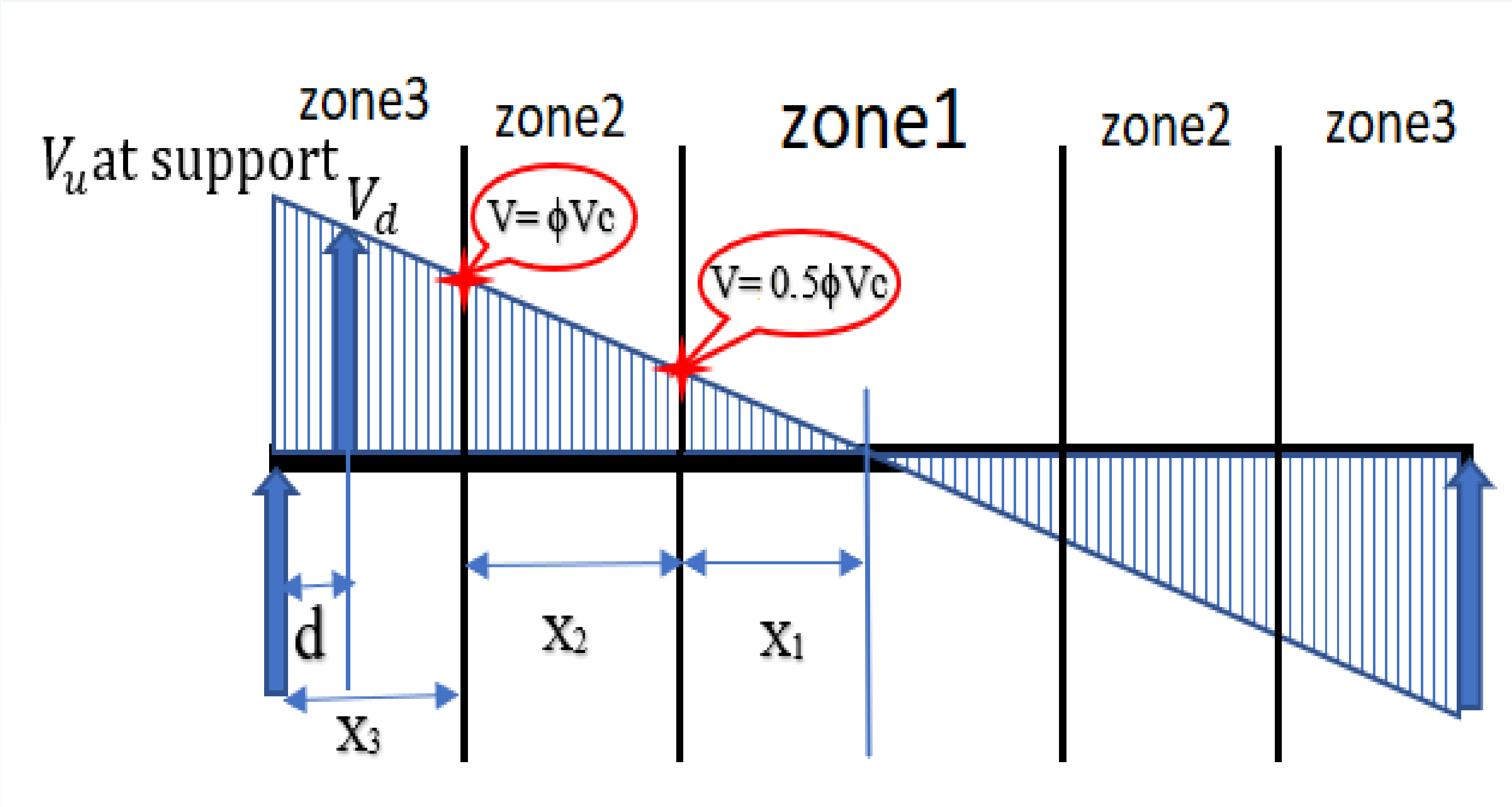
Zone 1: $V_n\ <\ 0.5\, V_{c,Eq.a}$, *no shear reinforcement is needed.*

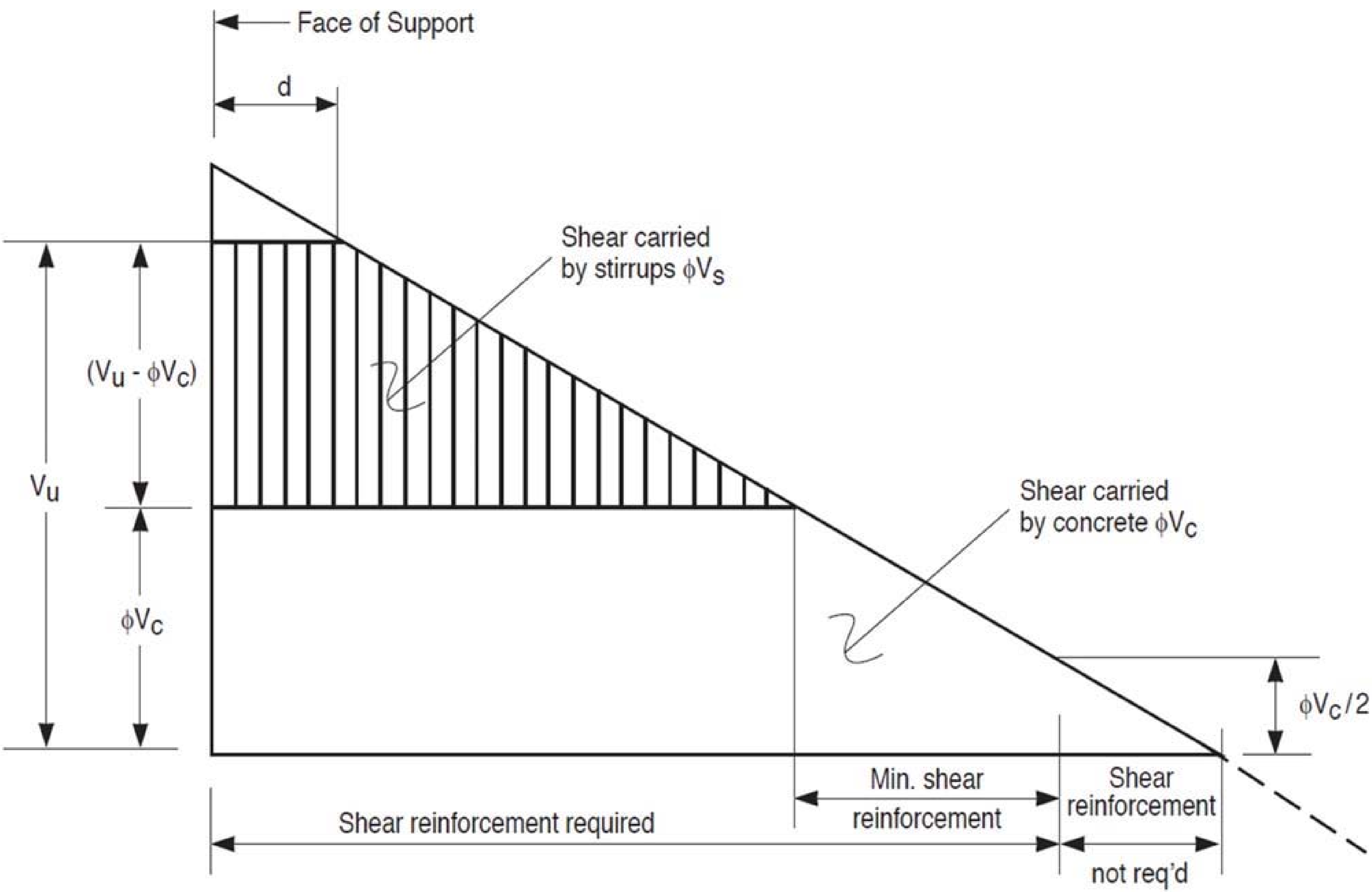
*Zone*2: $0.5\, V_{c,Eq.a}\ <\ V_n\ \le V_c$ *minimum shear reinforcement is required.*

*Zone*3: $V_n\ > V_c$ *shear reinforcement is required.*

Zone 1 and zone 2 can be consider as one zone with minimum shear reinforcement.

It is easy to locate these zones as shown below, for zone1, by determine the location of V = 0.5ϕVc (x_1) and for zone2, by determine the location of V= (x_2).


zone3
zone2
zone1
zone2
zone3
V_u at support
V_d
$V = \phi Vc$
$V = 0.5\phi Vc$
d
X_3
X_2
X_1


Face of Support
d
Shear carried
by stirrups ϕVs
(Vu - ϕVc)
Vu
ϕVc
Shear carried
by concrete ϕVc
ϕVc/2
Min. shear
reinforcement
Shear
reinforcement
not req'd
Shear reinforcement required

For zone1, $V = 0.5\phi Vc = \phi \times 77.5 = 58.13\ kN$, from similarity of triangles

$$\frac{V_{u,f}}{l/2} = \frac{0.5\phi V_c}{x_1}$$

$$x_1 = \frac{0.5\phi V_c\ l}{2\ V_{n,f}} = \frac{0.75 \times\ 77.5\ \times 4.9}{2 \times 407.8} = 0.35\ mm$$

For this distance of x_1 from center, no shear reinforcement is needed.

For zone2, $V = \phi Vc = 0.75 \times 155 = 116.25\ kN$, from similarity of triangles

$$\frac{V_{u,f}}{l/2} = \frac{\phi V_c}{x_1 + x_2}$$

$$x_1 + x_2 = \frac{\phi V_c\ l}{2\ V_{u,f}} = \frac{0.75 \times 155\ \times 4.9}{2 \times 407.8} = 0.7\ mm$$

$$x_2 = 0.7 - 0.35 = 0.35\ mm$$

$$For\ this\ distance\ of\ x_2, minimum\ shear\ reinforcement\ is\ required$$

$$S_{max} = \min\ (S_2\ and\ S_3) = 285\ mm \approx 275\ mm$$

∴ Use ϕ 10 mm @ 275 mm c/c

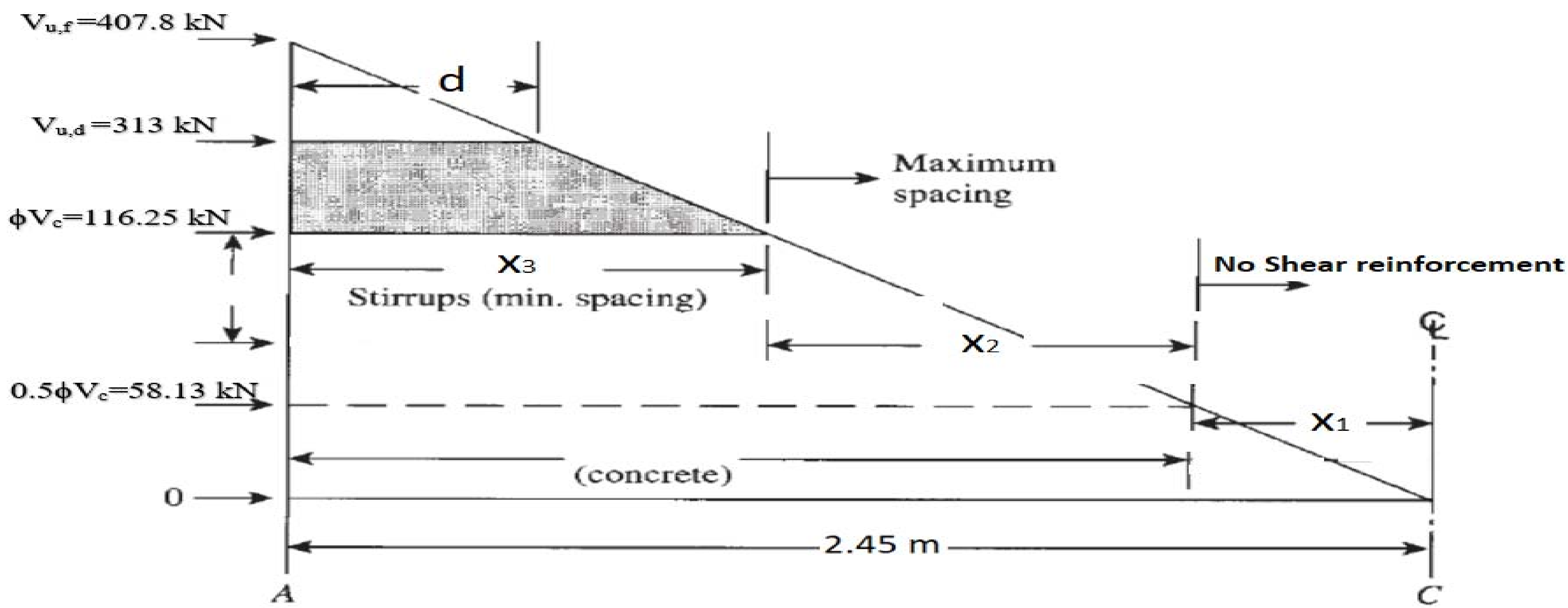
Actually, we can use min. shear reinforcement for $x_1 + x_2$.

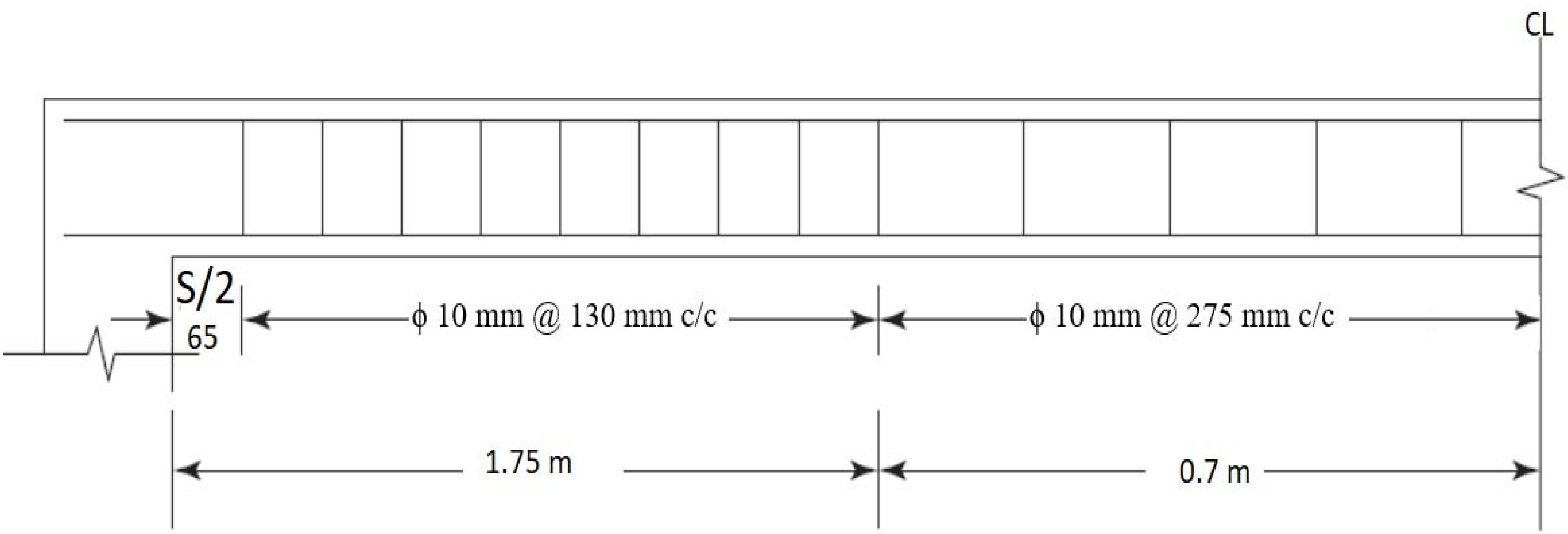
For zone 3,

$$x_3 = \frac{l}{2} - (x_1 + x_2) = \frac{4.9}{2} - 0.7 = 1.75\ mm$$

$$S_{max} = \min(S_1\ , S_2\ and\ S_3) = 143\ mm \approx 130\ mm$$

∴ Use ϕ 10 mm @ 130 mm c/c





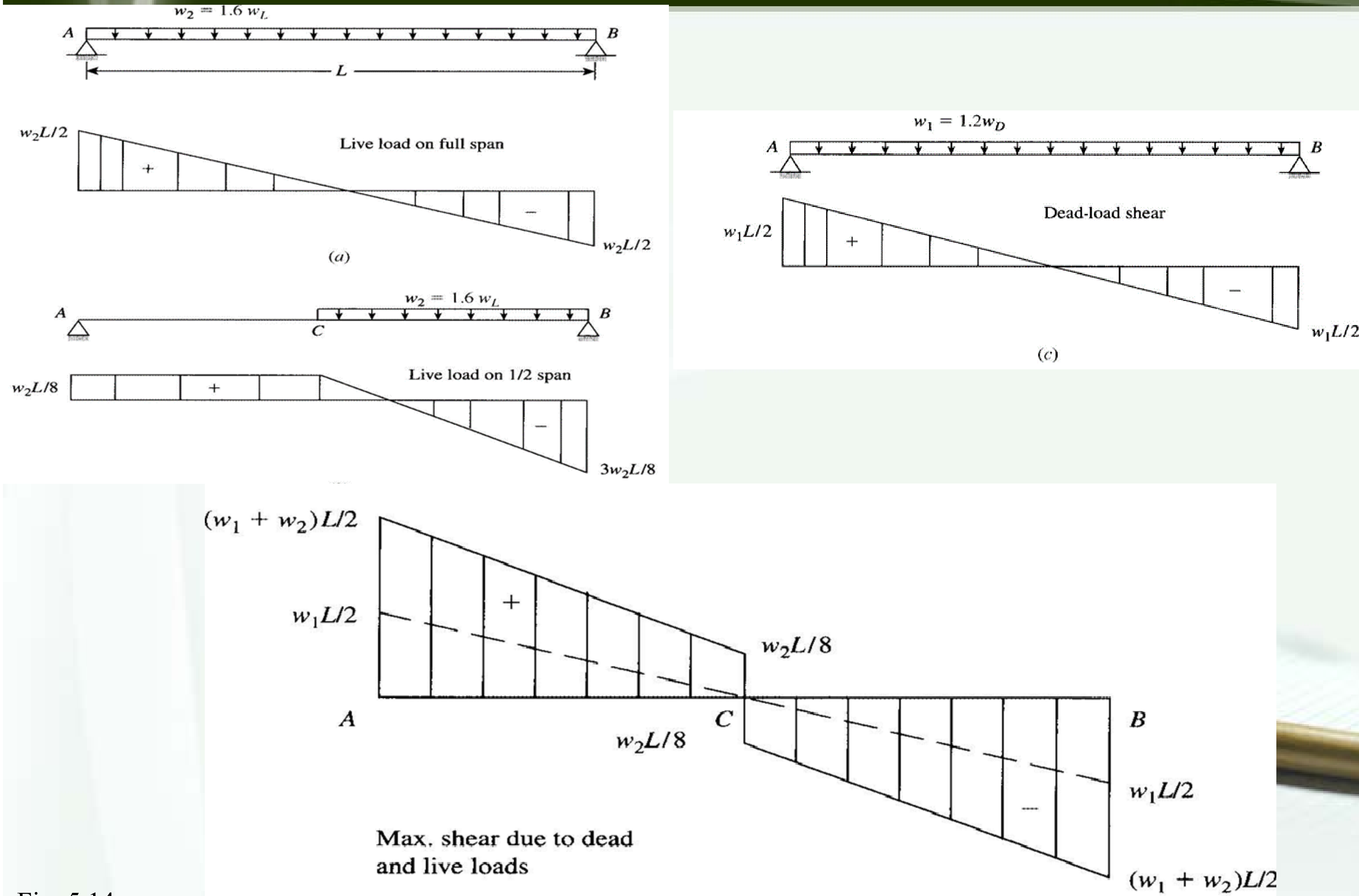
Distribution of stirrups.

SHEAR FORCE DUE TO LIVE LOADS

In example 2, it was assumed that the dead and live loads are uniformly distributed along the full span, producing zero shear at midspan. Actually, the dead load does exist along the full span, but the live load may be applied to the full span or part of the span, as needed to develop the maximum shear at midspan or at any specific section. Figure 5.15a shows a simply supported beam with a uniform load acting on the full span. The shear force varies linearly along the beam, with maximum shear acting at support A.

In the case of live load, $W_2 = 1.6W\ L$, the maximum shear force acts at support A when W_2 is applied on the full span, Fig. 5.14a. The maximum shear at midspan develops if the live load is placed on half the beam, BC (Fig. 5.14b), producing Vu at midspan equal to $W_2L/8$. Consequently, the design shear force is produced by adding the maximum shear force due to the live load (placed at different lengths of the span) to the dead-load shear force (Fig. 5.14c) to give the shear distribution shown in Fig. 5.14d. It is common practice to consider the maximum shear at support A to be $WuL/2 = (1.2WD + 1.6WL)L/2$, whereas Vu at midspan is $W_2L/8 = (1.6\ W\ L)L/8$ with a straight-line variation along AC and CB, as shown in Fig. 5.14d. The design for shear in this case will follow the same procedure explained in Example 2. If the approach is applied to the beam in Example 2, then

Vu (at A) = 407.8 kN and Vu (at midspan) = $(1.6 \times 54.75)(4.9/8) = 53.66$ kN.

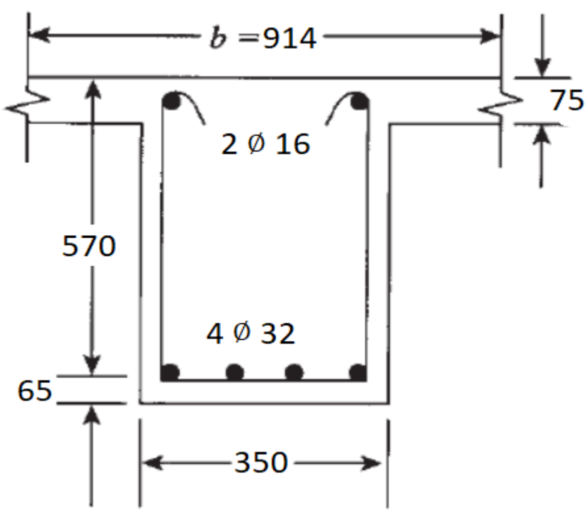


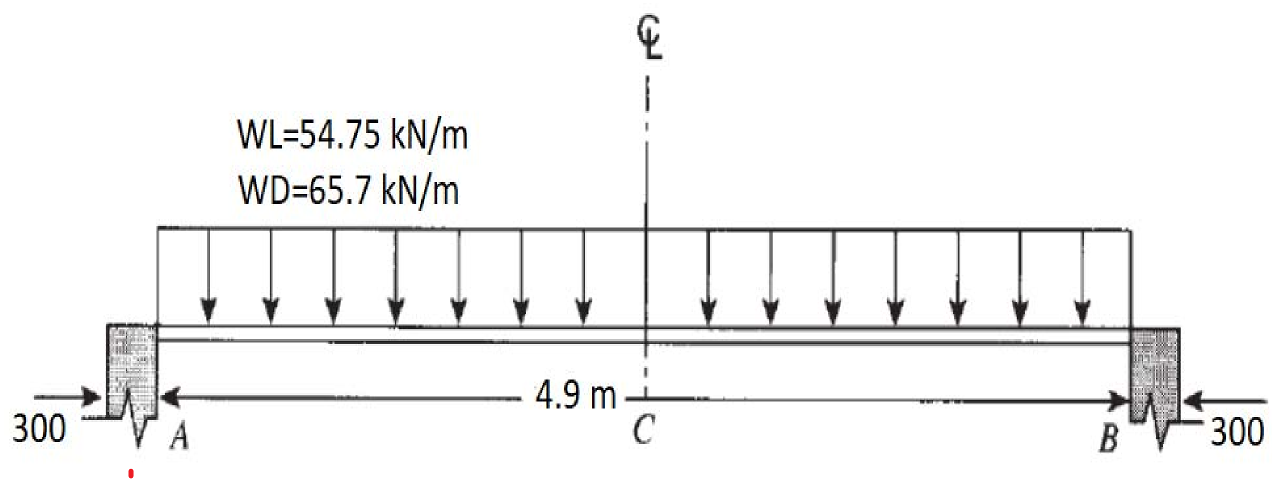
w2 = 1.6 wL
A
B
L
w2L/2
Live load on full span
+
−
w2L/2
(a)
w2 = 1.6 wL
A
C
B
w2L/8
Live load on 1/2 span
+
−
3w2L/8
w1 = 1.2wD
A
B
Dead-load shear
w1L/2
+
−
w1L/2
(c)
(w1 + w2)L/2
w1L/2
+
w2L/8
A
C
B
w2L/8
w1L/2
−
(w1 + w2)L/2
Max. shear due to dead
and live loads

Fig. 5.14

Example 3

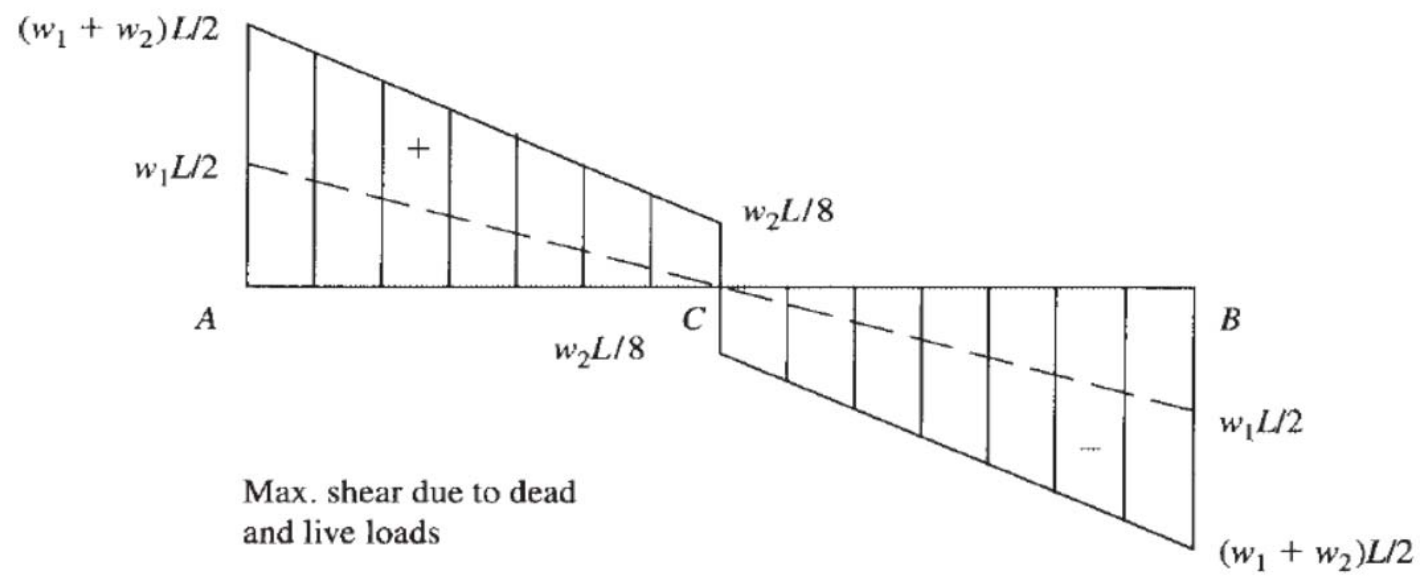
A 5.2 m, span simply supported beam has a clear span of 4.9 m and carries uniformly distributed dead and live loads of 65.7 kN/m and 54.75 kN/m, respectively. The dimensions of the beam section and steel reinforcement are shown in Fig. below. Check the section for shear and design the necessary shear reinforcement by taking the effect of placing of live load to produce maximum shear at mid-span. Given *fc'=21 MPa* normal-weight concrete and *fyt=420 MPa*.





Solution

As shown above in figure the maximum shear force will be



(d)

$W_1 = 1.2\ WD = 1.2(65.7) = 78.84\ kN/m$
$W_2 = 1.6\ WL = 1.6\ (54.75) = 87.6\ kN/m$

Calculate V_u (at face of support)

$$V_{u,f} = \frac{(W_1+W_2)\ l}{2} = \frac{(78.84 + 87.6)\ \times 4.9}{2} = 407.8\ kN$$

$$V_{u,m}\ (at\ midspan)\ = \frac{W_2 l}{8}\ (87.6)\ (4.9\ /\ 8)\ =\ 53.66\ kN$$

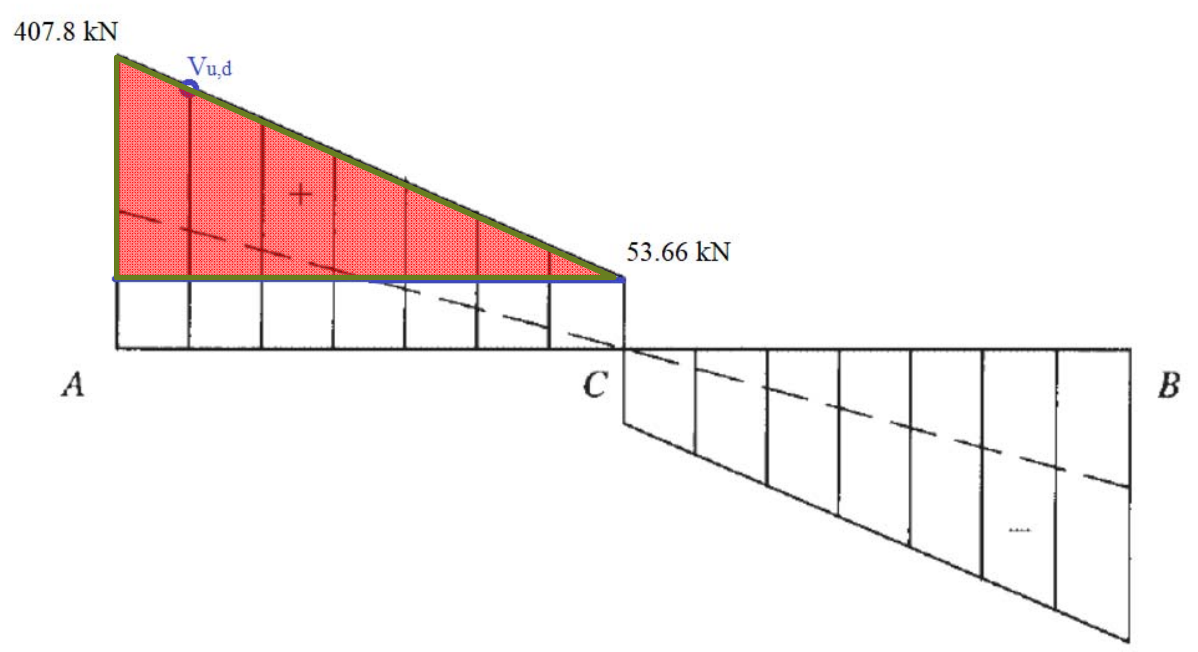
Calculate $V_{u,d}$ (at distance d from the face of the support) from similarity of triangles

$$\frac{V_{u,d} - 53.66}{\frac{l}{2} - d} = \frac{V_{u,f} - 53.66}{l/2}$$

$$\frac{V_{u,d} - 53.66}{\frac{4.9}{2} - 0.57} = \frac{407.8 - 53.66}{4.9/2}$$

$$V_{u,d} =\ 325.4\ kN$$

$$\therefore V_{n,d} = \frac{V_{u,d}}{\emptyset} = \frac{325.4}{0.75} =\ 434\ kN$$



$Calculate\ V_c$

$V_c = 0.17\,\lambda\sqrt{f_c'}\ b_w\, d = 0.17 \times 1 \times \sqrt{21} \times 350 \times 570$

$V_c = 155\ kN$

$Calculate\ \ 0.5\, V_c$

$0.5\, V_c = \frac{155}{2} = 77.5\ kN$

$V_{C1} = 0.33\sqrt{f_c'}\, b_w\, d = 0.33 \times \sqrt{21} \times 350 \times 570 = 302\ kN$

and $\quad V_{C2} = 0.66\sqrt{f_c'}\, b_w\, d = 2\, V_{C1} = 604\ kN$

$\therefore\ V_c\ (155\ kN) < V_n\ (434\ kN)\ \therefore\ shear\ reinforcement\ must\ be\ provided\ and\ calculate\ V_S$

$V_S = V_{n,d} - V_c$

$V_S = 434 - 155 = 279\ kN$

$V_S\ (279\ kN) < V_{C2}(604\ kN)$

$\therefore the\ dimensions\ of\ the\ sec.is\ OK$

Calculate the stirrups spacing, Use ϕ 10 mm, therefore $A_v = 157\ mm^2$

$$S_1 = \frac{A_v\, f_{yt} d}{V_s} = \frac{157\times\ 420\times 570}{279\times 10^3} = 134\ mm$$

$For\ V_S\ (279\ \ kN) <\ V_{C1}(302\ kN)$

$$\therefore S\ _2 = \frac{d}{2} \leq 600\ mm$$

$\therefore S\ _2 = \frac{570}{2} = 285\ mm \leq 600\ mm$ $\therefore S\ _2 = 285\ mm$ and :

$$S_3 = smaller\ of \left\{ \begin{matrix} \dfrac{A_v\ f_{yt}}{0.062\sqrt{f_c'}\ b_w} \\ \dfrac{A_v\ f_{yt}}{0.35\ b_w} \end{matrix} \right\} = min \left\{ \begin{matrix} \dfrac{157\times\ 420}{0.062\sqrt{21}\ \times 350} \\ \dfrac{157\times\ 420}{0.35\ \times 350} \end{matrix} \right\}$$

$S_3 =\ min \left\{ \begin{matrix} 663\ mm \\ 538\ mm \end{matrix} \right\}$ $\therefore S_3 =\ 538\ mm$

$S_{max} = \min(S_1\ , S_2\ and\ S_3) = 134\ mm$

∴ Use ϕ 10 mm @ 130 mm c/c

From shear diagram, the shear force on beam not constant and decrease to 53.66 kN in center of beam, therefore using the spacing (S= 130 mm) for all beam is not economic, because this value (S= 130 mm) determined according to maximum shear force at distance d from support. So, for such cases when shear force not constant, the beam can divide to 3 or 2 zones according to the following.

$Zone\ 1\text{:}\ V_n < 0.5\, V_{c,Eq.a},$ $\quad no\ shear\ reinforcement\ is\ needed.$

$Zone2\text{:}\ 0.5\, V_{c,Eq.a} < V_n \le V_c$ $\quad minimum\ shear\ reinforcement\ is\ required.$

$Zone3\text{:}\ V_n > V_c$ $\quad shear\ reinforcement\ is\ required.$

Zone 1 and zone 2 can be consider as one zone with minimum shear reinforcement.
It is easy to locate these zones as shown below, by determine the location of V= ϕVc (distance x1)

For zones 1 and 2, V=ϕVc = 0.75* 155 =116.25 kN, from similarity of triangles

$$\frac{V_{u,f} - 53.66}{l/2} = \frac{\phi V_c - 53.66}{x_1}$$

$$\frac{407.8 - 53.66}{4.9/2} = \frac{116.25 - 53.66}{x_1}$$

$$x_1 = 0.43\ mm$$

$For\ this\ distance\ of\ x_1, minimum\ shear\ reinforcement\ is\ required$

$S_{max} = \min(S_2\ and\ S_3) = 285\ mm \approx 280\ mm$

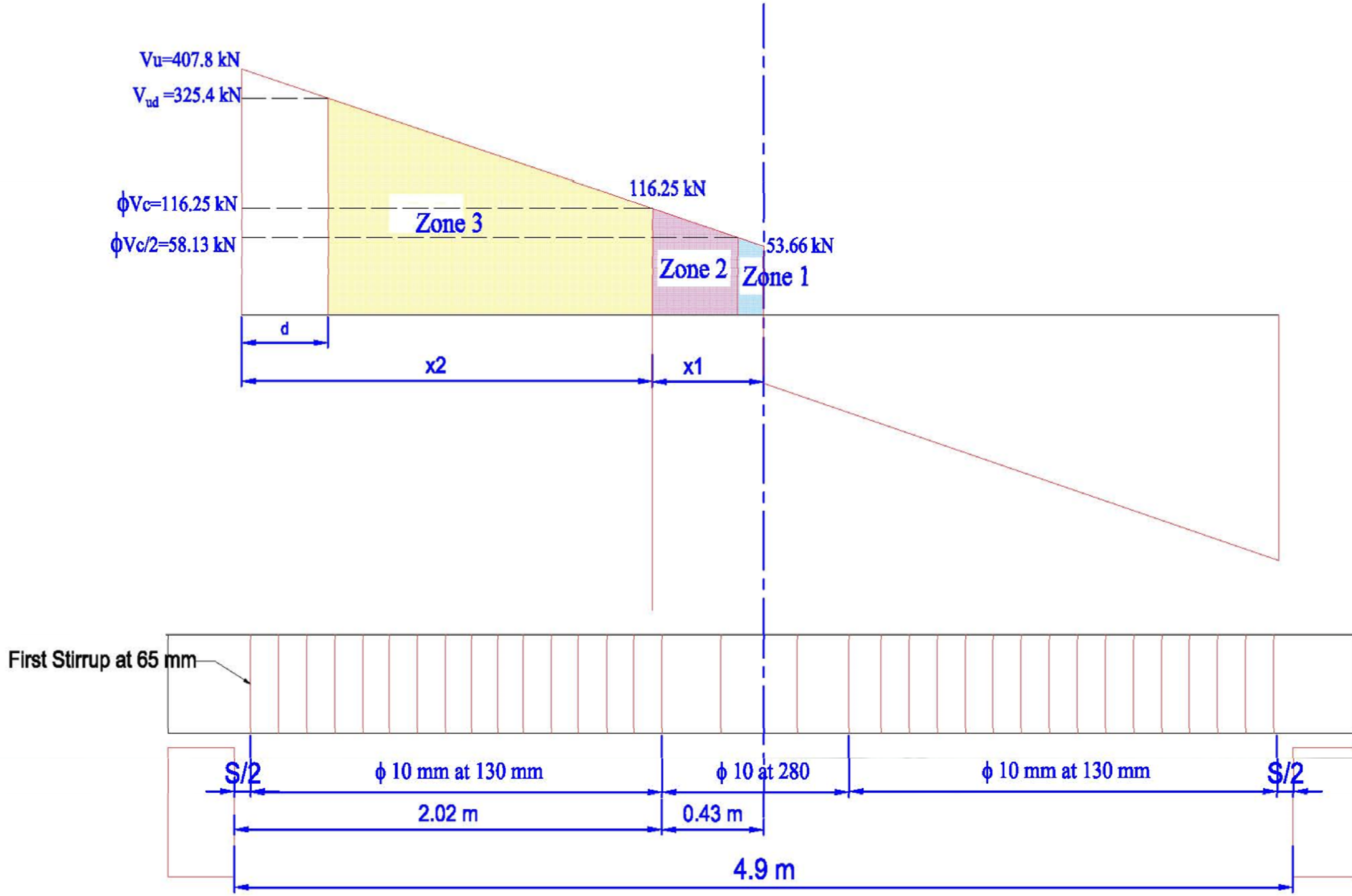
∴ Use ϕ 10 mm @ 280 mm c/c

For zone3,

$$x_2 = \frac{l}{2} - x_1 = \frac{4.9}{2} - 0.43 = 2.02\ m$$

$S_{max} = \min(S_1\ , S_2\ and\ S_3) = 134\ mm \approx 130\ mm$

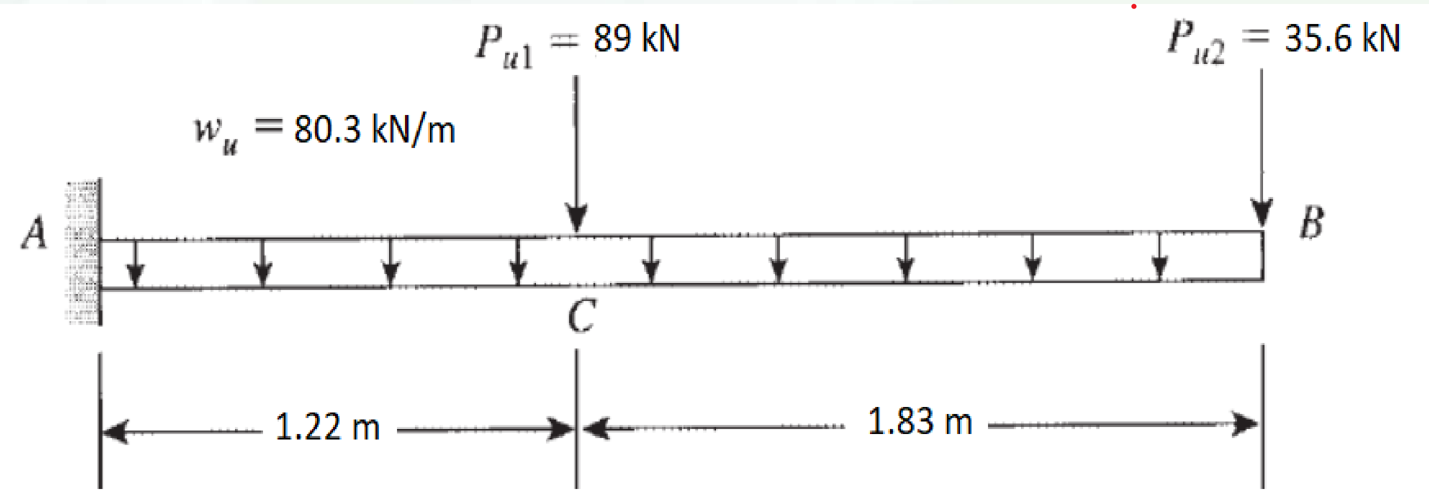
∴ Use ϕ 10 mm @ 130 mm c/c

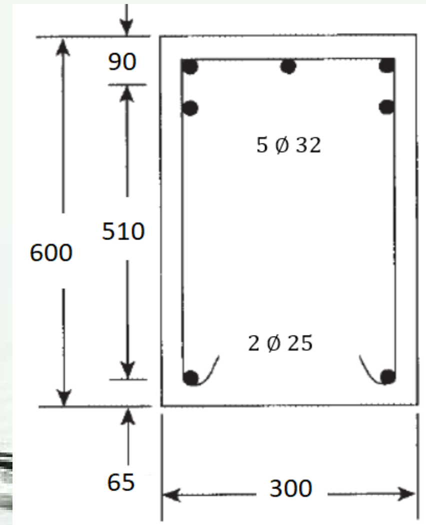


Vu=407.8 kN
V_{ud} =325.4 kN
ϕVc=116.25 kN
ϕVc/2=58.13 kN
116.25 kN
53.66 kN
Zone 3
Zone 2
Zone 1
d
x2
x1
First Stirrup at 65 mm
S/2
ϕ 10 mm at 130 mm
ϕ 10 at 280
ϕ 10 mm at 130 mm
S/2
2.02 m
0.43 m
4.9 m

Example 4

A 3.05 m -span cantilever beam has a rectangular section and carries uniform and concentrated factored loads (self-weight is included), as shown in Fig. below. Using $f_c' = 28\ MPa$, normal-weight concrete and $f_{yt} = 420\ MPa$, design the shear reinforcement required for the entire length of the beam according to the ACI Code.





Solution

Calculate the shear force along the beam due to external loads:

$V_{u,f}(at\ support) = 80.3(3.05) + 89 + 35.6$

$= 369.52\ kN$

$V_{u,d}(at\ d\ distance) = 369.52 - 80.3 \times 0.51$

$= 328.56\ kN$

$V_{u,1.22L}(at\ 1.22\ left) = 369.52 - 80.3 \times 1.22$

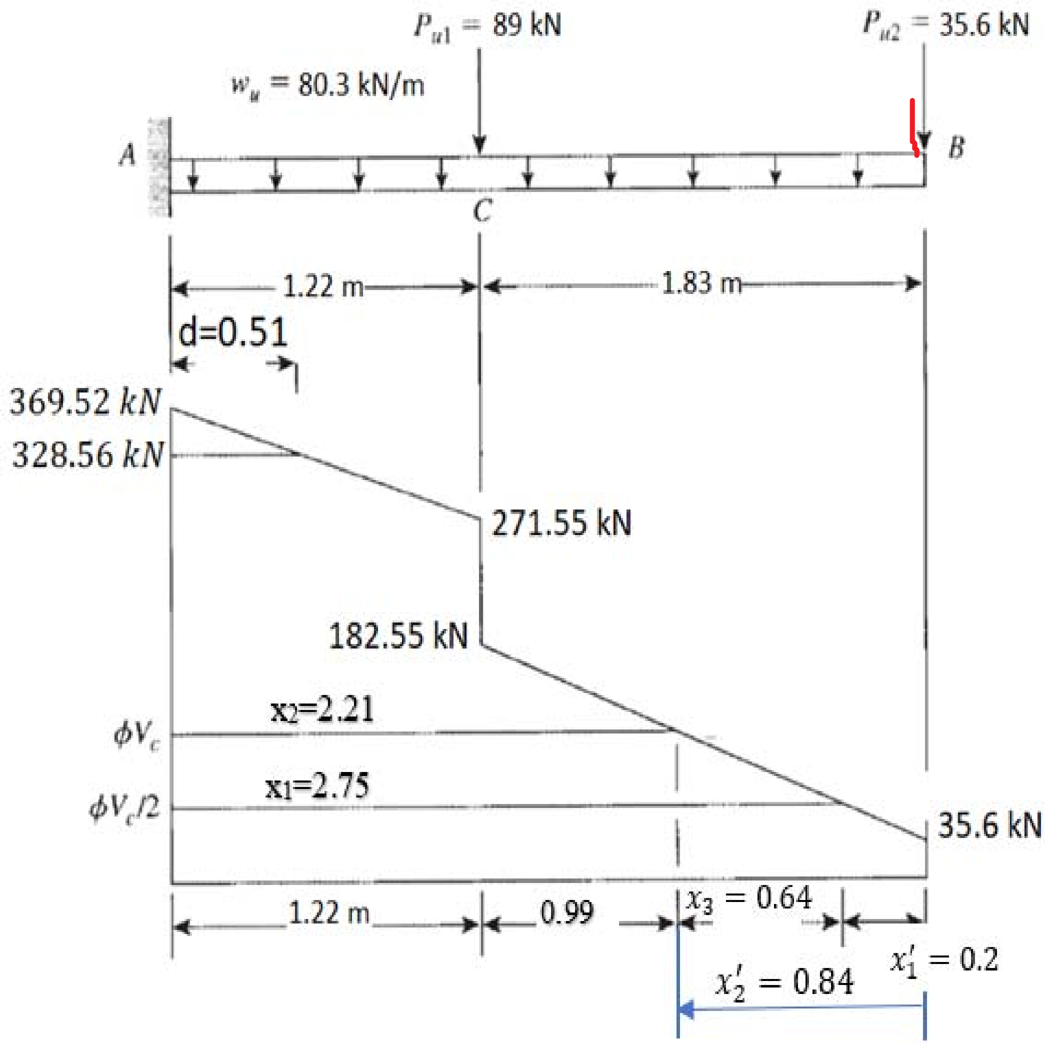
$= 271.55\ kN$

$V_{u,1.22R}(at\ 1.22\ right) = 271.55 - 89$

$= 182.55\ kN$

$V_{u,end}(at\ free\ end) = 35.6\ kN$

The shear diagram is shown below



$Calculate\ V_c$

$V_c = 0.17\,\lambda \sqrt{f_c'}\ b_w\, d = 0.17 \times 1 \times \sqrt{28} \times 300 \times 510$

$V_c = 137.6\ kN$

$Calculate\ \ 0.5\, V_c$

$0.5\, V_c = \frac{137.6}{2} = 68.8\ kN$

$V_{C1} = 0.33 \sqrt{f_c'}\, b_w\, d = 0.33 \times \sqrt{28} \times 300 \times 510 = 267.2\ kN$

and $\quad V_{C2} = 0.66 \sqrt{f_c'}\, b_w\, d = 2\, V_{C1} = 534.4\ kN$

$$V_n = \frac{V_{u,d}}{\emptyset} = \frac{328.56}{0.75} = 438.1\ kN$$

$\therefore\ V_c\ (137.6\ kN) < V_n\ (438.1\ kN)$

$\therefore\ shear\ reinforcement\ must\ be\ provided\ and\ calculate\ V_S$

$V_S = V_n - V_c$

$V_S = 438.1 - 137.6 = 300.5\ kN$

$V_S\ (300.5\ kN) < V_{C2}(534.4\ kN) \qquad \therefore the\ dimensions\ of\ the\ sec. is\ OK$

Calculate the stirrups spacing, Use ϕ 10 mm, therefore $A_v = 157\ mm^2$

$$S_1 = \frac{A_v\, f_{yt} d}{V_s} = \frac{157 \times 420 \times 510}{300.5 \times 10^3} = 112\ mm$$

$For\ V_S\ (300.5\ \ kN) > V_{C1}(267.2\ kN)$

$$\therefore S_{\ 2} = \frac{d}{4} \leq 300\ mm$$

$$\therefore S_{\ 2} = \frac{510}{4} = 127\ mm \leq 300\ mm \quad \therefore S_{\ 2} = \ 127\ mm$$, and

$$S_3 = smaller\ of \quad \begin{Bmatrix} \dfrac{A_v\ f_{yt}}{0.062\sqrt{f_c'}\ b_w} \\ \dfrac{A_v\ f_{yt}}{0.35\ b_w} \end{Bmatrix} = \ min \begin{Bmatrix} \dfrac{157 \times\ 420}{0.062\sqrt{28}\ \times 300} \\ \dfrac{157 \times\ 420}{0.35\ \times 300} \end{Bmatrix}$$

$$S_3 = \ min \begin{Bmatrix} 670\ mm \\ 628\ mm \end{Bmatrix} \quad \therefore S_3 = \ 628\ mm$$

$$S_{max} = \min(S_1\ , S_2\ and\ S_3) = 112\ mm \approx 110\ mm$$

∴ Use ϕ 10 mm @ 110 mm c/c

From shear diagram, the shear force on beam not constant and decrease to 35.6 kN at free end of beam, therefore using the spacing (S= 110 mm) for all beam is not economic, because this value (S= 110 mm) determined according to maximum shear force at distance d from support. So, for such cases when shear force not constant, the beam can divide to 3 or 2 zones according to the following.

$Zone\ 1:\ V_n\ <\ 0.5\,V_{c,Eq.a},$ $\quad$ *no shear reinforcement is needed.*

$Zone2:\ 0.5\,V_{c,Eq.a}\ <\ V_n\ \leq V_c$ $\quad$ *minimum shear reinforcement is required.*

$Zone3:\ \ V_n\ > V_c$ $\quad$ *shear reinforcement is required.*

Zone 1 and zone 2 can be consider as one zone with minimum shear reinforcement. (d/2, 600 mm)

It is easy to locate these zones as shown below, for zone1, by determine the location of V = 0.5ϕVc (x_1) and for zone2, by determine the location of V= ϕVc (x_2).

For zone1,

V=0.5ϕVc = 0.5 *0.75* 137.6 =51.6 kN,

$$35.6 + 80.3\,x_1' = 0.5\phi V_c$$

$$x_1' = \frac{51.6 - 35.6}{80.3} = 0.2\ m \quad from\ free\ end$$

$$\therefore x_1 = 3.05 - 0.2 = 2.75\ m \quad from\ support$$

For this distance of x_1' from free end, no shear reinforcement is needed.

For zone2, V=ϕVc = 0.75*137.6 =103.2 kN, from similarity of triangles

$$35.6 + 80.3\,x_2' = \phi V_c$$

$$x_2' = \frac{103.2 - 35.6}{80.3} = \underline{0.84\ m} \quad from\ free\ end$$

$$\therefore x_2 = 3.05 - 0.84 = 2.21\ m \quad from\ support$$

$For\ the\ distance\ \ x_3 = x_2' - x_1' = 0.64\ m\ ,$

$minimum\ shear\ reinforcement\ is\ required$

$S_3\ or\ S_2\ ,\ \ S_2 = d/2, 600\ mm$

$$S_2 = \frac{510}{2} = 255\ mm$$

∴ Use ϕ 10 mm @ 250 mm c/c

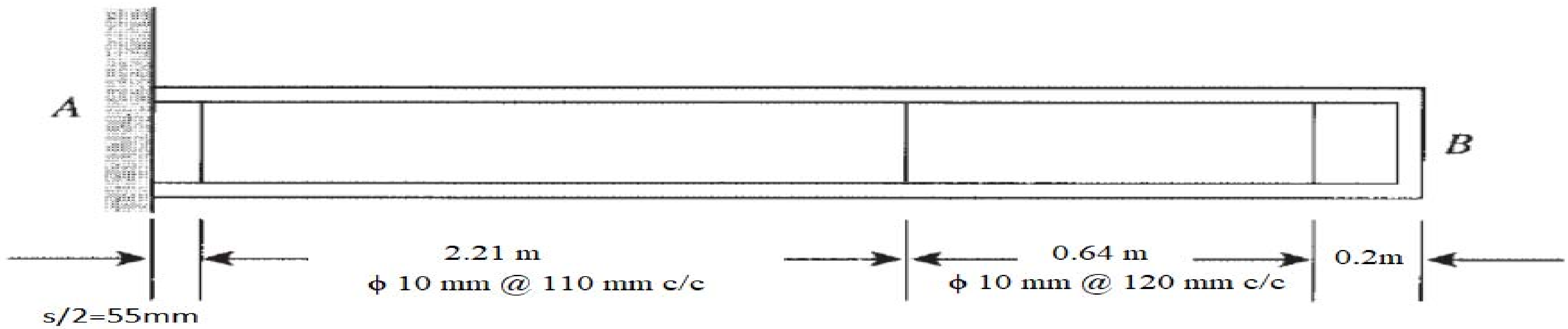
Actually, we can use min. shear reinforcement for all the distance x_2'.

For zone 3,

For the distance $x_2 = 2.21\ m$

$S_{max} = \min(S_1\ , S_2\ and\ S_3) = 112mm \approx 110\ mm$

∴ Use ϕ 10 mm @ 110 mm c/c



Distribution of stirrups.

Reinforced Concrete Design

Analysis and Design of One Way Concrete Slab

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Engineering College
Civil Engineering Department

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LOGO

Two way slab behavior

Dimension of Slab $L \times S$

$\frac{L}{S} < 2$ *with Uniform distributed load*

Supported on Four Edges

Considers two strip in two direction

Deflection for assumed simply

supported beam : $\Delta = \dfrac{5Wl^4}{384\,EI}$

If the two strip have same thickness

then deflection will be :

$\Delta ab = k\, W_{ab}\, S^4$

$\Delta cd = k\, W_{cd}\, l^4$

Where: W_{ab} and W_{cd} *is the transferred*

load by the strip ab and cd respectively

$$If\ Wu = W_{ab} + W_{cd}$$

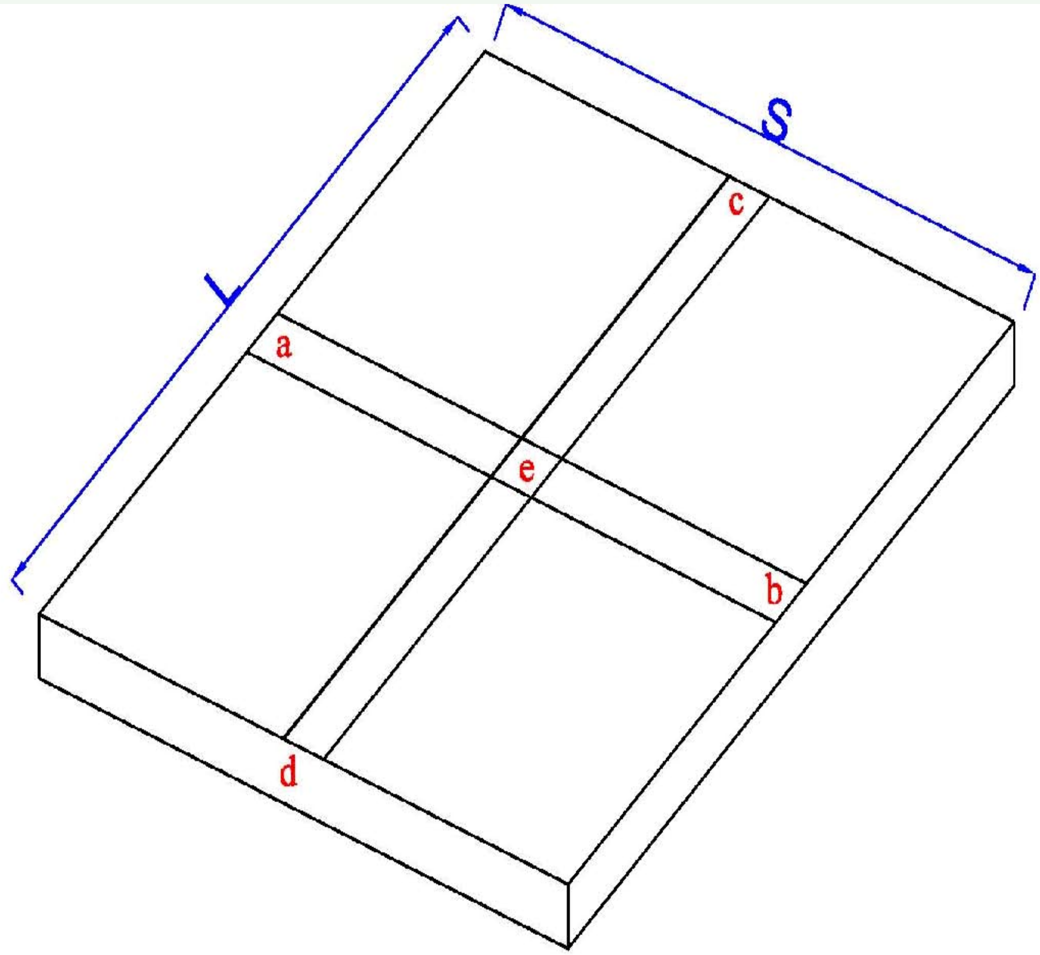
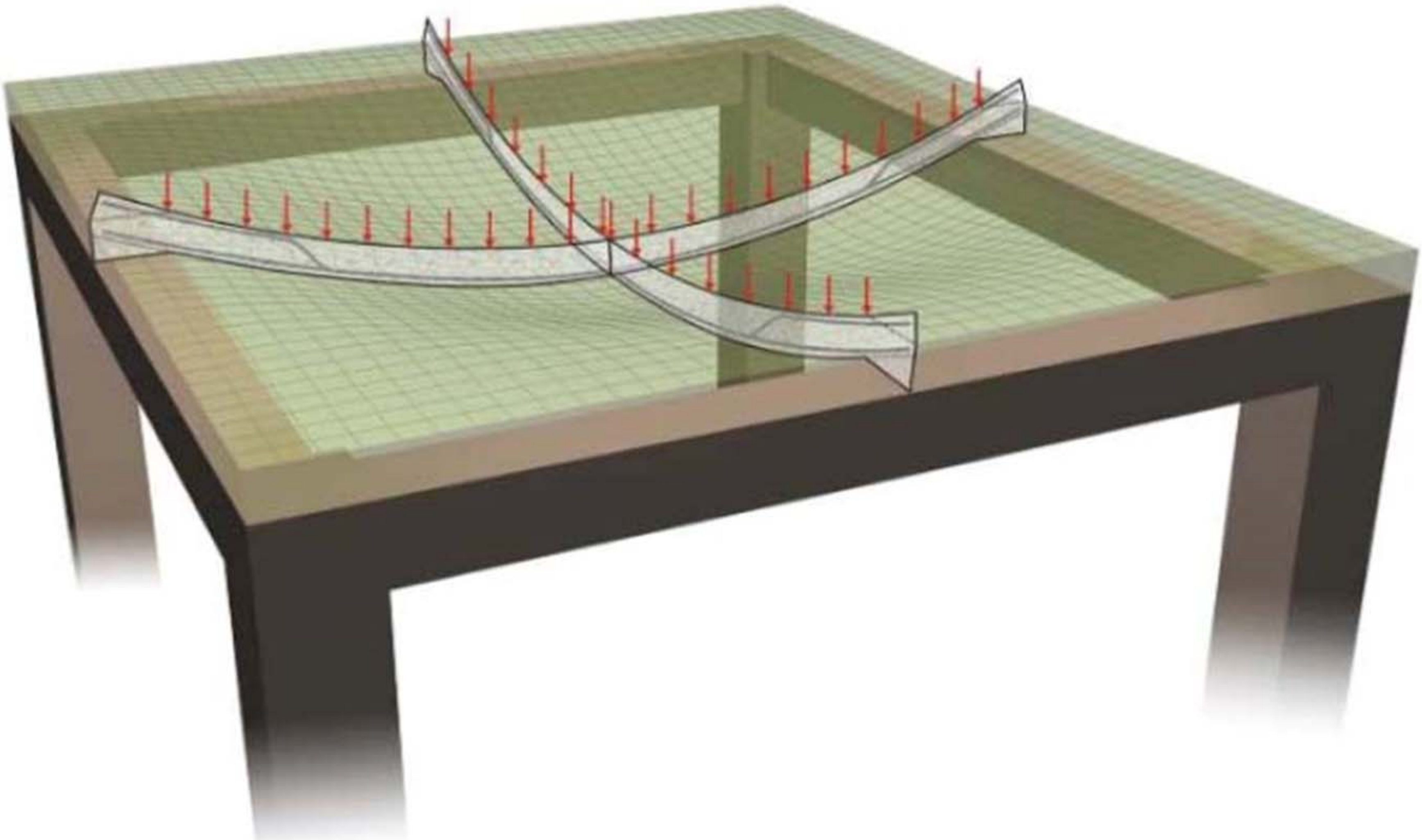


Fig. (1)



Behavior of a two-way slab

The Δ deflection at e are equal for both strip

$$k\,Wab\,S^4 = k\,Wcd\,l^4$$

$$Wab = \frac{L^4\,Wcd}{S^4} = \left(\frac{L}{S}\right)^4 Wcd$$

The transferred load into the short Direction = Load in Long Direction multiply by factor (L/S)^4

$If\ \left(\frac{L}{S}\right) = 1.5 \quad then \quad Wcd = 0.165\,W \quad and \quad Wab = 0.835\,W$

$If\ \left(\frac{L}{S}\right) = 2 \quad then \quad Wcd = 0.059\,W \quad and \quad Wab = 0.941\,W$

That's mean the short Direction resist the greater part of total applied load and when (L/S)>2 then the load transferred to the long Direction will be very small and can be neglected.

The analysis method assume :

-Uniform distributed load

-Live Load/ Dead Load ≤ 3 -Thickness of slab

ACI Code 1963 the h_{min} not less then 90 mm according to eq :

$$h_{min} = \frac{2(Ln + Sn)}{180} \geq 90\ mm$$

ACI Code 2014 present equation for slab with beams :
1-Table 8.3.1.1

$$h_{min} = \frac{Ln\left(0.8 + \frac{fy}{1400}\right)}{36 + 9\beta} \geq 90\ mm$$

where:

$$\beta = \frac{Ln}{Sn}$$

Ln , Sn : clear span of long and short direction respectively

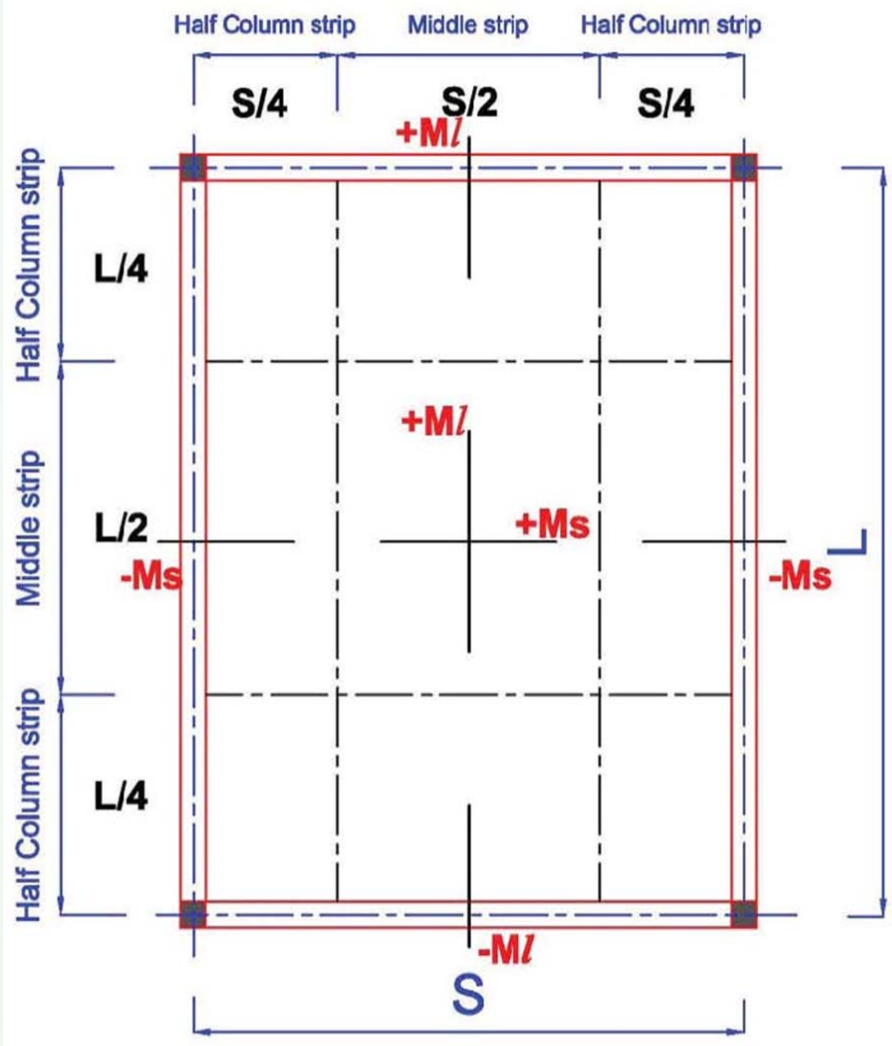


Fig. (2) Column and Middle Strip

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)[1]

f_y, MPa[2]	Without drop panels[3]			With drop panels[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams[4]		Without edge beams	With edge beams[4]	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

[1] ℓ_n is the clear span in the long direction, measured face-to-face of supports (mm).

[2] For f_y between the values given in the table, minimum thickness shall be calculated by linear interpolation.

[3] Drop panels as given in 8.2.4.

[4] Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if α_f is less than 0.8. The value of α_f for the edge beam shall be calculated in accordance with 8.10.2.7.

Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides

α_{fm}[1]	Minimum h, mm		
$\alpha_{fm} \le 0.2$	8.3.1.1 applies		(a)
$0.2 < \alpha_{fm} \le 2.0$	Greater of:	$\dfrac{\ell_n\left(0.8+\dfrac{f_y}{1400}\right)}{36+5\beta(\alpha_{fm}-0.2)}$	(b)[2],[3]
		125	(c)
$\alpha_{fm} > 2.0$	Greater of:	$\dfrac{\ell_n\left(0.8+\dfrac{f_y}{1400}\right)}{36+9\beta}$	(d)[2],[3]
		90	(e)

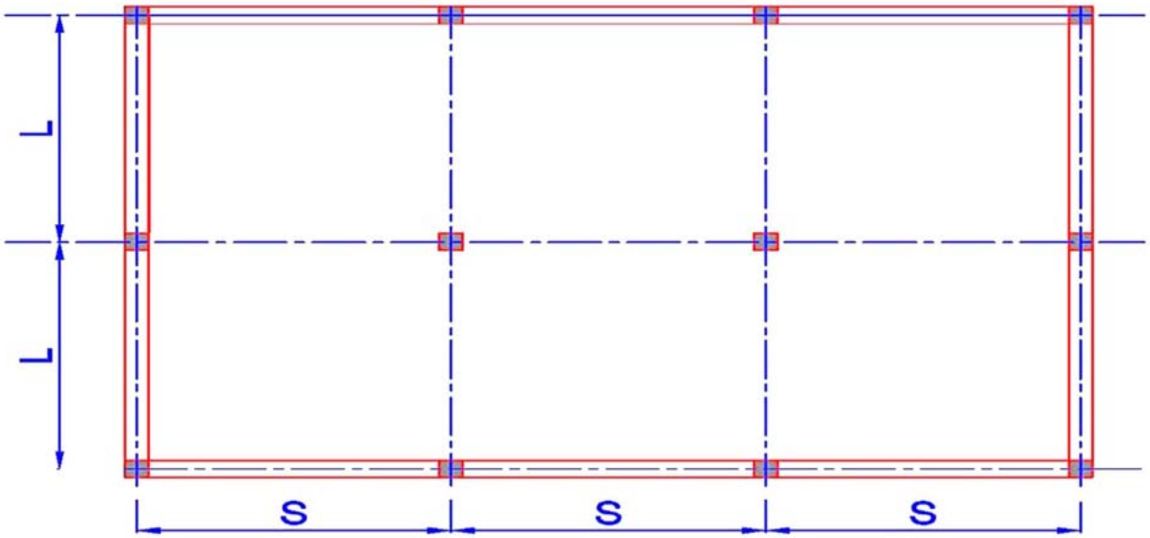
[1] α_{fm} is the average value of α_f for all beams on edges of a panel and α_f shall be calculated in accordance with 8.10.2.7.

[2] ℓ_n is the clear span in the long direction, measured face-to-face of beams (mm)

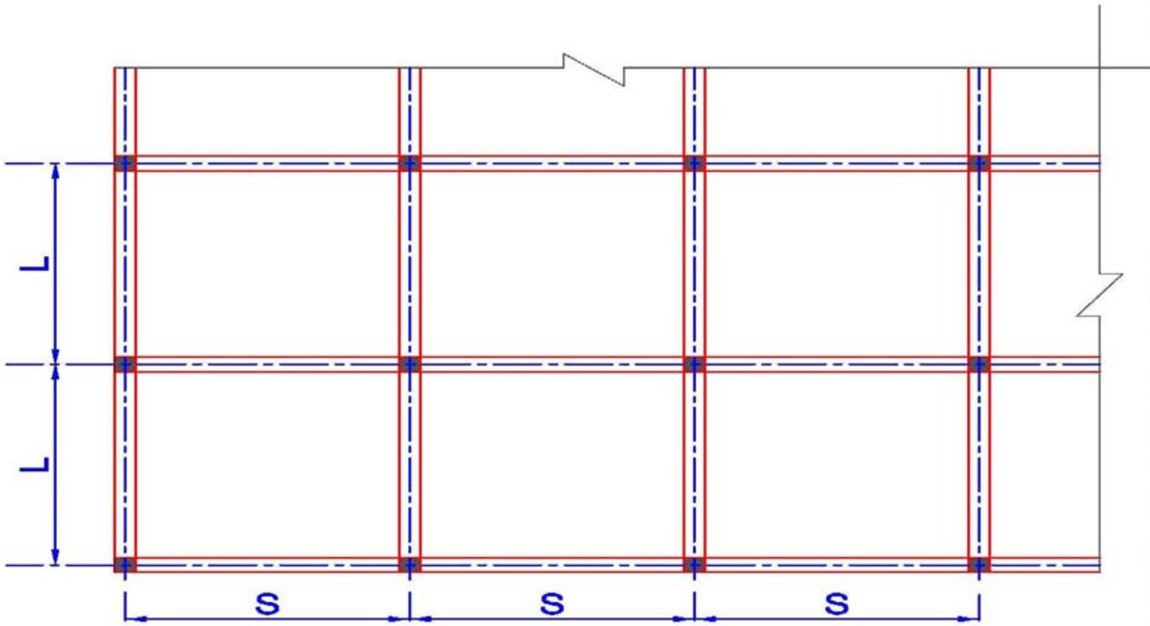
[3] β is the ratio of clear spans in long to short directions of slab

α_m: *is the average of* α_f *of all beams*

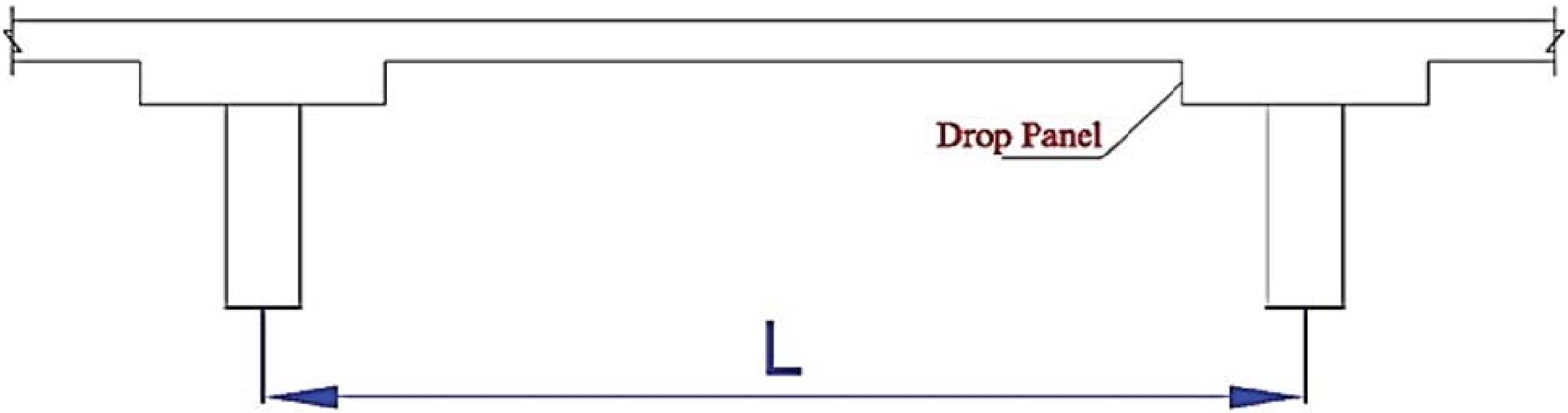
$$\alpha_f: \text{the flexural stiffness of beam/flexural stiffness of slab} = \frac{Eb\frac{Ib}{Lb}}{Es\frac{Is}{Ls}}$$

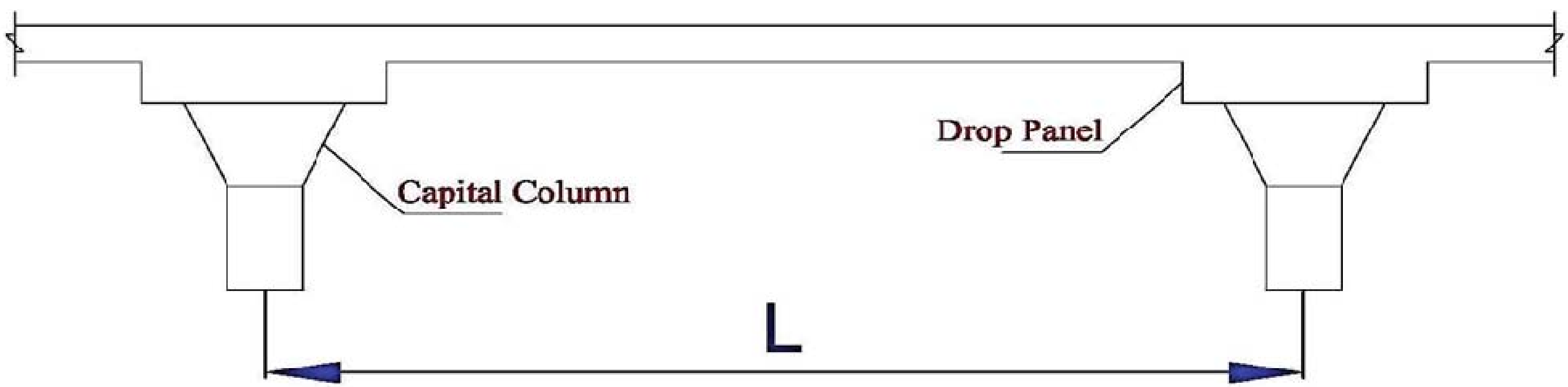


Slab with Edge Beam



Slab with Beam in All Direction





Slabs with Drop Panel

ACI Code suggest 3 methods to analyze the

Two-way slab

ACI Code suggest three methods to analyze the

Two-way slab since 1963

1-method 1

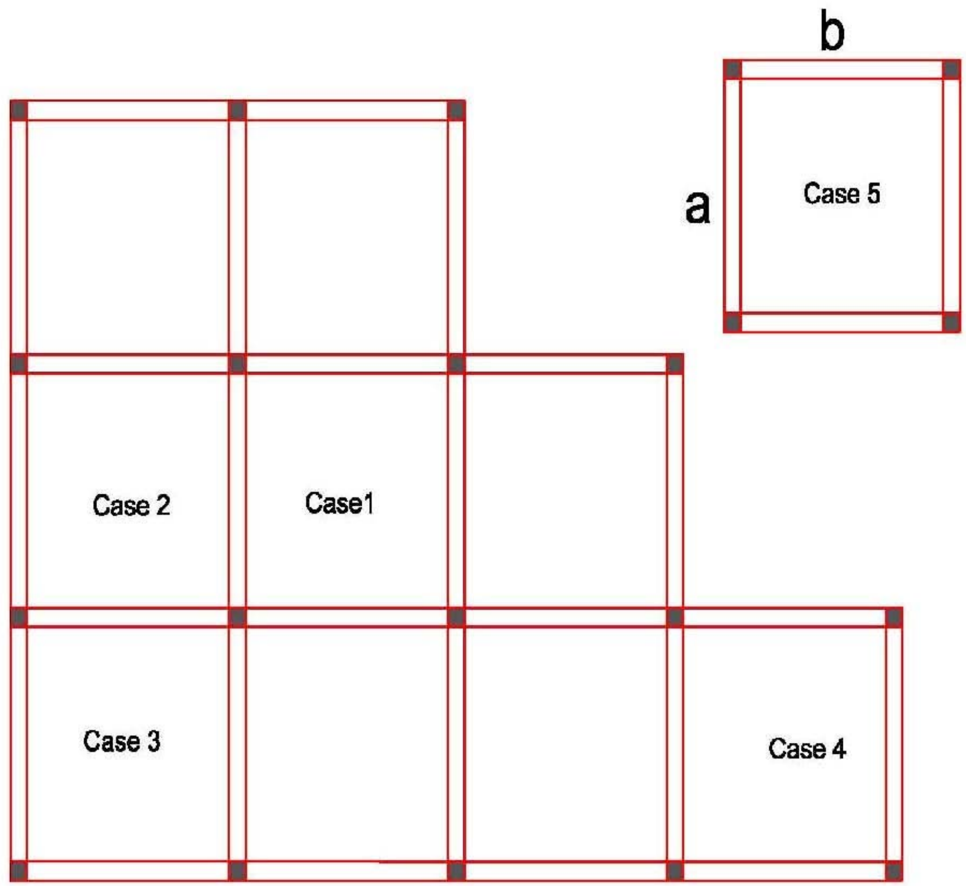
Method 2

The Moment at the middle strip :

$$M = C\,Wu\,S^2$$

C= is a factor can be found from tables

The Moment at the column strip = 2/3 M mid



Slab Cases

For Method 2

Where the negative moment on one side of a support is less than 80 percent of that on the other side, two-thirds of the difference shall be distributed in proportion to the relative stiffness of the slabs.

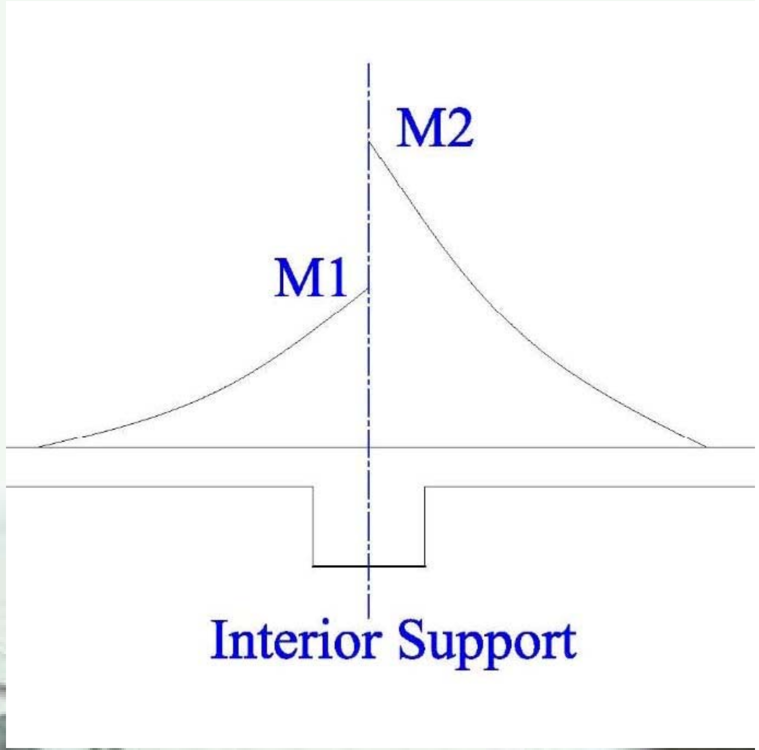
$$\frac{M2}{M1} \leq 0.8$$

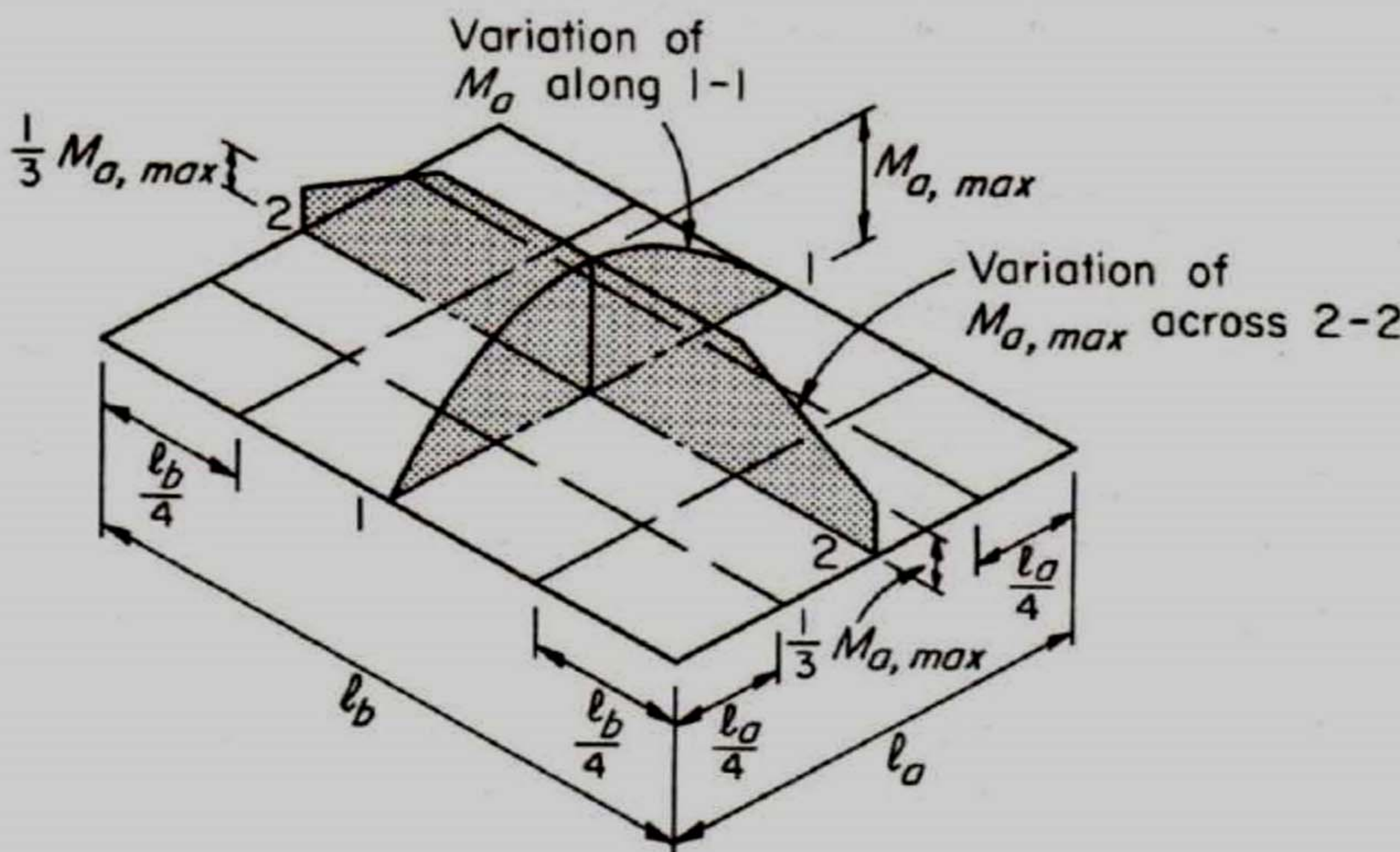
M Difference = M2-M1

2/3 M Difference Distributed for both side according to the slabs stiffness

While In Method 3 if $M1 \neq M2$,

The negative Moments in can be take is the maximum positive moment

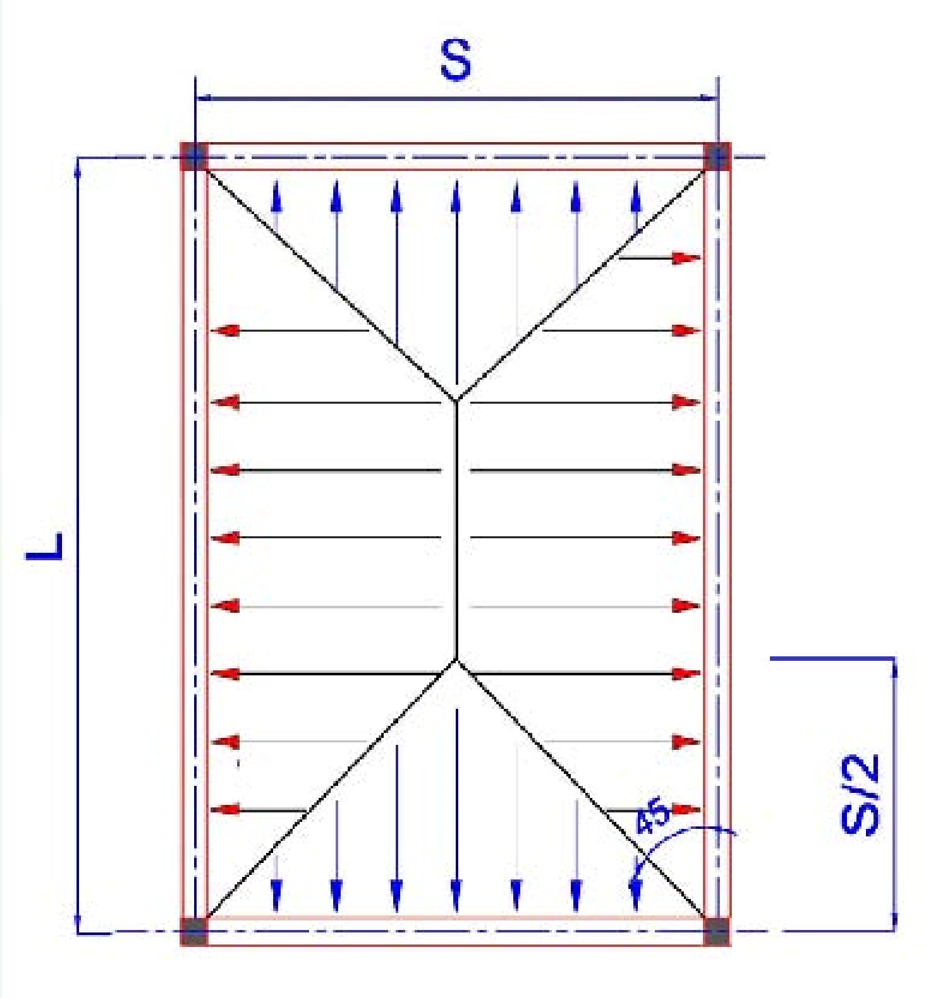




Variation of
M_a along 1-1
$\frac{1}{3} M_{a, max}$
2
$M_{a, max}$
1
Variation of
$M_{a, max}$ across 2-2
$\frac{\ell_b}{4}$
1
2
$\frac{\ell_a}{4}$
$\frac{1}{3} M_{a, max}$
ℓ_b
$\frac{\ell_b}{4}$
$\frac{\ell_a}{4}$
ℓ_a

Shear Force

The shear force on slab can be calculated according to the figure shown and transferred the equivalent load to the beams



Short Direction

$$W_{eq} = \frac{Wu\ La}{3} \qquad for\ moment$$

$$W_{eq} = \frac{Wu\ La}{4} \qquad for\ shear$$

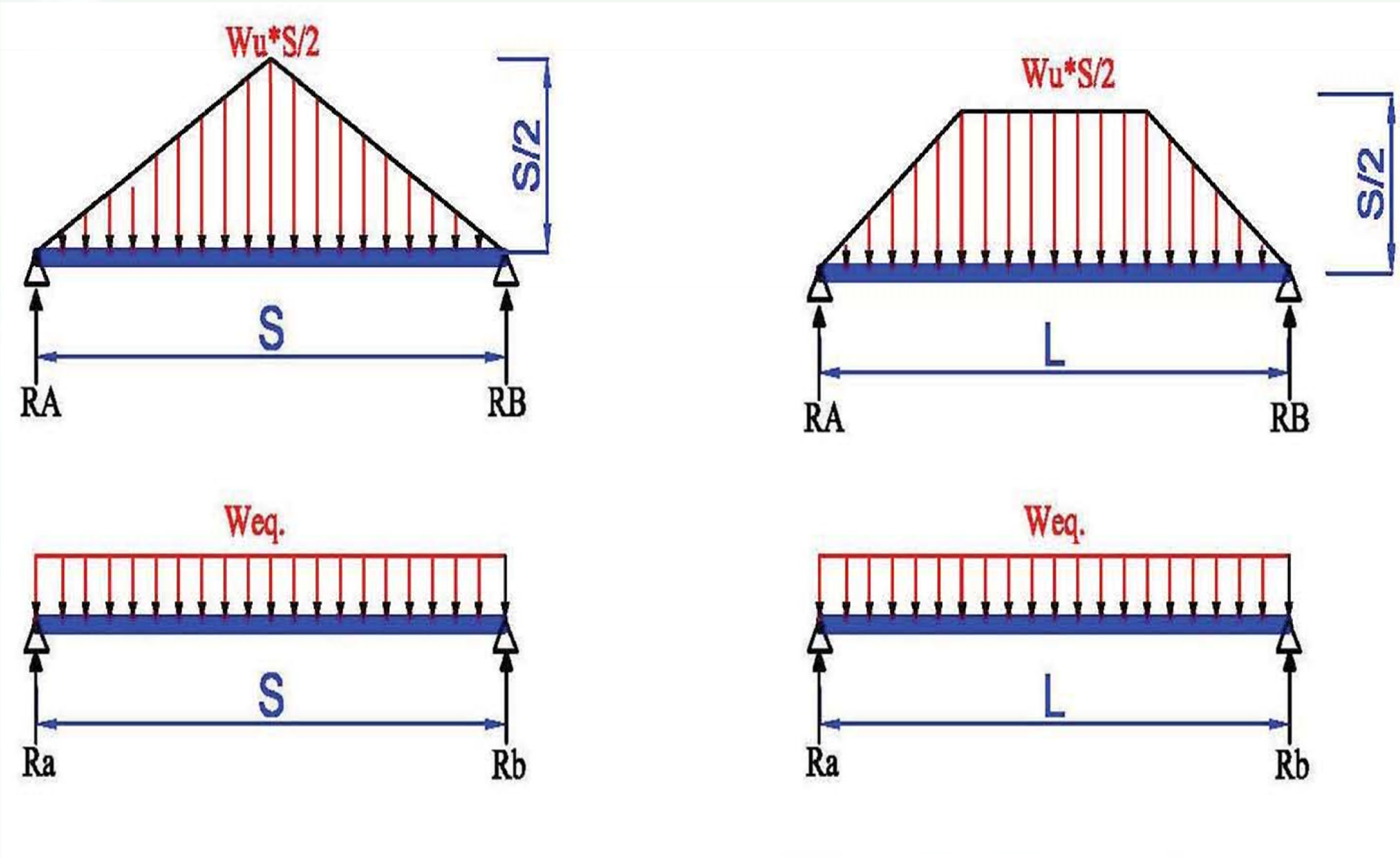
long Direction

$$Weq = \frac{wu\ S}{3}\left(\frac{3-m^2}{2}\right) \qquad for\ Moment$$

$$Weq = \frac{wu\ S}{4}(\ 2-m) \qquad for\ shear$$

$m = \ S/L \quad or \quad La/Lb$

$S\ ,L\ :\ length\ of\ span\ C/C\ in\ both\ direction$


Wu*S/2
S/2
S
RA
RB
Wu*S/2
S/2
L
RA
RB
Weq.
S
Ra
Rb
Weq.
L
Ra
Rb

Example (1) : An Interior Two way slab panel 6.0 m * 7.2m carry a live load 10 KN/m2. The slab thick 200 mm and is supported on beam 300 mm width and 900mm depth. Assume that the super imposed dead load equal to 3 KN/m2 . Determine the principal bending and shear in slab. Fy=420 MPA, fc=21MPa

Solution:

Method (2)

1- Minimum thickness

-ACI code 1963

$$h_{min} = \frac{2 \times (Sn + Ln)}{180} = \frac{2(5700 + 6900)}{180} = 140 \text{ mm}$$

-The ACI code 2014

• when the slab does not supported by beams

(interior panel) using ACI Table 8.3.1.1

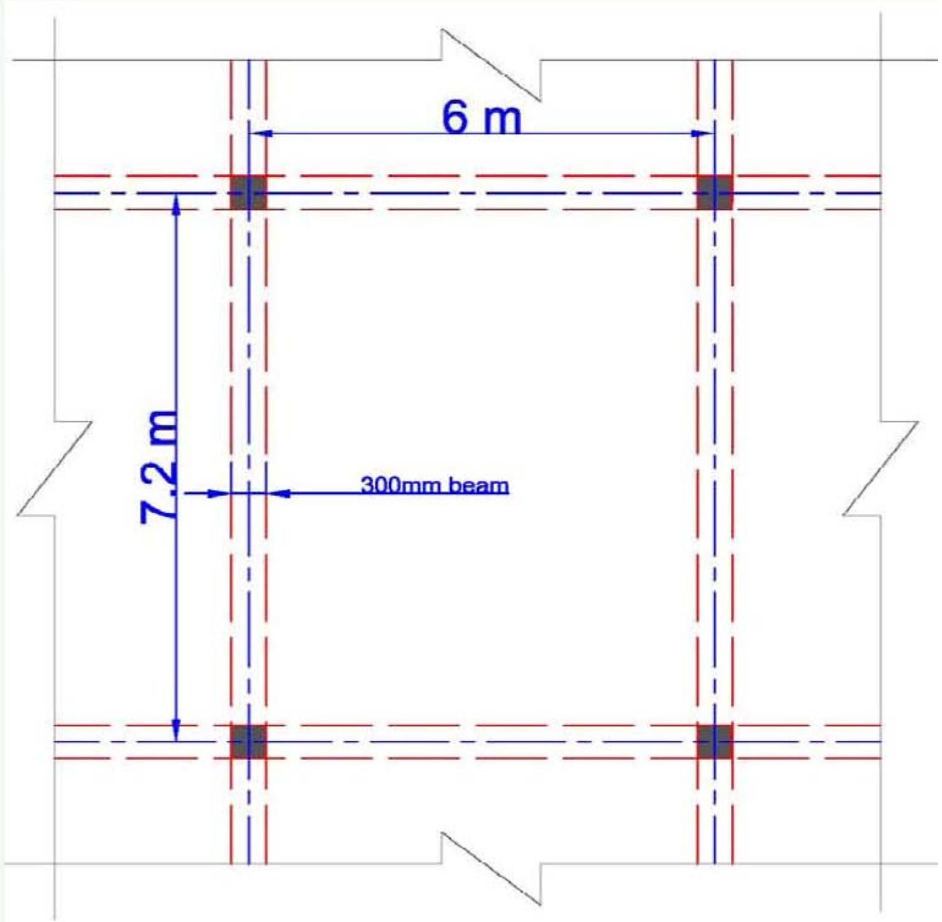
• For slab supported by beams : ($\alpha_m > 2$)

ACI code (table 8.3.1.2):

$$h_{min} = \frac{Ln\left(0.8 + \frac{fy}{1400}\right)}{36 + 9\beta} = \frac{6.9\left(0.8 + \frac{420}{1400}\right)}{36 + 9 \times \left(\frac{6.9}{5.7}\right)} \geq 90\ mm$$

$$= 161.8\ mm \geq 90\ mm$$

We will use h = 200mm (as mention in Example)



$Self\ Wt\ of\ slab\ =\ t\ *1*1*\ \ c = 0.2*1*1*24 = 4.8\ KN/m2$
$Wu =\ 1.2WD + 1.6WL$
$Wu = 1.2(4.8+3) + 1.6\ *\ 10 = 25.36\ KN/m2$
$m = \frac{S}{L} = \frac{6}{7.2} = 0.833\ \ or\ \ m = \left(\frac{Sn}{Ln} = \frac{5.7}{6.9} =\ 0.83\right) no\ big\ difference\ \ \ (for\ method\ 2\ use\ L\ and\ S\ center\ to\ center)$
From Table m lies between 0.8 and 0.9 for interior panel CASE I
Moment factors for Short Direction

Factor	0.8	0.833	0.9	Moment
$-C$	0.048	0.04536*	0.040	Negative moment
$+C$	0.036	0.03402	0.030	Positive moment

$$*C = \frac{(0.9-0.833)\times 0.048 + (0.833-0.8)\times 0.04}{(0.9-0.8)} =\ 0.04536$$

$-Mu = c\ Wu.S^2 = 0.05436 \times 25.36 \times 6^2 = 41.41\ KN.m/m$
$+Mu = c\ Wu.S^2 = 0.03402 \times 25.36 \times 6^2 = 31.06\ KN.m/m$

METHOD 2—TABLE I—MOMENT COEFFICIENTS

Moments	Short span: Values of m						Long span, all values of m
	1.0	0.9	0.8	0.7	0.6	0.5 and less	
Case 1—Interior panels							
Negative moment at—							
Continuous edge	0.033	0.040	0.048	0.055	0.063	0.083	0.033
Discontinuous edge	—	—	—	—	—	—	—
Positive moment at midspan	0.025	0.030	0.036	0.041	0.047	0.062	0.025
Case 2—One edge discontinuous							
Negative moment at—							
Continuous edge	0.041	0.048	0.055	0.062	0.069	0.085	0.041
Discontinuous edge	0.021	0.024	0.027	0.031	0.035	0.042	0.021
Positive moment at midspan	0.031	0.036	0.041	0.047	0.052	0.064	0.031
Case 3—Two edges discontinuous							
Negative moment at—							
Continuous edge	0.049	0.057	0.064	0.071	0.078	0.090	0.049
Discontinuous edge	0.025	0.028	0.032	0.036	0.039	0.045	0.025
Positive moment at midspan	0.037	0.043	0.048	0.054	0.059	0.068	0.037
Case 4—Three edges discontinuous							
Negative moment at—							
Continuous edge	0.058	0.066	0.074	0.082	0.090	0.098	0.058
Discontinuous edge	0.029	0.033	0.037	0.041	0.045	0.049	0.029
Positive moment at midspan	0.044	0.050	0.056	0.062	0.068	0.074	0.044
Case 5—Four edges discontinuous							
Negative moment at—							
Continuous edge	—	—	—	—	—	—	—
Discontinuous edge	0.033	0.038	0.043	0.047	0.053	0.055	0.033
Positive moment at midspan	0.050	0.057	0.064	0.072	0.080	0.083	0.050

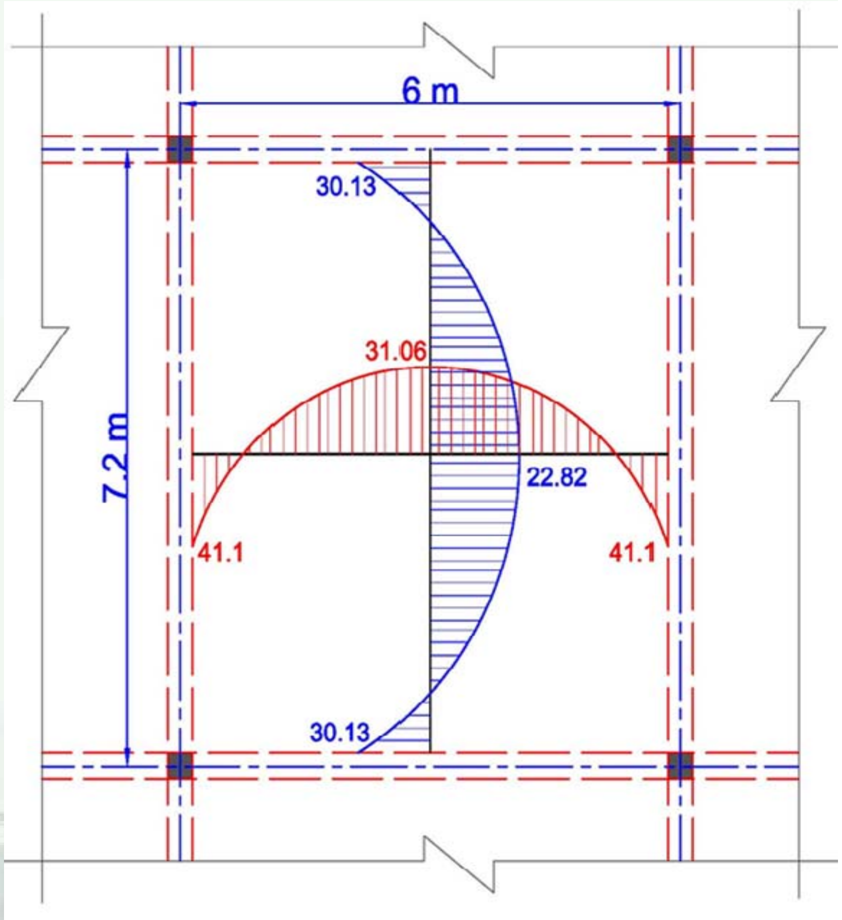
Moment factors for Long Direction

-C= 0.033 negative moment factor

+C=0.025 Positive moment factor

$-Mu = c\ Wu.S^2 = 0.033 \times 25.36 \times\ 62 = 30.13\ KN.m/m$
$+Mu = c\ Wu.S^2 = 0.025 \times 25.36 \times\ 62 = 22.82\ KN.m/m$

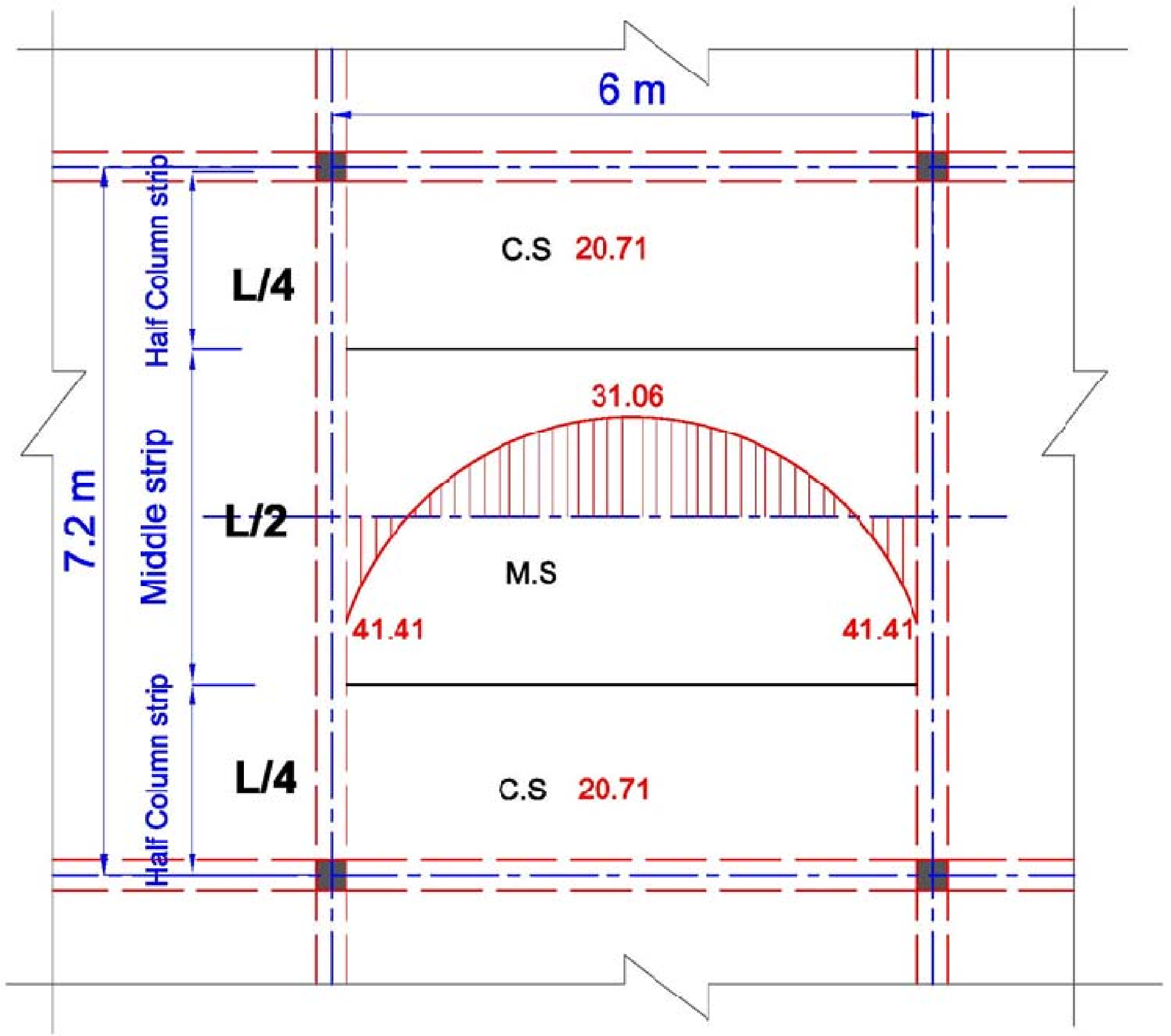
Moment at column strip will be 2/3 from middle strip moment in both direction



Shear in Slab

$$Vu = \frac{Wu \times S}{2}$$
$$= \frac{25.36 \times 6}{2} = 76.08\ KN/m$$

Moment diagram
(KN.m/m)
In Short Direction
Middle and Column Strip



Loads on Beams

Bending Moments

1-long Direction

$$W_{eq} = \frac{wu\,S}{3}\left(\frac{3-m^2}{2}\right) = \frac{25.36\times 6}{3}\left(\frac{3-0.833^2}{2}\right) = 58.47\ KN/m \quad from\ one\ side$$

$$W_{eq} = \frac{wu\,S}{4}(2-m) \qquad for\ shear$$

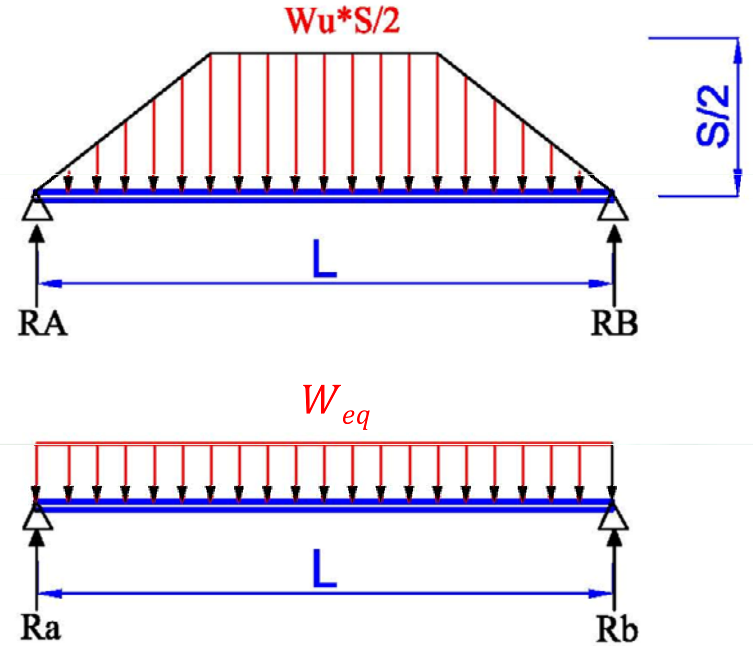
There is two slab transferred load to the beam

$W_{eq} = 2\times 58.47 = 116.94\ KN/m \quad (from\ both\ side)$

$Self\ weight\ of\ drop\ beam\ part = 1.2\times(h-t)\times b\times 1\times\gamma c$

$= 1.2\times(0.9-0.2)\times 1\times 0.3\times 24 = 6.05\ KN/m$

$Total\ Wu_b = 116.94 + 6.05 = 122.99\ KN/m$



2- Short Beam

$$Weq = \frac{Wu\,S}{3}$$

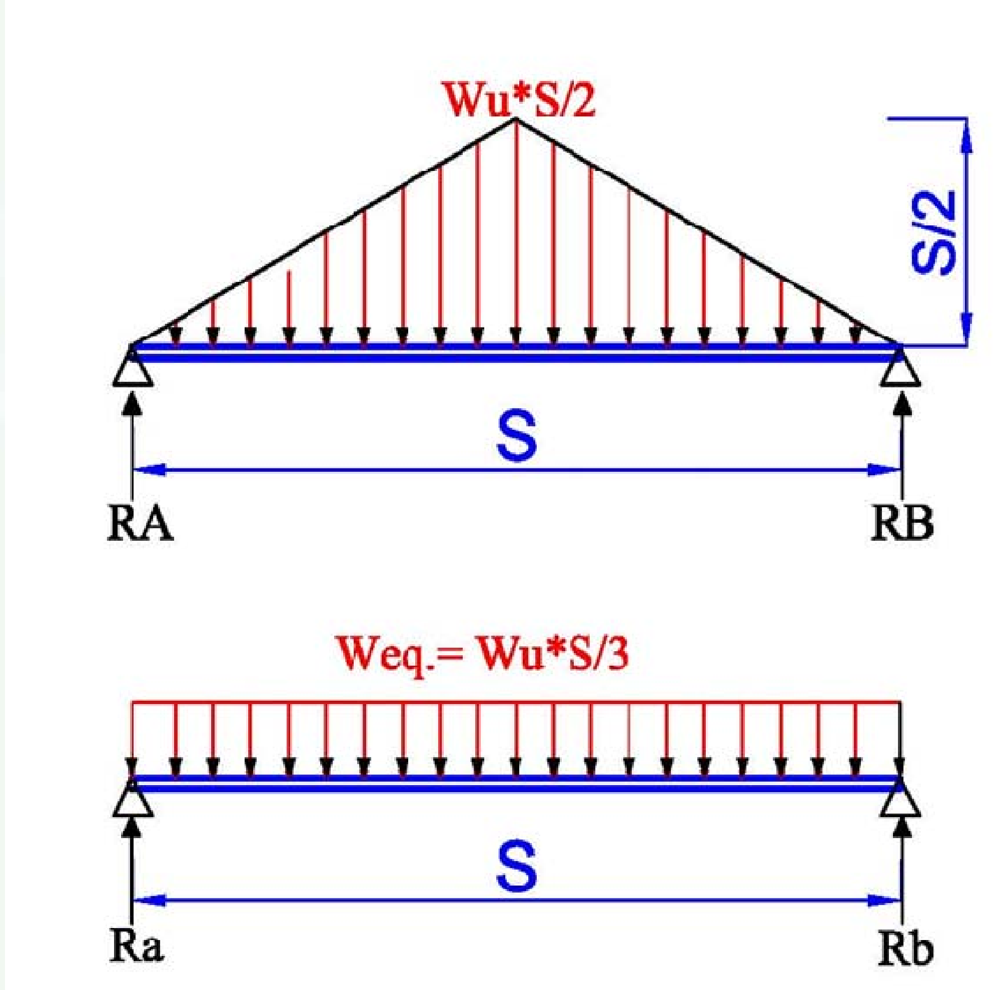
$$= \frac{25.36 \times 6}{3} = 50.6\,KN/m \quad from\ one\ Side$$

There is two slab transferred load to the beam

$$Weq = 2 \times 50.6 = 101.2\,KN/m$$

$$Self\ weight\ of\ drop\ beam\ part = 6.05\,KN/m$$

$$Wua = 101.2 + 6.05 = 107.25\,KN/m$$



Beam Moment Calculation

Using Factored for interior panel for beams

1- Long Direction

$Wub = 122.99\ KN/m$

$$-M = \frac{1}{11}\,(Wub \times L^2) = \frac{1}{11} \times (\,122.99 \times 6.92) = 532.32\ KN.m$$

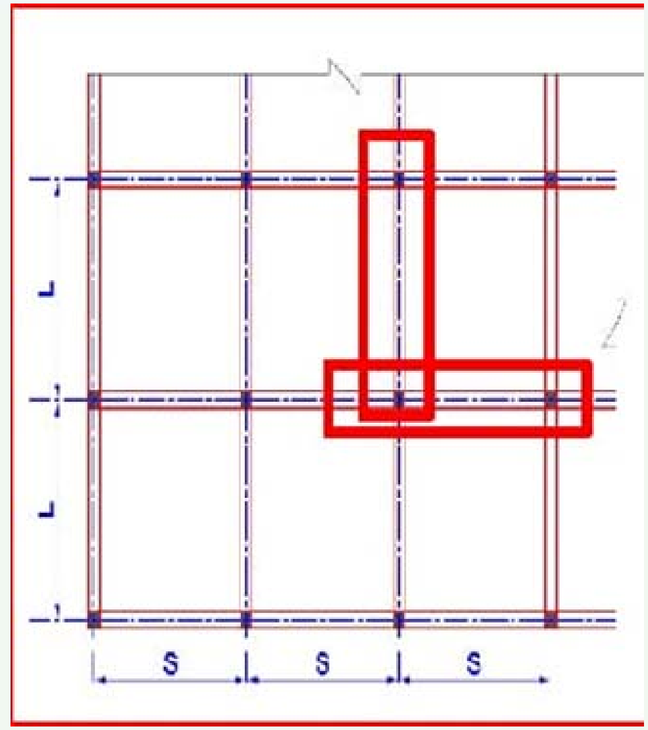
$$+M = \frac{1}{16}\,(Wub \times L^2) = \frac{1}{16} \times (\,122.99 \times 6.92) = 365.97\ KN.m$$

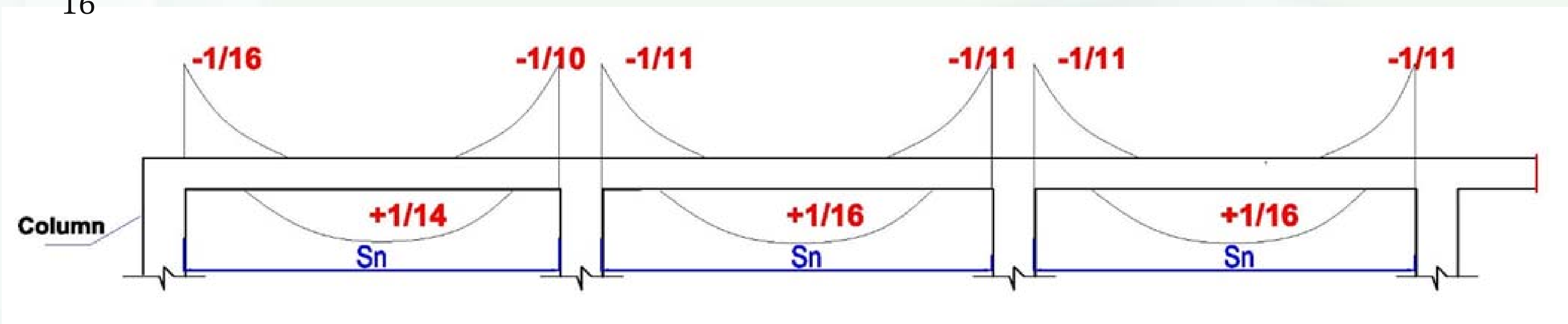
2- Short Direction

$Wub = 107.25\ KN/m$

$$-M = \frac{1}{11}\,(Wua \times S^2) = 1/11 \times (\,107.25 \times 5.72) = 316.78\ KN.m$$

$$+M = \frac{1}{16}\,(Wua \times S^2) = 1/16 \times (\,107.25 \times 5.72) = 217.8 KN.m$$





B.M Factors

Shear in Beams

1-Long direction

$W_{ub} = Wu \times S/4 \times (2 - m)$

$= 25.36 \times 6/4 \times (2 - 0.833) = 44.38\ KN/m$

From both side have load

$2 \times 44.38 = 88.76\ KN/m$

$Self\ weight\ of\ Beam = 6.05\ KN/m$

W_{ub}= 88.76+6.05= 94.81 KN/m

Shear force at support

$$Vu = \frac{Wu \times L}{2} = \frac{94.81 \times 7.2}{2} = 341.22\ KN$$

1-Short direction

$$W_{ua} = \frac{Wu\ S}{4} \quad for\ shear$$

$$= \frac{25.36 \times 6}{4} = 38.04\ KN/m$$

From both side have load and adding self weight of beam

$Wu\ a = 2 \times 38.04 + 6.05 = 82.13\ KN/m$

Shear force at support

$$Vu = \frac{Wu \times S}{2} = \frac{82.81 \times 6}{2} = 246.4\ KN$$

Method 3

ACI code using method 3 and denoted to long direction as b and short direction with a and considering the live load effect.

- Negative Moment

1- Short direction (a)

$$-M_a = C_{a\ neg}\ Wu\ L_a^2$$

2- Long direction (b)

$$-M_b = C_{b\ neg}\ Wu\ L_b^2$$

Where:

Wu : total uniform factored load (D.L + L.L)

C_a: *Moment coefficient from table*

C_b: *Moment coefficient from table*

L_a: *clear span for short direction*

L_b: *clear span for short direction*

Positive Moment

$1 - \ Short\ direction\ (\ a)$

$$+Ma_{\ D.L} = C_{a\ DL} \times Wu_{\ DL} \times La^2$$

$$+Ma_{\ L.L} = C_{a\ LL} \times Wu_{\ LL} \times La^2$$

$$+Ma = +Ma_{\ D.L} + Ma_{\ L.L}$$

$2 - \ Long\ direction\ (\ b)$

$$+Mb_{D.L} = C_{b\ DL} \times Wu_{DL} \times Lb^2$$

$$+Mb_{L.L} = C_{b\ LL} \times Wu_{LL} \times Lb^2$$

$$+Mb = +Mb_{\ D.L} + Mb_{\ L.L}$$

Note:

When two negative moment at support are different for continuous slab, can take average Moment:

$$-M = \frac{M\ left + Mright}{2}$$

Item	Moment Direction	
	Short Direction S or (a)	Long Direction L or (b)
Negative Moment (-M)	$-M_a = C_{a\ neg}\ Wu\ La^2$	$-M_b = C_{b\ neg}\ Wu\ Lb^2$
Positive Moment (+M)	$+Ma_{D.L} = C_{a\ DL} \times Wu_{DL} \times La^2$ $+Ma_{L.L} = C_{a\ LL} \times Wu_{LL} \times La^2$	$+Mb_{D.L} = C_{b\ DL} \times Wu_{DL} \times Lb^2$ $+Mb_{L.L} = C_{b\ LL} \times Wu_{LL} \times Lb^2$
	$+Ma = +Ma\ D.L + Ma\ L.L$	$+Mb = +Mb\ D.L + Mb\ L.L$

Example (2) : (as in Ex. 1) An Interior Two way slab panel 6.0 m * 7.2m carry a live load 10 KN/m^2. The slab thick 200 mm and is supported on beam 300 mm width and 900mm depth. Assume that the super imposed dead load equal to 3 KN/m^2 . Determine the principal bending and shear in slab. Fy=280 MPa, fc=21MPa

Sol.

Wu= 25.36 KN/m^2 (exa. 1)

Interior panel continues from all side (Case 2) Table 1

$$\frac{La}{Lb} = \frac{(6-0.3)}{(7.2-0.3)} = 0.826 \quad (or\ a/b\ )$$

1- Negative Moment Factors

- ***Short Direction (by interpolation)***

0.8	0.065
0.826	
0.85	0.06

$$Ca\ neg. = \frac{0.06 \times (0.826 - 0.8) + 0.065 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.0624$$

-Long Direction (by interpolation)

0.8	0.027
0.826	
0.85	0.031

$$Cb\ neg. = \frac{0.031 \times (0.826 - 0.8) + 0.027 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.02908$$

METHOD 3—TABLE I—COEFFICIENTS FOR NEGATIVE MOMENTS IN SLABS*

$$\left.\begin{array}{l} M_{A\,neg} = C_{A\,neg} \times w \times A^2 \\ M_{B\,neg} = C_{B\,neg} \times w \times B^2 \end{array}\right\} \text{where } w = \text{total uniform dead plus live load}$$

Ratio $m = \frac{A}{B}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{A\,neg}$		0.045		0.050	0.075	0.071		0.033	0.061
	$C_{B\,neg}$		0.045	0.076	0.050			0.071	0.061	0.033
0.95	$C_{A\,neg}$		0.050		0.055	0.079	0.075		0.038	0.065
	$C_{B\,neg}$		0.041	0.072	0.045			0.067	0.056	0.029
0.90	$C_{A\,neg}$		0.055		0.060	0.080	0.079		0.043	0.068
	$C_{B\,neg}$		0.037	0.070	0.040			0.062	0.052	0.025
0.85	$C_{A\,neg}$		0.060		0.066	0.082	0.083		0.049	0.072
	$C_{B\,neg}$		0.031	0.065	0.034			0.057	0.046	0.021
0.80	$C_{A\,neg}$		0.065		0.071	0.083	0.086		0.055	0.075
	$C_{B\,neg}$		0.027	0.061	0.029			0.051	0.041	0.017
0.75	$C_{A\,neg}$		0.069		0.076	0.085	0.088		0.061	0.078
	$C_{B\,neg}$		0.022	0.056	0.024			0.044	0.036	0.014
0.70	$C_{A\,neg}$		0.074		0.081	0.086	0.091		0.068	0.081
	$C_{B\,neg}$		0.017	0.050	0.019			0.038	0.029	0.011
0.65	$C_{A\,neg}$		0.077		0.085	0.087	0.093		0.074	0.083
	$C_{B\,neg}$		0.014	0.043	0.015			0.031	0.024	0.008
0.60	$C_{A\,neg}$		0.081		0.089	0.088	0.095		0.080	0.085
	$C_{B\,neg}$		0.010	0.035	0.011			0.024	0.018	0.006
0.55	$C_{A\,neg}$		0.084		0.092	0.089	0.096		0.085	0.086
	$C_{B\,neg}$		0.007	0.028	0.008			0.019	0.014	0.005
0.50	$C_{A\,neg}$		0.086		0.094	0.090	0.097		0.089	0.088
	$C_{B\,neg}$		0.006	0.022	0.006			0.014	0.010	0.003

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

METHOD 3—TABLE 2—COEFFICIENTS FOR DEAD LOAD POSITIVE MOMENTS IN SLABS*

$$\left.\begin{aligned} M_{A\ pos\ DL} &= C_{A\ DL} \times w \times A^2 \\ M_{B\ pos\ DL} &= C_{B\ DL} \times w \times B^2 \end{aligned}\right\} \text{ where } w = \text{total uniform dead load}$$

Ratio $m = \frac{A}{B}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{A\ DL}$	0.036	0.018	0.018	0.027	0.027	0.033	0.027	0.020	0.023
	$C_{B\ DL}$	0.036	0.018	0.027	0.027	0.018	0.027	0.033	0.023	0.020
0.95	$C_{A\ DL}$	0.040	0.020	0.021	0.030	0.028	0.036	0.031	0.022	0.024
	$C_{B\ DL}$	0.033	0.016	0.025	0.024	0.015	0.024	0.031	0.021	0.017
0.90	$C_{A\ DL}$	0.045	0.022	0.025	0.033	0.029	0.039	0.035	0.025	0.026
	$C_{B\ DL}$	0.029	0.014	0.024	0.022	0.013	0.021	0.028	0.019	0.015
0.85	$C_{A\ DL}$	0.050	0.024	0.029	0.036	0.031	0.042	0.040	0.029	0.028
	$C_{B\ DL}$	0.026	0.012	0.022	0.019	0.011	0.017	0.025	0.017	0.013
0.80	$C_{A\ DL}$	0.056	0.026	0.034	0.039	0.032	0.045	0.045	0.032	0.029
	$C_{B\ DL}$	0.023	0.011	0.020	0.016	0.009	0.015	0.022	0.015	0.010
0.75	$C_{A\ DL}$	0.061	0.028	0.040	0.043	0.033	0.048	0.051	0.036	0.031
	$C_{B\ DL}$	0.019	0.009	0.018	0.013	0.007	0.012	0.020	0.013	0.007
0.70	$C_{A\ DL}$	0.068	0.030	0.046	0.046	0.035	0.051	0.058	0.040	0.033
	$C_{B\ DL}$	0.016	0.007	0.016	0.011	0.005	0.009	0.017	0.011	0.006
0.65	$C_{A\ DL}$	0.074	0.032	0.054	0.050	0.036	0.054	0.065	0.044	0.034
	$C_{B\ DL}$	0.013	0.006	0.014	0.009	0.004	0.007	0.014	0.009	0.005
0.60	$C_{A\ DL}$	0.081	0.034	0.062	0.053	0.037	0.056	0.073	0.048	0.036
	$C_{B\ DL}$	0.010	0.004	0.011	0.007	0.003	0.006	0.012	0.007	0.004
0.55	$C_{A\ DL}$	0.088	0.035	0.071	0.056	0.038	0.058	0.081	0.052	0.037
	$C_{B\ DL}$	0.008	0.003	0.009	0.005	0.002	0.004	0.009	0.005	0.003
0.50	$C_{A\ DL}$	0.095	0.037	0.080	0.059	0.039	0.061	0.089	0.056	0.038
	$C_{B\ DL}$	0.006	0.002	0.007	0.004	0.001	0.003	0.007	0.004	0.002

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

METHOD 3—TABLE 3—COEFFICIENTS FOR LIVE LOAD POSITIVE MOMENTS IN SLABS*

$$\left.\begin{aligned} M_{A\,pos\,LL} &= C_{A\,LL} \times w \times A^2 \\ M_{B\,pos\,LL} &= C_{B\,LL} \times w \times B^2 \end{aligned}\right\} \text{ where } w = \text{total uniform live load}$$

Ratio $m = \frac{A}{B}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	$C_{A\,LL}$	0.036	0.027	0.027	0.032	0.032	0.035	0.032	0.028	0.030
	$C_{B\,LL}$	0.036	0.027	0.032	0.032	0.027	0.032	0.035	0.030	0.028
0.95	$C_{A\,LL}$	0.040	0.030	0.031	0.035	0.034	0.038	0.036	0.031	0.032
	$C_{B\,LL}$	0.033	0.025	0.029	0.029	0.024	0.029	0.032	0.027	0.025
0.90	$C_{A\,LL}$	0.045	0.034	0.035	0.039	0.037	0.042	0.040	0.035	0.036
	$C_{B\,LL}$	0.029	0.022	0.027	0.026	0.021	0.025	0.029	0.024	0.022
0.85	$C_{A\,LL}$	0.050	0.037	0.040	0.043	0.041	0.046	0.045	0.040	0.039
	$C_{B\,LL}$	0.026	0.019	0.024	0.023	0.019	0.022	0.026	0.022	0.020
0.80	$C_{A\,LL}$	0.056	0.041	0.045	0.048	0.044	0.051	0.051	0.044	0.042
	$C_{B\,LL}$	0.023	0.017	0.022	0.020	0.016	0.019	0.023	0.019	0.017
0.75	$C_{A\,LL}$	0.061	0.045	0.051	0.052	0.047	0.055	0.056	0.049	0.046
	$C_{B\,LL}$	0.019	0.014	0.019	0.016	0.013	0.016	0.020	0.016	0.013
0.70	$C_{A\,LL}$	0.068	0.049	0.057	0.057	0.051	0.060	0.063	0.054	0.050
	$C_{B\,LL}$	0.016	0.012	0.016	0.014	0.011	0.013	0.017	0.014	0.011
0.65	$C_{A\,LL}$	0.074	0.053	0.064	0.062	0.055	0.064	0.070	0.059	0.054
	$C_{B\,LL}$	0.013	0.010	0.014	0.011	0.009	0.010	0.014	0.011	0.009
0.60	$C_{A\,LL}$	0.081	0.058	0.071	0.067	0.059	0.068	0.077	0.065	0.059
	$C_{B\,LL}$	0.010	0.007	0.011	0.009	0.007	0.008	0.011	0.009	0.007
0.55	$C_{A\,LL}$	0.088	0.062	0.080	0.072	0.063	0.073	0.085	0.070	0.063
	$C_{B\,LL}$	0.008	0.006	0.009	0.007	0.005	0.006	0.009	0.007	0.006
0.50	$C_{A\,LL}$	0.095	0.066	0.088	0.077	0.067	0.078	0.092	0.076	0.067
	$C_{B\,LL}$	0.006	0.004	0.007	0.005	0.004	0.005	0.007	0.005	0.004

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.

$Ma = Ca.neg.Wu.la^2 = 0.0624 \times 25.36 \times (5.7)^2 = 51.41\ KN.m/m$

$-Mb = Cb.neg.Wu.lb^2 = 0.02908 \times 25.36 \times (6.9)^2 = 35.03\ KN.m/m$

2- Positive Moment

Short Direction

-Factors of Dead Load (from Table 2)

0.8 0.026

0.826 $Ca.DL = \dfrac{0.024 \times (0.826 - 0.8) + 0.026 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.02496$

0.85 0.024

$Self\ Wt\ of\ slab = t \times 1 \times 1 \times \gamma c = 0.2 \times 1 \times 1 \times 24 = 4.8 KN/m2$

$Wu_D = 1.2\ (4.8 + 3) = 9.36\ KN/m2$

$+Ma_{DL} = 0.02496 \times 9.36 \times 5.72 = 7.6\ KN.m/m$

-Factors of Live Load (from Table 2)

0.8 0.041

0.826 $Ca.LL = \dfrac{0.037 \times (0.826 - 0.8) + 0.041 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.03892$

0.85 0.037

$Wu\ LL = 1.6 \times 10 = 16\ KN/m2$

$+Ma\ LL = 0.03892 \times 16 \times 5.72 = 20.23\ KN.m/m$

$+Ma = Ma\ DL + Ma\ LL = 7.6 + 20.23 = 27.83\ KN.m/m$

Long Direction

-*Factors of Dead Load* (*from Table* 3)

0.8 0.011

0.826 $Cb.DL = \dfrac{0.012 \times (0.826 - 0.8) + 0.011 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.01148$

0.85 0.012

$Wu_D = 9.36\ KN/m^2$

$+Mb_{DL} = 0.01148 \times 9.36 \times 6.92 = 5.12\ KN.m/m$

−*Factors of Live Load* (*from Table* 3)

0.8 0.017

0.826 $Cb.LL = \dfrac{0.024 \times (0.826 - 0.8) + 0.026 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.01804$

0.85 0.019

$Wu_{LL} = 16\ KN/m^2$

$+Mb_{LL} = 0.01804 \times 16 \times 6.92 = 13.81\ KN.m/m$

$+Mb = Mb_{DL} + Mb_{LL} = 5.12 + 13.78 = 18.93\ KN.m/m$

Shear On Slab

-Short Direction (from Table 4)

0.8 0.71

0.826 $C_{wa} = \dfrac{0.66 \times (0.826 - 0.8) + 0.71 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.684$

0.85 0.66

$Wa = 0.684 * 25.36 = 17.35\ KN/m2$

$Vu = Wa \times La/2 = 17.35 \times \dfrac{5.7}{2} = 49.43\ KN/m$

$\boldsymbol{Long\ Direction}\ (\ from\ Table\ 4)$

0.8 0.29

0.826 $C_{wb} = \dfrac{0.34 \times (0.826 - 0.8) + 0.29 \times (0.85 - 0.826)}{(0.85 - 0.8)} = 0.316$

0.85 0.34

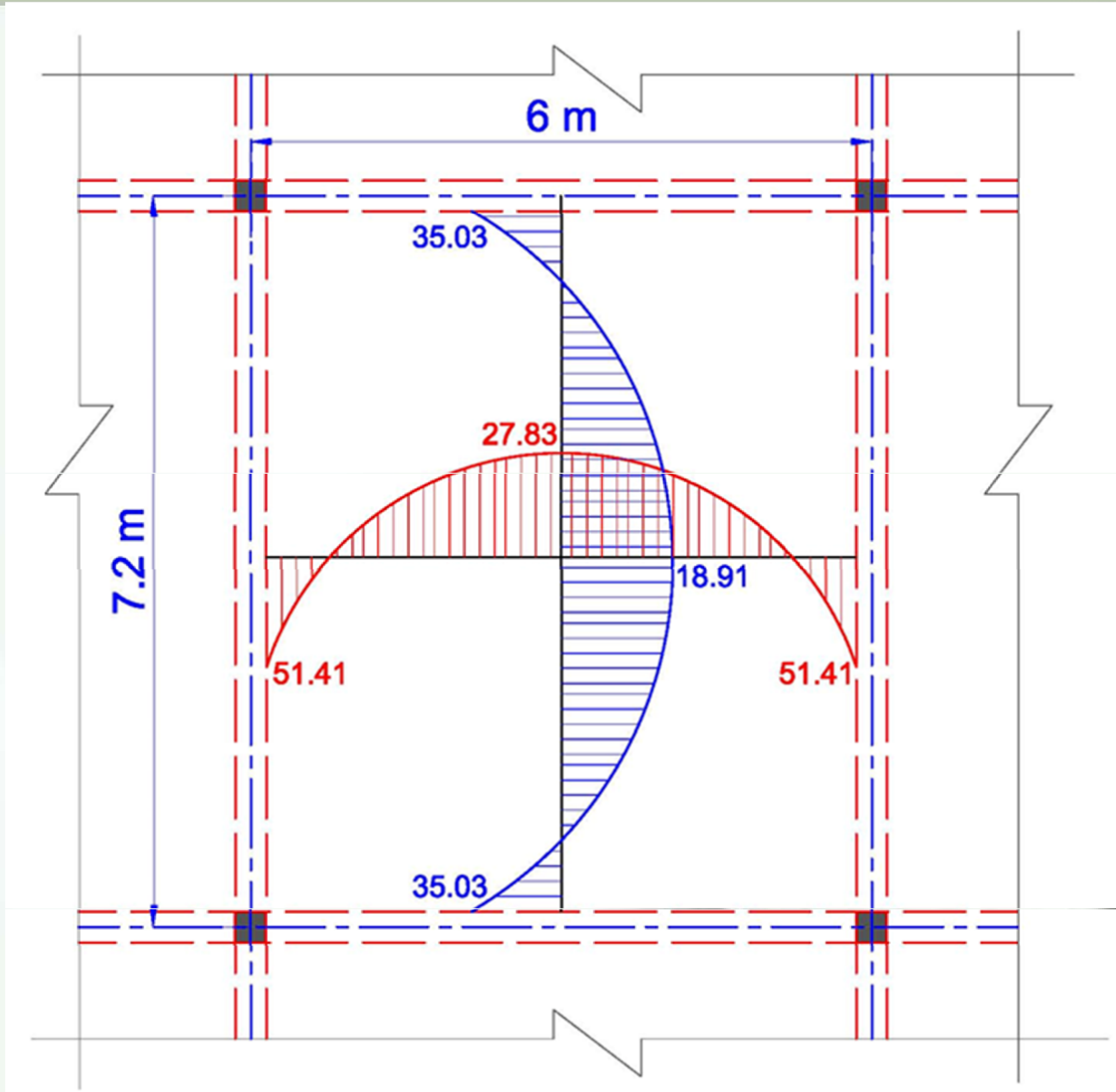
$Wb == 0.316 \times 25.36 = 8.01\ KN/m2$

$Vu = Wb \times \dfrac{Lb}{2} = 8.01 \times 6.9/2 = 27.65 KN/m$

METHOD 3—TABLE 4—RATIO OF LOAD *w* IN *A* and *B* DIRECTIONS FOR SHEAR IN SLAB AND LOAD ON SUPPORTS*

Ratio $m = \frac{A}{B}$		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
1.00	W_A	0.50	0.50	0.17	0.50	0.83	0.71	0.29	0.33	0.67
	W_B	0.50	0.50	0.83	0.50	0.17	0.29	0.71	0.67	0.33
0.95	W_A	0.55	0.55	0.20	0.55	0.86	0.75	0.33	0.38	0.71
	W_B	0.45	0.45	0.80	0.45	0.14	0.25	0.67	0.62	0.29
0.90	W_A	0.60	0.60	0.23	0.60	0.88	0.79	0.38	0.43	0.75
	W_B	0.40	0.40	0.77	0.40	0.12	0.21	0.62	0.57	0.25
0.85	W_A	0.66	0.66	0.28	0.66	0.90	0.83	0.43	0.49	0.79
	W_B	0.34	0.34	0.72	0.34	0.10	0.17	0.57	0.51	0.21
0.80	W_A	0.71	0.71	0.33	0.71	0.92	0.86	0.49	0.55	0.83
	W_B	0.29	0.29	0.67	0.29	0.08	0.14	0.51	0.45	0.17
0.75	W_A	0.76	0.76	0.39	0.76	0.94	0.88	0.56	0.61	0.86
	W_B	0.24	0.24	0.61	0.24	0.06	0.12	0.44	0.39	0.14
0.70	W_A	0.81	0.81	0.45	0.81	0.95	0.91	0.62	0.68	0.89
	W_B	0.19	0.19	0.55	0.19	0.05	0.09	0.38	0.32	0.11
0.65	W_A	0.85	0.85	0.53	0.85	0.96	0.93	0.69	0.74	0.92
	W_B	0.15	0.15	0.47	0.15	0.04	0.07	0.31	0.26	0.08
0.60	W_A	0.89	0.89	0.61	0.89	0.97	0.95	0.76	0.80	0.94
	W_B	0.11	0.11	0.39	0.11	0.03	0.05	0.24	0.20	0.06
0.55	W_A	0.92	0.92	0.69	0.92	0.98	0.96	0.81	0.85	0.95
	W_B	0.08	0.08	0.31	0.08	0.02	0.04	0.19	0.15	0.05
0.50	W_A	0.94	0.94	0.76	0.94	0.99	0.97	0.86	0.89	0.97
	W_B	0.06	0.06	0.24	0.06	0.01	0.03	0.14	0.11	0.03

*A cross-hatched edge indicates that the slab continues across or is fixed at the support; an unmarked edge indicates a support at which torsional resistance is negligible.


6 m
7.2 m
35.03
27.83
18.91
51.41
51.41
35.03

Shear On Beams

-Short Direction

the load transfer from slab with long direction (on short beam)

$= \ 27.65 \ KN/m$

$and \ there \ is \ two \ slab \ from \ both \ side$

$= \ 2 \times 27.65 = 55.3 \ KN$

$Selfwt.of \ beam \ = \ 6.05 \ KN/m$

$Total \ Wua = \ 55.3 + 6.05 = \ 61.35 \ KN/m$

$$Vu = \frac{Wua \times La}{2} = \frac{61.35 \times 5.7}{2} = 174.85KN$$

-Long Direction

the load transfer from slab with short direction (on long beam)

$= \ 49.43 \ KN/m$

$and \ there \ is \ two \ slab \ from \ both \ side$

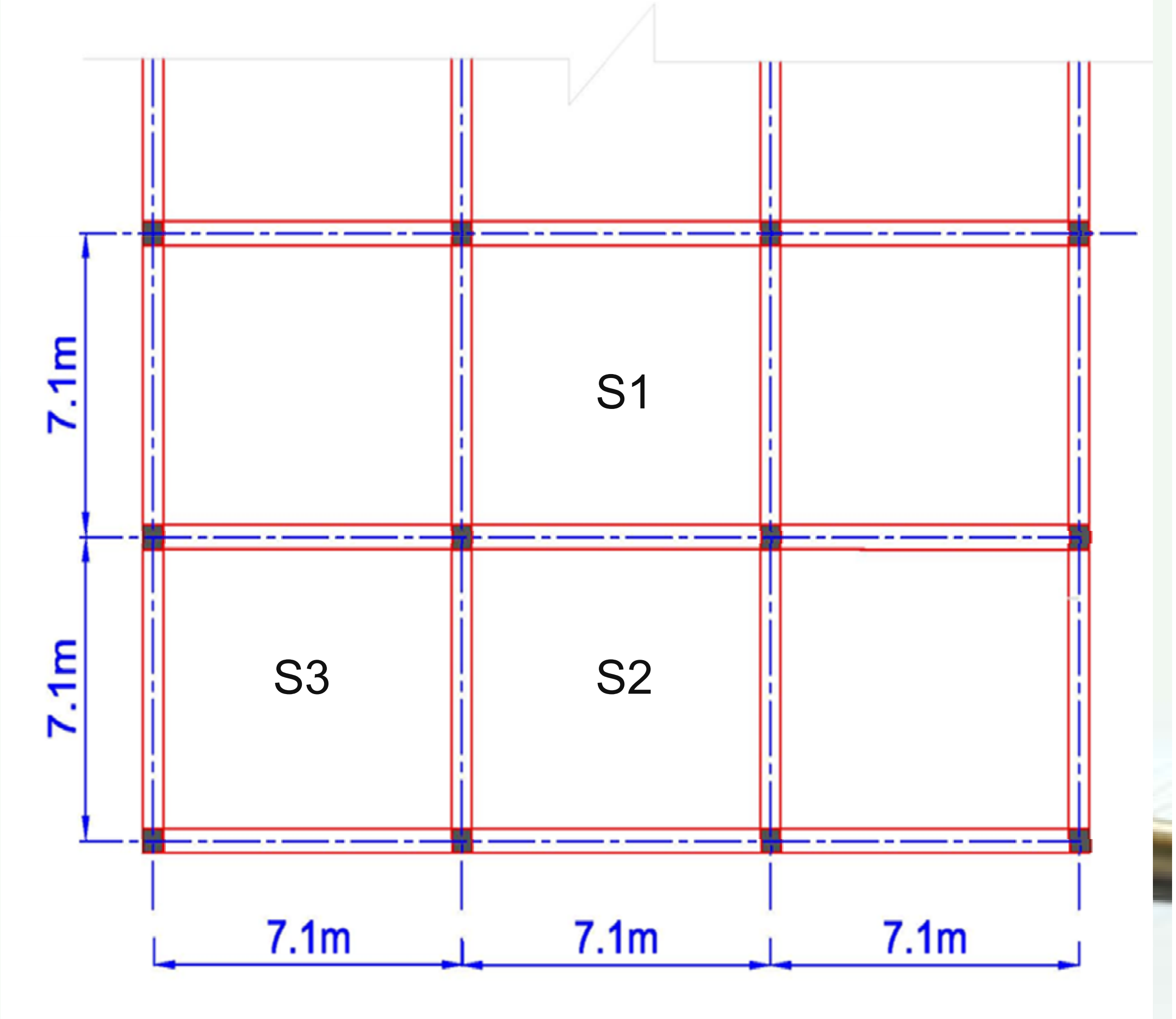
$= \ 2 \times 49.43 \ = 98.86 \ KN$

$Selfwt.of \ beam \ = \ 6.05 \ KN/m$

$Total \ Wub = \ 98.86 + 6.05 = \ 104.91KN/m$

$$Vu = \frac{Wub \times Lb}{2} = \frac{104.91 \times 6.9}{2} = 361.93KN$$

Example (3) : An Apartment building is designed using 6.1*6.1 m Two way slabs system. The live load 2 KN/m2 , the superimposed load (partition loads) is 1.5 KN/m2 and the floor finish load is 2 KN/m2. Design a typical panels. Assume f'c=21MPa, fy =280 Mpa. The column dimension 300* 300 mm and the supporting beams are 300 mm width . Also Design the interior beam.



Sol.

-Slab Thickness

ACI Code 1963 allowed slab thickness

not less than 90 mm

$$t_{min} = \frac{2\,(L + S)}{180} \geq 90\ mm$$

$$= \frac{2(5.8 + 5.8)}{180} = 128.9\ mm$$

$ACI\ Code\ 2014\ allowed\ using\ equation$

$where\ \alpha\ m \geq 2$

$$\beta = \frac{Ln}{Sn} = 1.0$$

$$t_{min} = \frac{5.8 \times \left[0.8 + \left(\frac{280}{1400}\right)\right]}{36 + 9 \times 1}$$

$= 141.5mm\ Use\ t\ or\ h = 150\ mm$

Load On Slab

$D.L\ of\ slab\ =\ 0.15 \times 1 \times 1 \times 24 = 3.6\ KN/m^2$

$Floor\ Finishing\ =\ 2\ KN/m^2$

$Partitions\ =\ 1.5\ KN/m^2$

$Total\ DL\ =\ 7.1\ KN/m^2$

$L.L\ =\ 2\ KN/m^2$

$Wu\ =\ 1.2\ DL +\ 1.6\ LL$

$=\ 1.2 \times 7.1\ +\ 1.6 \times 2 =\ 11.72\ KN/m^2$

Using Method 2

$M =\ ceof. \times Wu \times S$

$From\ table\ 1\ of\ Method\ 2$

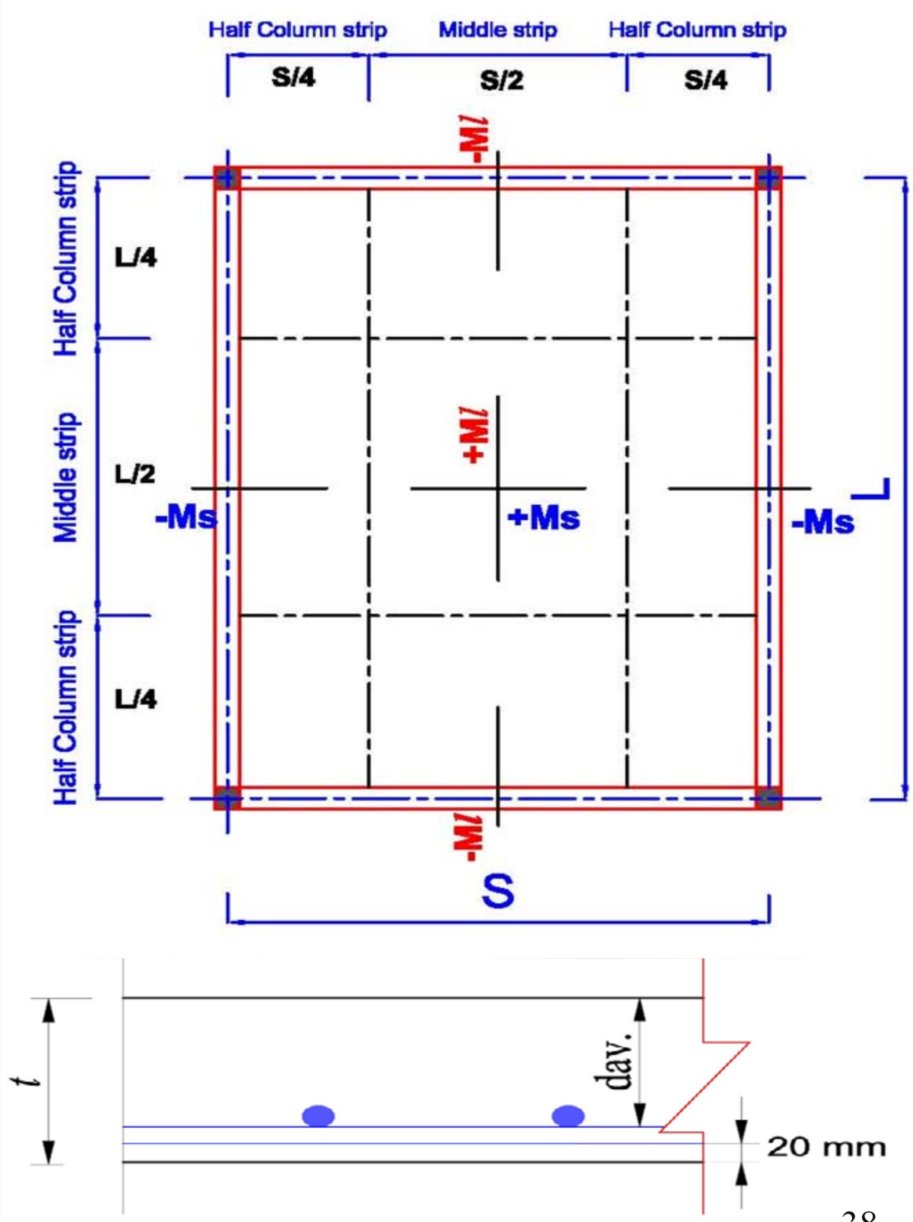
$dav. = h - cover\ - \phi =\ 150 - 20 - 10 \qquad (use\ \phi 10\ mm)$

$=\ 120\ mm$

Notes

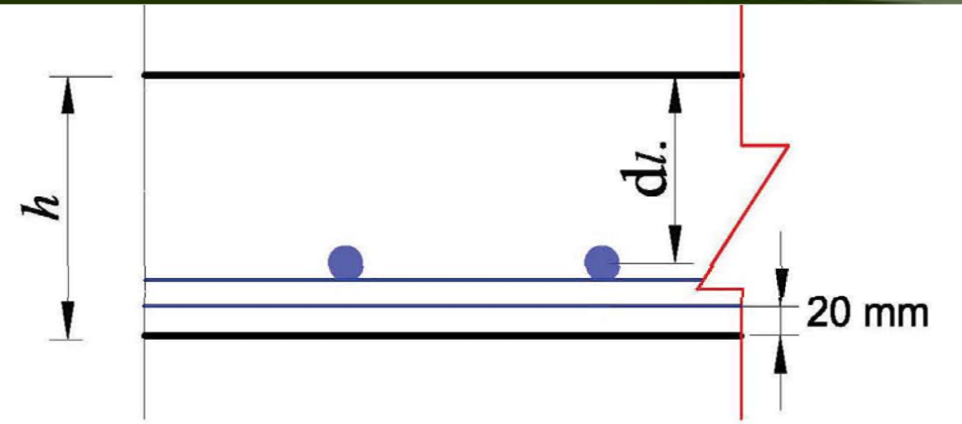
1- For square panel use d average

2- rectangular panel the steel in short direction at bottom layer (large M, d the greater) and for long direction the steel at top layer (d shorter)



Long span

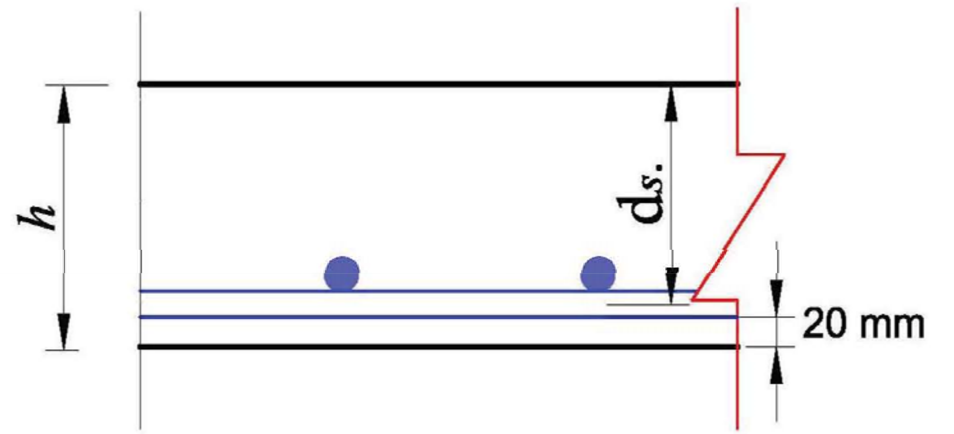
$$d_L = h - 20 - \phi_s - \frac{\phi_L}{2}$$



(a) Effective depth in long direction

short span

$$d_S = h - 20 - \frac{\phi s}{2}$$



(b) Effective depth in short direction

$Choose\ (S3)\ = CASE\ 3$

$$From\ Table\ (1) m\ = \frac{S}{L} = \ 1$$

$Moment\ factors\ for\ both\ Direction\ (\ square\ panel\ )$

$-C\ =\ 0.049$ $\quad$ $Negative\ moment\ Factor\ Discontinuous\ edge$

$-C\ =\ 0.025$ $\quad$ $Negative\ moment\ Factor\ Continuous\ edge$

$+C\ =\ 0.037$ $\quad$ $Positive\ moment\ Factor\ Midspan$

$-Mu\ =\ c\ Wu.S^2 = 0.049 \times 11.72 \times 6.12 =\ 21.37\ KN.m/m$ $\quad$ $Cont.$

$-Mu\ =\ c\ Wu.S^2 = 0.025 \times 11.72 \times 6.12 =\ 10.9\ KN.m/m$ $\quad$ $Discont.$

$+Mu\ =\ c\ Wu.S^2 = 0.037 \times 11.72 \times 6.12 = 16.14\ KN.m/m$ $\quad$ $Mid\ span$

$Mid\ Span$

$$R = \frac{Mu}{\rho b d^2}$$

$$R = \frac{16.14 \times 10^6}{(\ 0.9 \times 1000 \times 120^2)} = 1.245$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR}{fy}}\right)$$

$$m\ = \frac{fy}{0.85 * fc} =\ 15.68$$

$$\rho\ = \left(\frac{1}{15.68}\right) \times \left(\ 1 - \ \sqrt{1 - \frac{(2 \times 15.68 \times 1.245)}{280}}\right) = 0.00461$$

Method 2
Table 1

METHOD 2—TABLE 1—MOMENT COEFFICIENTS

Moments	Short span, Values of m: 1.0	0.9	0.8	0.7	0.6	0.5 and less	Long span, all values of m
Case 1—Interior panels							
Negative moment at—							
Continuous edge	0.033	0.040	0.048	0.055	0.063	0.083	0.033
Discontinuous edge	—	—	—	—	—	—	—
Positive moment at midspan	0.025	0.030	0.036	0.041	0.047	0.062	0.025
Case 2—One edge discontinuous							
Negative moment at—							
Continuous edge	0.041	0.048	0.055	0.062	0.069	0.085	0.041
Discontinuous edge	0.021	0.024	0.027	0.031	0.035	0.042	0.021
Positive moment at midspan	0.031	0.036	0.041	0.047	0.052	0.064	0.031
Case 3—Two edges discontinuous							
Negative moment at—							
Continuous edge	0.049	0.057	0.064	0.071	0.078	0.090	0.049
Discontinuous edge	0.025	0.028	0.032	0.036	0.039	0.045	0.025
Positive moment at midspan	0.037	0.043	0.048	0.054	0.059	0.068	0.037
Case 4—Three edges discontinuous							
Negative moment at—							
Continuous edge	0.058	0.066	0.074	0.082	0.090	0.098	0.058
Discontinuous edge	0.029	0.033	0.037	0.041	0.045	0.049	0.029
Positive moment at midspan	0.044	0.050	0.056	0.062	0.068	0.074	0.044
Case 5—Four edges discontinuous							
Negative moment at—							
Continuous edge	—	—	—	—	—	—	—
Discontinuous edge	0.033	0.038	0.043	0.047	0.053	0.055	0.033
Positive moment at midspan	0.050	0.057	0.064	0.072	0.080	0.083	0.050

$As = \rho . b . d = 0.00461 \times 1000 \times 120 = 553\ mm^2/m$

$Use\ \phi\ 10\ mm$

$$S = \frac{78 \times 1000}{553} = 142\ mm$$

$Use\ \phi\ 10\ mm\ at\ 140\ mm\ c/c$

$As \min = \rho . b . h\ (mm^2)$

$\rho min = 0.0018$

$As \min. = 0.0018 \times 1000 \times 150 = 270\ mm\ 2/\ m < As\ Provide\ (OK)$

$S \max = 2 \times h = 300\ or\ 450\ mm \quad at\ critical\ section$ ACI (8.7.2.2)

$Use\ \phi\ 10\ mm\ @\ 140\ mm$

$$Column\ Strip\ Moment = \frac{2}{3}\ M\ mid = 16.14 \times \frac{2}{3} = 10.76\ KN.m/m$$

$Or\ can\ use\ the\ spacing\ of$

$1.5 * middle\ strip\ spacing = 213\ mm\ C/C < 2\ h = 300\ mm$

$Use\ Use\ \phi\ 10\ mm\ @\ 210\ mm$

Negative Moment

$-Discontinues\ edge$

- M= 10.9 KN.m/m

$$R = \frac{Mu}{\phi b d^2}$$

$$R = \frac{10.9 \times 10^6}{0.9 \times 1000 \times 120^2} = 0.841$$

$$m = \frac{fy}{0.85 \times fc} = 15.68$$

$$\rho = \frac{1}{15.68}\left(1 - \sqrt{1 - \frac{(2 \times 15.68 \times 0.841}{280}}\right) = 0.00308$$

$As = \rho . b . d = 0.00308 \times 1000 \times 120 = 370\ mm^2/m$

$Use\ \phi\ 10\ mm$

$$S = \frac{78 \times 1000}{370} = 211mm\ , \qquad Use\ \phi\ 10\ mm\ at\ 210\ mm\ c/c$$

−Continuous Edge

$$R = \frac{Mu}{\phi b d 2}$$

$$= \frac{21.37 \times 10^6}{(0.9 \times 1000 \times 120^2)} = 1.649$$

$$\rho = \left(\frac{1}{15.68}\right) \times \left(1 - \sqrt{1 - \frac{2 \times 15.68 \times 1.649}{280}}\right)$$

$= 0.006189$

$As = \rho . b . d = 0.006189 \times 1000 \times 120 = 742 mm^2/m$

$Use\ \phi\ 10\ mm$

$S = 78 \times 1000 / 742 = 105mm\ Use\ \phi\ 10\ mm\ at\ 100\ mm\ c/c$

Note: The reinforcement detail for long Direction same as of short direction cause the panel is square (L = S)

Check for Shear

The shear force on slab can be calculated according to (same in both direction):

$$V = \frac{Wu.S}{2} \qquad at\ center\ of\ support$$

$$= \frac{11.72 \times 6.1}{2} = 35.75\ KN/m$$

$$Vud = Vu - Wu \times \frac{0.3}{2} - Wu \times d$$

$$= 35.75 - 11.72 \times \frac{0.3}{2} - 11.72 \times 0.12 = 32.59\ KN/m$$

$$\phi\ Vc = \phi \times 0.17 \sqrt{f'c} \times b \times d = 0.75 \times 0.17 \times \sqrt{21} \times 1000 \times 120 = 70.11\ KN/m$$

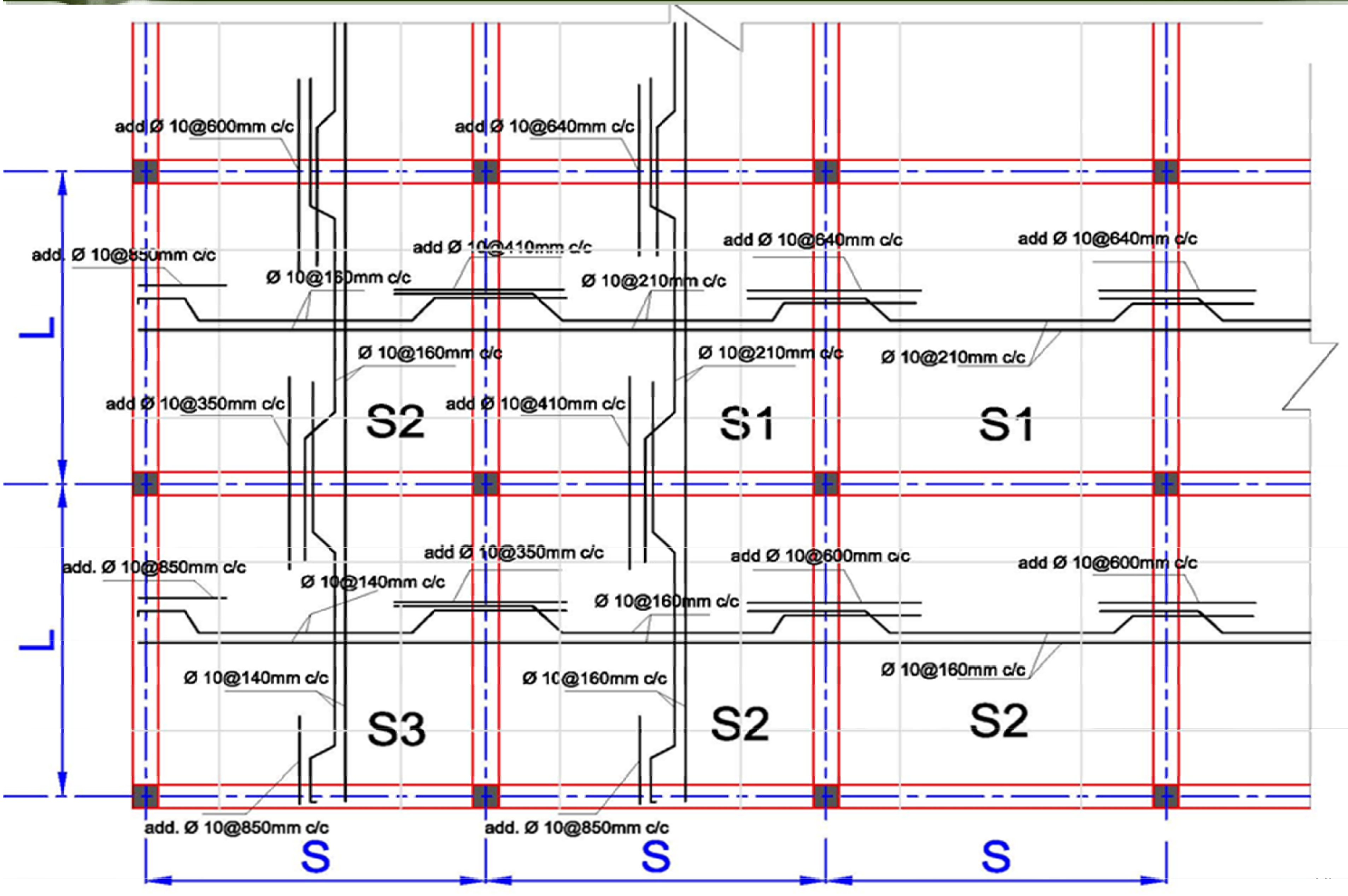
$$\phi\ Vc > Vud \qquad (\ OK\ section\ is\ safe\ )$$

No.	Detail	Interior Panal (S1)					
		Short Span			Long Span		
		(-M) Cont.	(+M) Mid	(-M) Discont.	(-M) Discont.	(+M) Mid	(-M) Discont.
1	Mu x 106 (N.mm/m)	14.40	10.90	14.40	14.40	10.90	14.40
2	d (mm)	120	120	120	120	120	120
3	m=	15.69	15.69	15.69	15.69	15.69	15.69
4	Rn=	1.111	.841	1.111	1.111	0.841	1.111
5	ρ=r.b.h (mm2)	0.0041	0.0031	0.0041	0.0041	0.0031	0.0041
6	As (calculated)	492.0	369.4	492.0	492.0	369.4	492.0
7	As(min)= 0.0018 b.h	270	270	270	270	270	270
8	As(choosed)=	492	369	492	492	369	492
9	S=1000*Ab/As (mm)	160	213	160	160	213	160
10	S(max)= 2*h=300 or 450 mm	300	300	300	300	300	300
11	S(choosed)=	160	212.6	159.6	160	213	160
12	Use S=	150	210	150	150	210	150

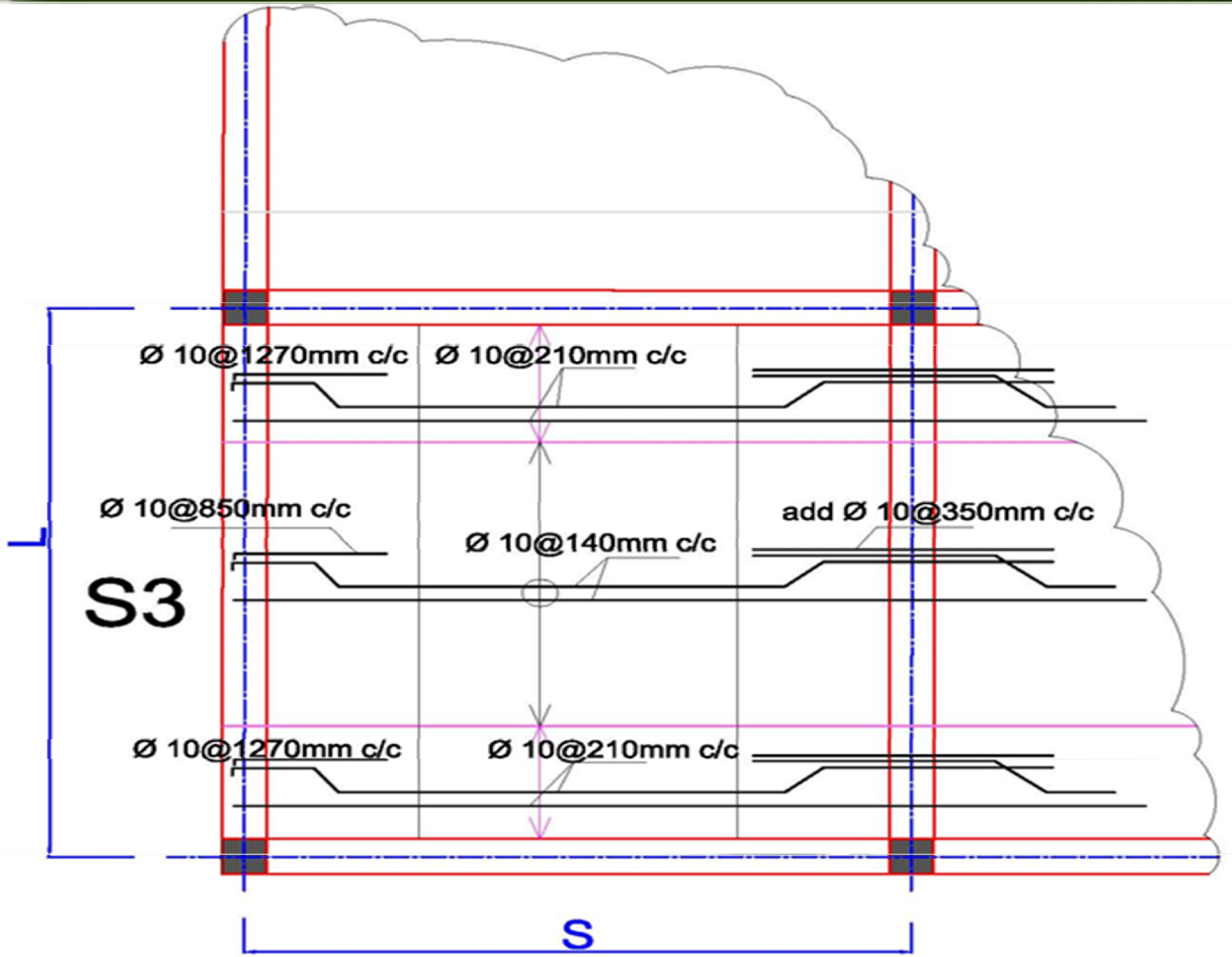
By Prof. Dr. Haleem K. Hussain

No.	Detail	Interior Panal (S2)					
		Short Span			Long Span		
		(-M) Cont.	(+M) Mid	(-M) Discont.	(-M) Discont.	(+M) Mid	(-M) Discont.
1	Mu x 106 (N.mm/m)	9.16	13.52	17.88	17.88	13.52	17.88
2	d (mm)	120	120	120	120	120	120
3	m=	15.69	15.69	15.69	15.69	15.69	15.69
4	Rn=	0.707	1.043	1.380	1.380	1.043	1.380
5	As =ρ.b.h (mm2)	0.0026	0.0038	0.0051	0.0051	0.0038	0.0051
6	As (calculated)	309	461	616	616	461	616
7	As(min)= 0.0018 b.h	270	270	270	270	270	270
8	As(choosed)=	309	461	616	616	461	616
9	S=1000*Ab/As (mm)	254	170	127	127	170	127
10	S(max)= 2*h=300 or 450 mm	300	300	300	300	300	300
11	S(choosed)=	254.0	170.4	127.5	127	170.4	127.5
12	Use S=	250	160	120	120	160	120

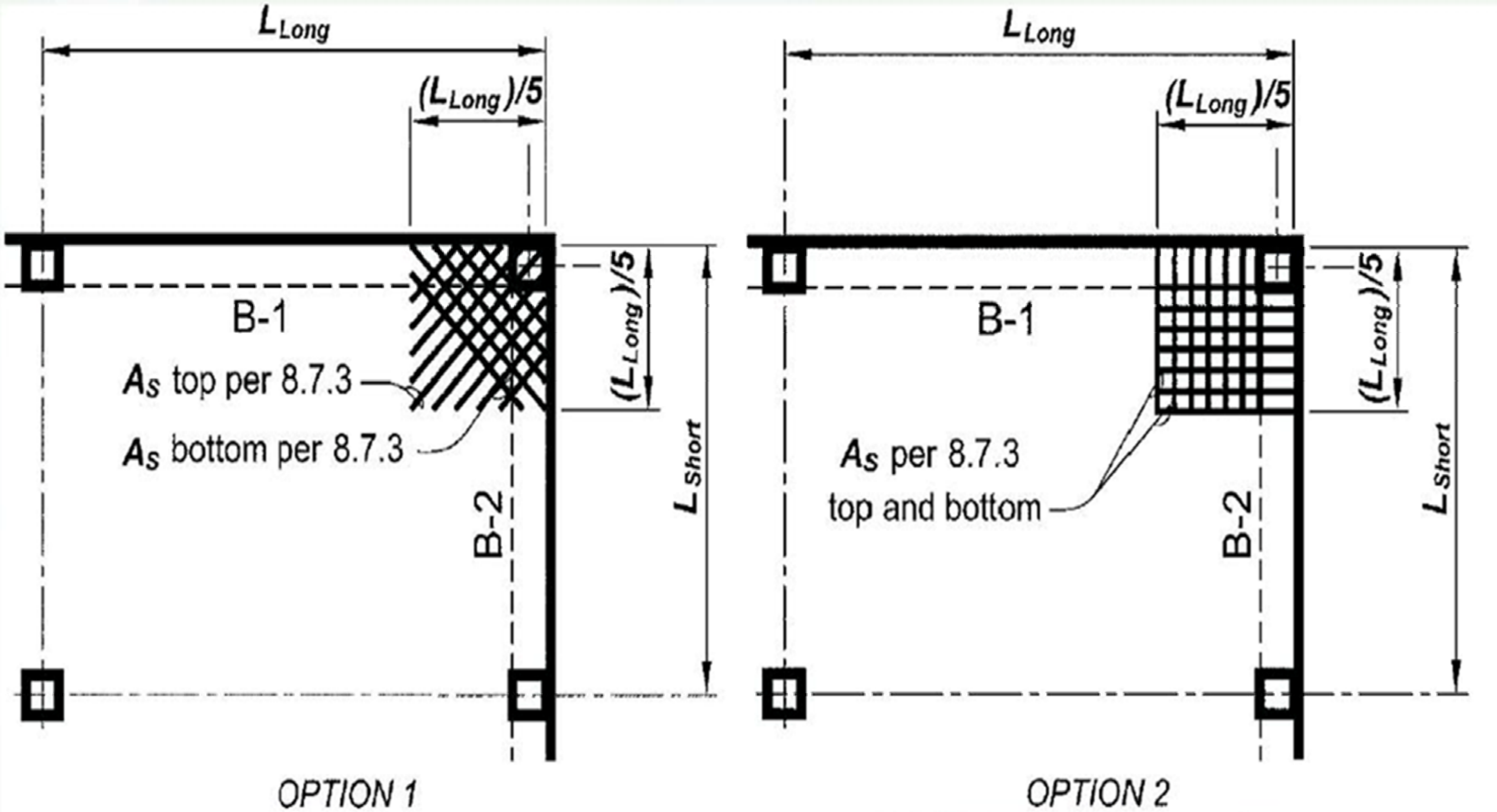
No.	Detail	Interior Panal (S3)					
		Short Span			Long Span		
		(-M) Cont.	(+M) Mid	(-M) Discont.	(-M) Discont.	(+M) Mid	(-M) Discont.
1	Mu x 106 (N.mm/m)	10.90	16.14	21.37	10.90	16.14	21.37
2	d (mm)	120	120	120	120	120	120
3	m=	15.69	15.69	15.69	15.69	15.69	15.69
4	Rn=	0.841	1.245	1.649	0.841	1.245	1.649
5	As =ρ.b.h (mm2)	0.0031	0.0046	0.0062	0.0031	0.0046	0.0062
6	As (calculated)	369	554	743	369	554	743
7	As(min)= 0.0018 b.h	270	270	270	270	270	270
8	As(choosed)=	369	554	743	369	554	743
9	S=1000*Ab/As (mm)	213	142	106	213	142	106
10	S(max)= 2*h=300 or 450 mm	300	300	300	300	300	300
11	S(choosed)=	212.6	141.8	105.7	213	141.8	105.7
12	Use S=	210	140	100	210	140	100

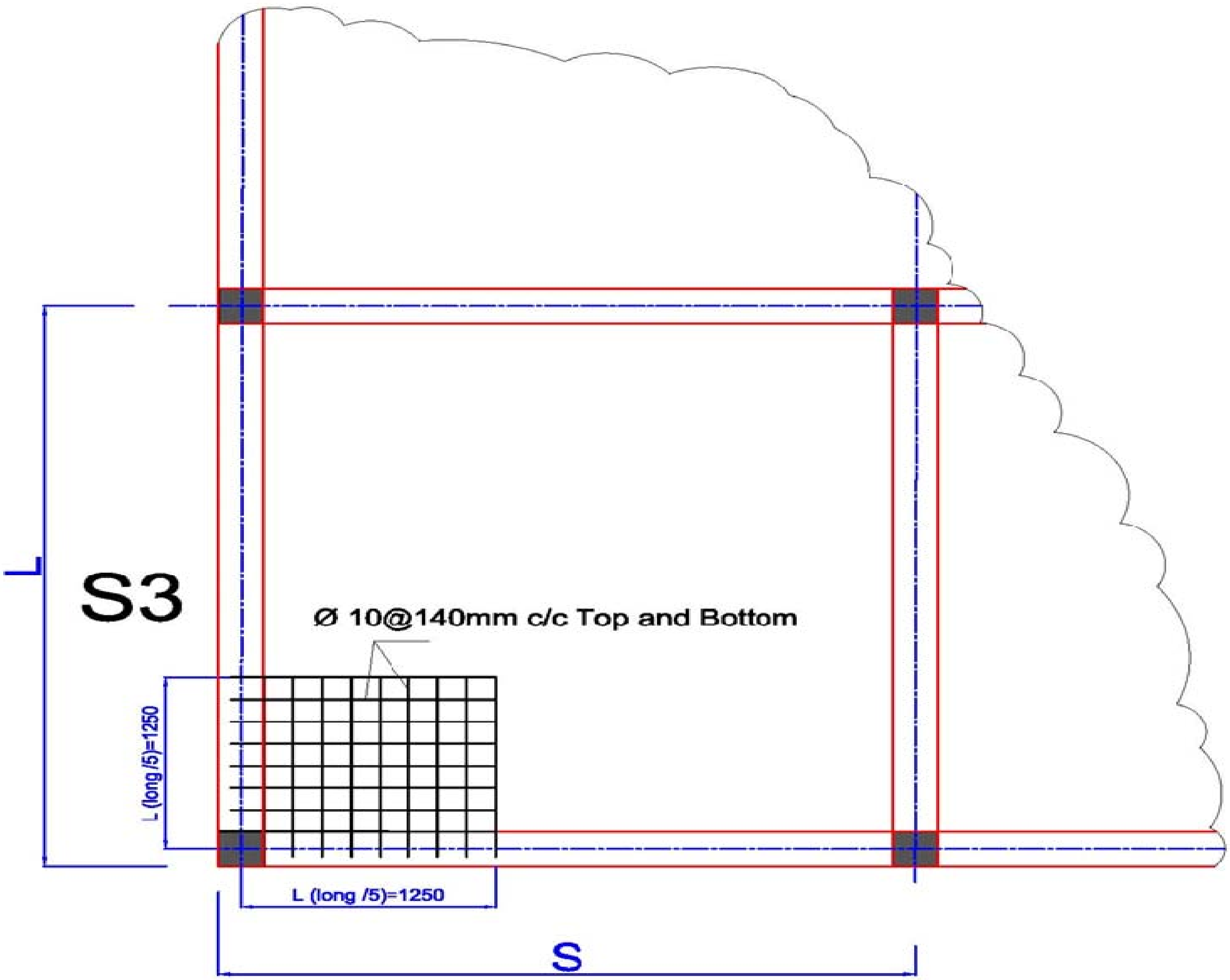


add Ø 10@600mm c/c
add Ø 10@640mm c/c
add. Ø 10@850mm c/c
Ø 10@160mm c/c
add Ø 10@410mm c/c
Ø 10@210mm c/c
add Ø 10@640mm c/c
add Ø 10@640mm c/c
Ø 10@160mm c/c
Ø 10@210mm c/c
Ø 10@210mm c/c
add Ø 10@350mm c/c
S2
add Ø 10@410mm c/c
S1
S1
L
add. Ø 10@850mm c/c
add Ø 10@350mm c/c
Ø 10@140mm c/c
add Ø 10@600mm c/c
add Ø 10@600mm c/c
Ø 10@160mm c/c
Ø 10@160mm c/c
Ø 10@140mm c/c
Ø 10@160mm c/c
S3
S2
S2
L
add. Ø 10@850mm c/c
add. Ø 10@850mm c/c
S
S
S


Ø 10@1270mm c/c
Ø 10@210mm c/c
Ø 10@850mm c/c
add Ø 10@350mm c/c
Ø 10@140mm c/c
L
S3
Ø 10@1270mm c/c
Ø 10@210mm c/c
S

Corner slab reinforcement detail




S3
Ø 10@140mm c/c Top and Bottom
L (long /5)=1250
L (long /5)=1250
L
S

Thank You

………….To be Continued