MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH

SHATT AL-ARAB UNIVERSITY COLEGE

MEDICAL INSTRUMENTATION ENGINEERING TECHINQUE

For

Students of first semester

_By_
Lecturer: Dr. Abduljabbar H. Hammadi
Ph. D. ELectrical Engineering

Fundamental of electrical engineering (DC)

Department of Medical Instrumentation Engineering Techniques

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Chapter one

Objective: after the end of courses the student will be able transform between units and definitions of main terms in circuit.

1- SI units:

The system of units used in engineering and science is the (International system of units), usually abbreviated to SI units, and is based on the metric system.

Quantity	Quantity Symbol	Unit	Unit Symbol
Length	l	metre	m
Mass	m	kilogram	kg
Time	t	second	s
Velocity	v	metres per second	m/s or $m\,s^{-1}$
Acceleration	a	metres per second squared	m/s^2 or $m\,s^{-2}$
Force	F	newton	N
Electrical charge or quantity	Q	coulomb	C
Electric current	I	ampere	A
Resistance	R	ohm	Ω
Conductance	G	siemen	S
Electromotive force	E	volt	V
Potential difference	V	volt	V
Work	W	joule	J
Energy	E (or W)	joule	J
Power	P	watt	W

SI units may be made larger or smaller by using prefixes which denote multiplication or division by a particular amount.

Prefix	Name	Meaning
M	mega	multiply by 1 000 000 (i.e. $\times 10^6$)
k	kilo	multiply by 1000 (i.e. $\times 10^3$)
m	milli	divide by 1000 (i.e. $\times 10^{-3}$)
μ	micro	divide by 1 000 000 (i.e. $\times 10^{-6}$)
n	nano	divide by 1 000 000 000 (i.e. $\times 10^{-9}$)
p	pico	divide by 1 000 000 000 000 (i.e. $\times 10^{-12}$)

Charge: The unit of charge is the coulomb (C) where one coulomb is one ampere second. (1 coulomb = 6.24 x 10^{18} electrons).

This charge, in coulombs $Q = It$ where *I* is the current in amperes and *t* is the time in seconds. For example, if a current of 5 A flows for 2 minutes, find the quantity of electricity transferred.

Quantity of electricity $Q = It$ coulombs

$$I = 5A, t = 2 \times 60 = 120s, hence\ Q = 5 \times 120 = 600C$$

Force: The unit of forceis the newton (N) where one newton is one kilogram metre per second squared. Thus force, in newtons $F = ma$, where m is the mass in kilograms and *a* is the acceleration in metres per second squared. Gravitational force, or weight, is *mg*, where $g = 9.81\ m/s^2$.

For example: A mass of 5000 g is accelerated at 2 m/s^2 by a force. Determine the force needed. Force = mass x acceleration = 5kg x 2m/s^2 = 10 kg m/s^2 = 10 N.

Work: The unit of work or energy is the joule (J) where one joule is one newton metre. Thus work done on a body, in joules, *W*=*Fs* where *F* is the force in newtons and *s* is the distance in metres moved by the body in the direction of the force.

Power: The unit of power is the watt (*W*) where one watt is one joule per second. Power is defined as the rate of doing work or transferring energy. Thus, power, in watts, *P* = *W*/*t*, where *W* is the work done or energy transferred, in joules, and *t* is the time, in seconds. Thus, energy, in joules, *W*=*Pt*

Example: A mass of 1000 kg is raised through a height of 10 m in 20 s. What is (a) the work done and (b) the power developed?

(a) Work done = force × distance

and force = mass × acceleration

Hence, work done $= (1000\,\text{kg} \times 9.81\,\text{m/s}^2) \times (10\,\text{m})$

$= 98\,100\,\text{Nm}$

$= \mathbf{98.1\,kNm}$ or $\mathbf{98.1\,kJ}$

(b) $\text{Power} = \dfrac{\text{work done}}{\text{time taken}} = \dfrac{98100\,\text{J}}{20\,\text{s}}$

$= 4905\,\text{J/s} = \mathbf{4905\,W}$ or $\mathbf{4.905\,kW}$

Now try the following exercise:

1 What quantity of electricity is carried by 6.24×10^{21} electrons? [1000 C]

2 In what time would a current of 1 A transfer a charge of 30 C? [30 s]

3 A current of 3 A flows for 5 minutes. What charge is transferred? [900 C]

4 How long must a current of 0.1 A flow so as to transfer a charge of 30 C? [5 minutes]

5 What force is required to give a mass of 20 kg an acceleration of 30 m/s^2? [600 N]

6 Find the accelerating force when a car having a mass of 1.7 Mg increases its speed with a constant acceleration of 3 m/s^2 [5.1 kN]

7 A force of 40 N accelerates a mass at 5 m/s^2. Determine the mass. [8 kg]

8 Determine the force acting downwards on a mass of 1500 g suspended on a string. [14.72 N]

9 A force of 4 N moves an object 200 cm in the direction of the force. What amount of work is done? [8 J]

10 A force of 2.5 kN is required to lift a load. How much work is done if the load is lifted through 500 cm? [12.5 kJ]

11 An electromagnet exerts a force of 12 N and moves a soft iron armature through a distance of 1.5 cm in 40 ms. Find the power consumed. [4.5 W]

12 A mass of 500 kg is raised to a height of 6 m in 30 s. Find (a) the work done and (b) the power developed. [(a) 29.43 kNm (b) 981 W]

Electrical potential and e.m.f.: The unit of electric potential is the volt (V), where one volt is one joule per coulomb. One volt is defined as the difference in potential between two points in a conductor which, when carrying a current of one ampere, dissipates a power of one watt, i.e.

$$Volts = \frac{watts}{amperes} = \frac{Joules/second}{amperes} = \frac{Joules}{amperes\ second} = \frac{Joules}{Coulombs}$$

A change in electric potential between two points in an electric circuit is called apotential difference.

Resistance and conductance: The unit of electric resistance is the ohm (Ω), where one ohm is one volt per ampere. It is defined as the resistance between two points in a conductor when a constant electric potential of one volt applied at the two points produces a current flow of one ampere in the conductor. Thus, resistance, in ohms

$$R = \frac{V}{I}$$

The reciprocal of resistance is called conductance and is measured in siemens (S). Thus

conductance, in siemens

$$G = \frac{1}{R}$$

Electrical power and energy: When a direct current ofIamperes is flowing in an electric circuit and the voltage across the circuit is *V* volts, then power, in watts $P=VI$

Electrical energy = Power x time = *VIt* joules Although the unit of energy is the joule, when dealing with large amounts of energy, the unit used is the kilowatt hour (kWh) where

$$1kWh = 1000\ watt\ hour = 1000 \times 3600\ watt\ seconds\ or\ joules = 3600000\ J$$

Example: An electric heater consumes 1.8 MJ when connected to a 250 V supply for

30 minutes. Find the power rating of the heater and the current taken from the supply.

$$\text{Power} = \frac{\text{energy}}{\text{time}} = \frac{1.8 \times 10^6\ \text{J}}{30 \times 60\ \text{s}}$$
$$= 1000\ \text{J/s} = 1000\ \text{W}$$

i.e. **power rating of heater = 1 kW**

$$\text{Power } P = VI, \text{ thus } I = \frac{P}{V} = \frac{1000}{250} = 4\ \text{A}$$

Hence the current taken from the supply is 4 A.

Now try the following exercise:

1 Find the conductance of a resistor of resistance (a) 10 Ω (b) 2 kΩ (c) 2 mΩ
[(a) 0.1 S (b) 0.5 mS (c) 500 S]

2 A conductor has a conductance of 50 μS. What is its resistance? [20 kΩ]

3 An e.m.f. of 250 V is connected across a resistance and the current flowing through the resistance is 4 A. What is the power developed? [1 kW]

4 450 J of energy are converted into heat in 1 minute. What power is dissipated? [7.5 W]

5 A current of 10 A flows through a conductor and 10 W is dissipated. What p.d. exists across the ends of the conductor? [1 V]

6 A battery of e.m.f. 12 V supplies a current of 5 A for 2 minutes. How much energy is supplied in this time? [7.2 kJ]

7 A d.c. electric motor consumes 36 MJ when connected to a 250 V supply for 1 hour. Find the power rating of the motor and the current taken from the supply. [10 kW, 40 A]

Ohms law :

This law applies to electric conduction through good conductors and may be stated as follows :
The ratio of potential difference (V) between any two points on a conductor to the current (I) flowing between them , is constant ,
in other words ,

$$\frac{V}{} = \text{constant}, \text{ or } \qquad \frac{V}{I} = R\ I$$

Resistance : It is defined as the property of a material due to which it oppose the flow of electrons through it . The unit of resistance is ohm (Ω) . The resistance (R) offered by a conductor depends on the following factors :

1. **It varies directly as its length (L) .**
2. **It varies inversely as the cross sectional area (A) of the conductor .**
3. **It depends on the nature of the material .**
4. **It also depends on the temperature of conductor .**

Neglecting the last factor for the time being , we can say that :

$$R \ \alpha \ \frac{L}{A}$$

$$R = \rho \ \frac{L}{A}$$

Where :

R is the resistance of the conductor (Ω) . L is the length of the conductor (m) .
A is the cross sectional area of the conductor (m^2).
ρ is a constant depending on the nature of the material of the conductor and known as its specific resistance (Ω .m) .

Example : Calculate the resistance of 1 km cable composed of 19 strands of similar alloy conductors , each strand being 1.32 mm in diameter . Resistivity of alloy may be taken as 1.72×10^{-8} Ω . m .

Sol.

$$A = \frac{\pi d^2}{4}$$

$$= \frac{3.14 \times (1.32 \times 10^{-3})^2}{4} = 13.67 \times 10^{-7}\ m^2$$

Total cross sectional area of the cable = $19 \times 13.67 \times 10^{-7}\ m^2$

$$R = \rho \frac{L}{A}$$

$$= \frac{1.72 \times 10^{-8} \times 1000}{19 \times 13.67 \times 10^{-7}} = 0.66\ \Omega$$

Open and short circuit in series circuits :

Objective:the studenta able to regocnize between open andshort cct.and the deffrence in draw of cct.

1. Open circuit :

In this case there is no current flows through the circuit as shown in fig. 1 .

$$I = 0$$

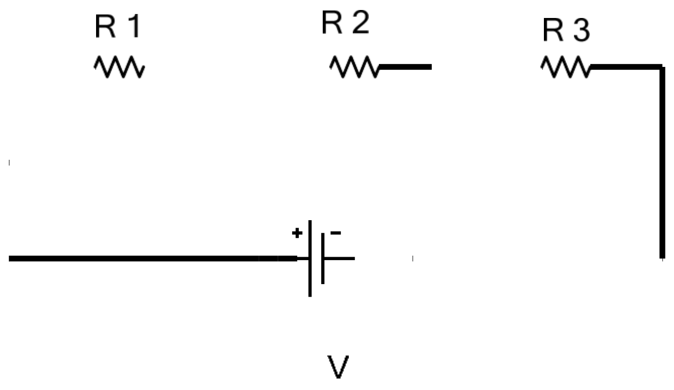


Fig. 1

2. Short circuit :

If the resistance is short circuited , the current will flow through the short circuit (no current flows through the shorted resistance)
as shown in fig. 2 .

$$I = \frac{V}{R_1 + R_2}$$

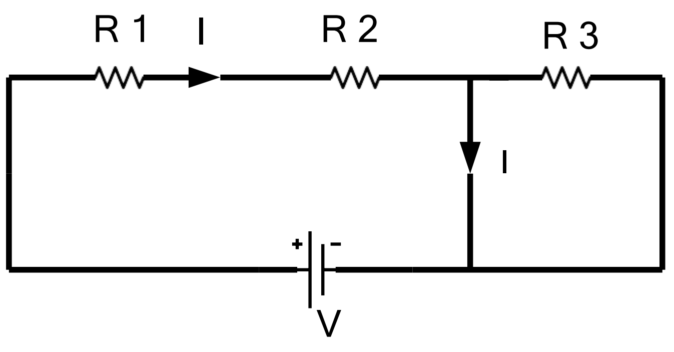


Fig.2

<u>Open and short circuit in parallel circuits</u> :

1. **Open circuit :**

In this case , there is no current flow in the open branch as shown in fig. 3 .

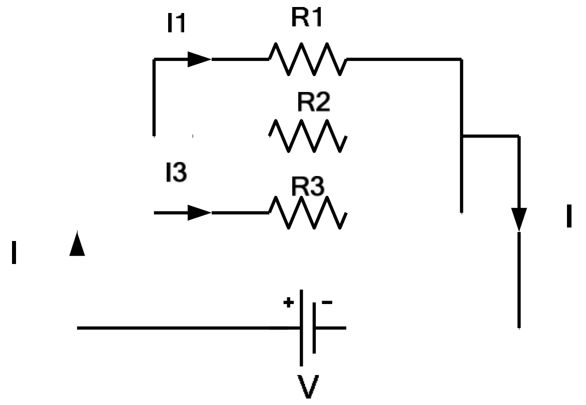


Fig. 3

$I_2 = 0$

$$I = I_1 + I_3$$

2. **short circuit :**

In this case , there is no current flow through R_1 , R_2 and R_3 because the total current (I) pass through the short circuit as shown in fig. 4 .

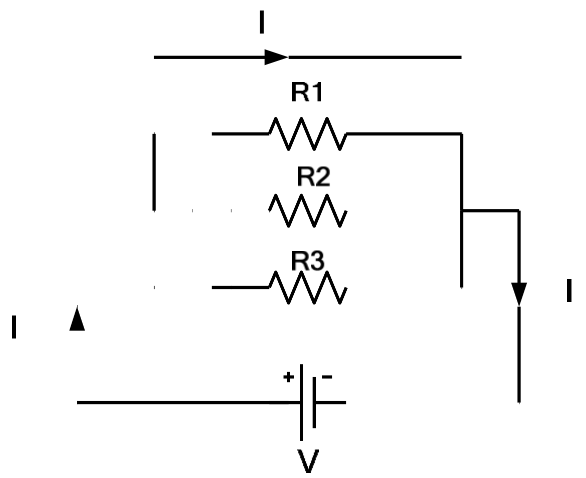


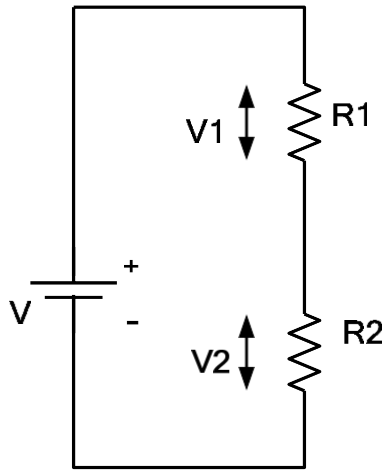
Fig. 4

$$I = \frac{V}{r_i}$$

Where r_i is the internal resistance of the battery .

Voltage divider rule (V.D.R) **:**

In series circuits , voltage across any resistance could be obtained in terms of total voltage as follows :Voltage across resistance equal to the total voltage multiply by the value of this resistance divided by the sum of all resistances .



$$V_1 = V \times \frac{R_1}{R_1 + R_2}$$

$$V_2 = V \times \frac{R_2}{R_1 + R_2}$$

Example : Using voltage divider rule (V.D.R.) ,find V_1 , V_2 , V_3 And V ' from fig. 5.

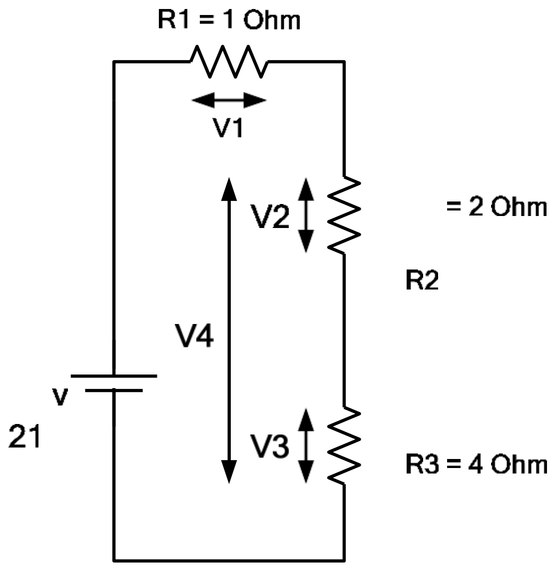


Fig. 5

$$V_1 = V \times \frac{R_1}{R_1 + R_2 + R_3} = 21 \times \frac{1}{1 + 2 + 4} = 3 \text{ v}$$

$$V_2 = V \times \frac{R_2}{R_1 + R_2 + R_3} = 21 \times \frac{2}{1 + 2 + 4} = 6 \text{ v}$$

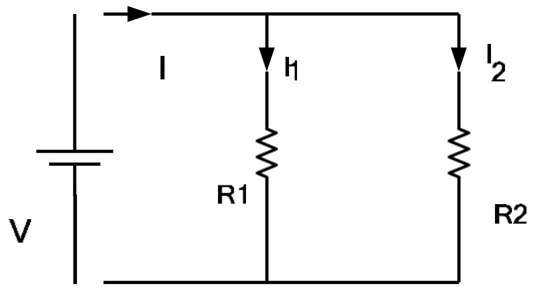
$$V_3 = V \times \frac{R_3}{R_1 + R_2 + R_3} = 21 \times \frac{4}{1 + 2 + 4} = 12 \text{ v}$$

$$V_4 = V \times \frac{R_2 + R_3}{R_1 + R_2 + R_3} = 21 \times \frac{2 + 4}{1 + 2 + 4} = 18 \text{ v}$$

Current divider rule (C.D.R) :

Objective:the student able to find current in parallel case.

In parallel circuits , branch current could be obtained in terms of the total current as follows :
Branch current equal to the total current multiply by the resistance of other branch divided by the sum of all resistances .



$$I_1 = I \times \frac{R_2}{R_1 + R_2}$$

$$I_2 = I \times \frac{R_1}{R_1 + R_2}$$

Example : Using current divider rule (C.D.R.) , calculate I_1 , I_2 and I_3 from fig. 6 .

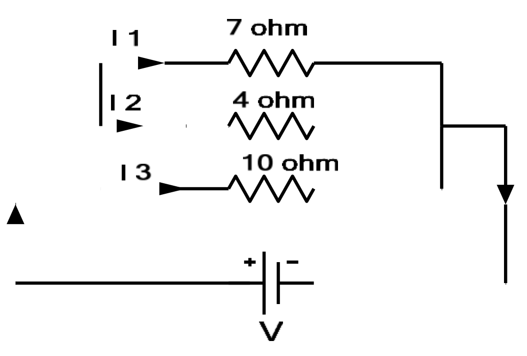


Fig. 6

To find I_1 , the other resistances are (4 // 10) .

$$\frac{4 \times 10}{4 + 10} = 2.857\ \Omega$$

$$I_1 = 20 \times \frac{2.857}{7 + 2.857} = 5.796\,A$$

To find I_2 , the other resistances are (7 // 10) .

$$\frac{7 \times 10}{7 + 10} = 4\ \Omega$$

$$I_2 = 20 \times \frac{4}{4 + 4} = 10\ A$$

To find I_3 , the other resistances are (7 // 4) .

$$\frac{7 \times 4}{7 + 4} = 2.545\ \Omega$$

$$I_3 = 20 \times \frac{2.545}{2.545 + 10} = 4.057\ A$$

Kirchhoffs laws :

Objective: the student able yo find voltages and currents by easy method.

1. kirchhoffs voltage law :

The algebraic sum of voltages in any closed loop is zero .

$\sum V = 0$

Now , from fig. 1 , there are three equations according to kirchhoffs voltage law .

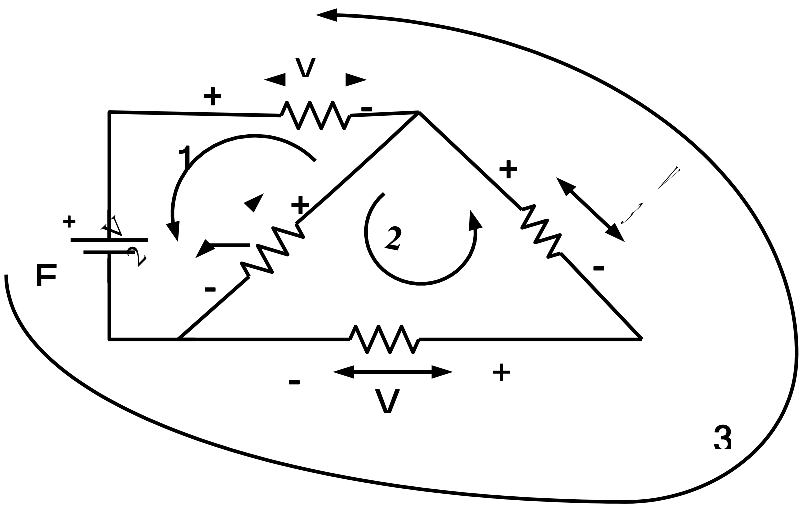


Fig. 1

Loop 1 :

$E - V_1 - V_2 = 0$
$E = V_1 + V_2$ ------- (1)

Loop 2:

$V_2 - V_3 - V_4 = 0$
$V_2 = V_3 + V_4$ -------- (2)

Loop 3 :

$E - V_1 - V_3 - V_4 = 0$
$E = V_1 + V_3 + V_4$ --------- (3)

<u>Example</u> : For the circuit shown in fig. 2 ,using kirchhoffs voltage law ,find V_1 and V_2 .

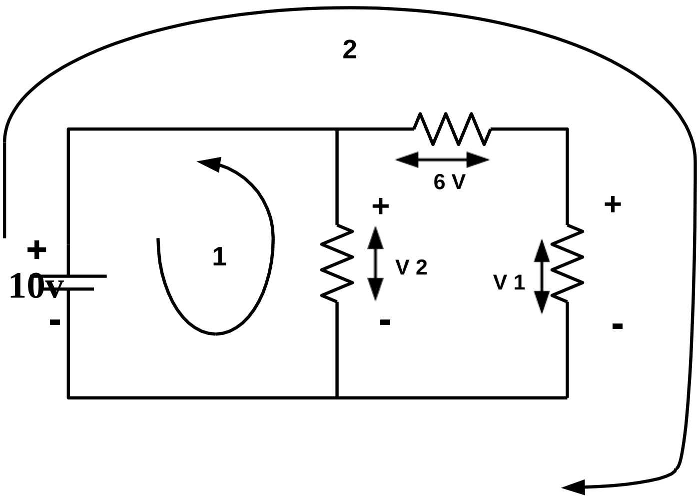


Fig. 2

Loop 1 :

$10 - V_2 = 0$ V_2
$= 10$ v

Loop 2 :

$$-10 + 6 + V_1 = 0$$
$$V_1 = 10 - 6 = 4\ v$$

2. kirchhoffs current law :

In any electrical network , the algebraic sum of currents meeting at a point (junction) is zero as shown in fig. 3 .

$$\sum I = 0$$

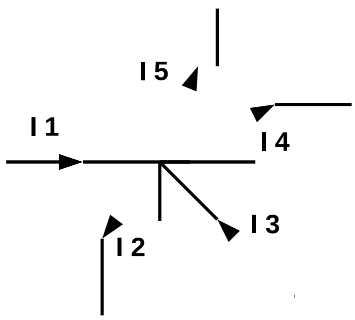


Fig. 3

$$I_1 + I_3 = I_2 + I_4 + I_5$$

$$I_1 + I_3 - I_2 - I_4 - I_5 = 0$$

<u>Example</u> : Using kirchhoffs current law , find I_5 from fig. 4 .

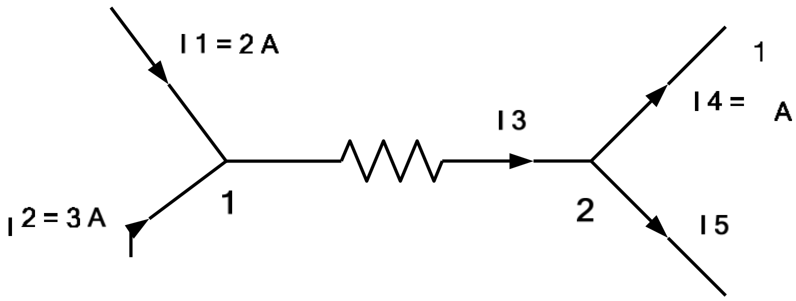


Fig. 4

At node 1 :

$I_1 + I_2 = I_3$
2+ 3 = 5 A , therefore I_3 = 3A At node

2 :

$I_3 = I_4 + I_5$
$5 \quad = 1 \quad + I_5$
$I_5 = 5 - 1 = 4$ A

<u>Example</u> : Using kirchhoffs law , find I_1 , I_2 and I_3 for the circuit shown in fig. 5 .

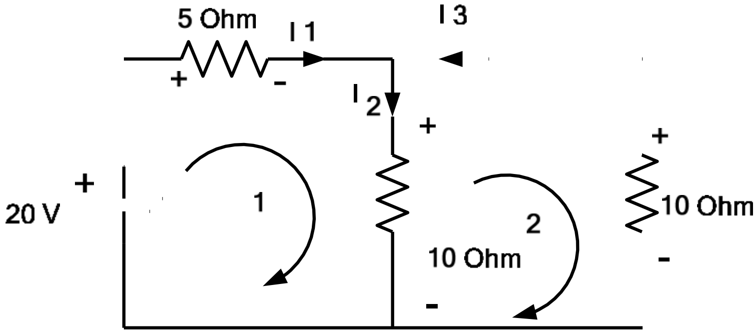


Fig. 5

$I_1 = I_2 + I_3$ ---------- (1)

Loop 1 :

$-20 + 5 I_1 + 10 I_2 = 0$
$5 I_1 + 10 I_2 = 20$
$I_1 + 2 I_2 = 4$ --------- (2)

Loop 2 :

$-10 I_2 + 10 I_3 = 0$
$I_2 = I_3$ -------------- (3)

From Equ . (2)

$I_1 = 4 - 2 I_2$ ------------- (4)

Sub. Equ. (3) and (4) in (1)

$4 - 2 I_2 = I_2 + I_2$
$4 I_2 = 4$
$I_2 = 1$ A

From Equ . (4)

$I_1 = 4 - (2 \times 1) = 2$ A

$I_3 = I_2$
$I_3 = 1$A

Maxwells method :

In this method loop current is used instead of branch currents as in kirchhoffs laws . Here , the current in different meshes are assigned continuous paths so that they do not split at a junction into branch current . Basically , this method consists of writing loop voltage equation in terms of the unknown loop currents .

Example : Using Maxwells method , calculate all currents for the circuit shown in fig. 6 .

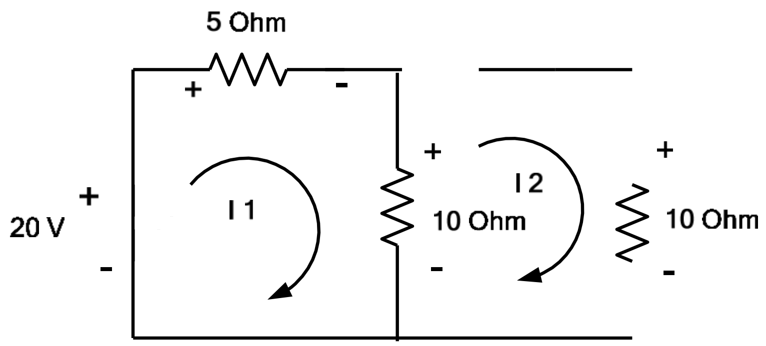


Fig. 6

Loop 1 :

$- 20 + 5 I_1 + 10 (I_1 – I_2)=0$

$15 I_1 – 10 I_2 = 20$

$3 I_1 – 2 I_2 = 4$ --------- (1)

Loop 2 :

$10 (I_2 – I_1) + 10 I_2 = 0$

$10 I_2 – 10 I_1 + 10 I_2 = 0$

$20 I_2 = 10 I_1$

$I_1 =2 I_2$ -------------- (2)

Sub. Equ. (2) in (1)

$3 (2 I_2) – 2 I_2 = 4$

$6 I_2 - 2I_2 = 4$

$I_2 = 1 A$

$I_1 = 2 I_2$

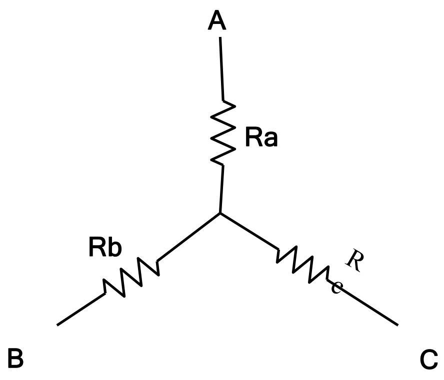
$I_1 = 2A$

Now , branch current will be calculated as follows : The

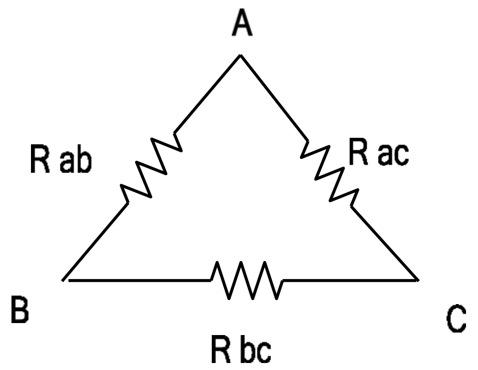
current through 5 Ω resistor $I_{5\Omega} = I_1 = 2$ A .
The current through 10 Ω resistor $I_{10\,\Omega} = I_1 - I_2 = 2 - 1 = 1$ A . The current through a 10 Ω resistor $I_{10\,\Omega} = I_2 = 1$ A .

Star delta transformation :

In solving complicated networks , it is necessary to transform from star to delta or from delta to star as shown below .



star connection



delta connection

1. Convert from star to delta :

$$R_{ab} = R_a + R_b + \frac{R_a \times R_b}{R_C}$$

$$R_{ac} = R_a + R_c + \frac{R_a \times R_c}{R_b}$$

$$R_{bc} = R_b + R_c + \frac{R_b \times R_c}{R_a}$$

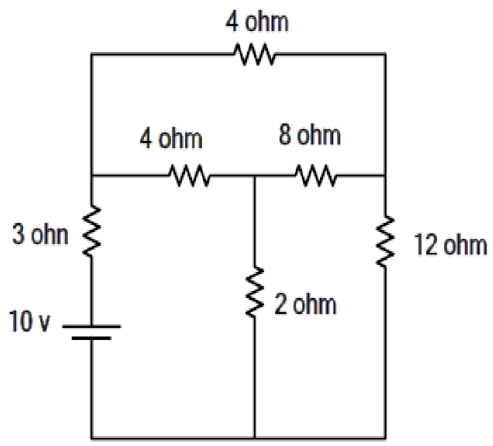
Convert from delta to star :

$$R_a = \frac{R_{ab} \times R_{ac}}{R_{ab} + R_{ac} + R_{bc}}$$

$$R_b = \frac{R_{ab} \times R_{bc}}{R_{ab} + R_{ac} + R_{bc}}$$

$$R_c = \frac{R_{ac} \times R_{bc}}{R_{ab} + R_{ac} + R_{bc}}$$

<u>Example</u> : **For the circuit shown in fig. 1 , find the total resistance**



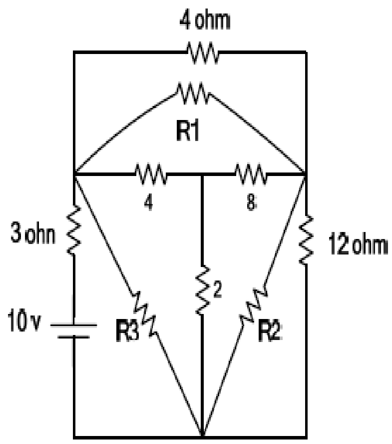


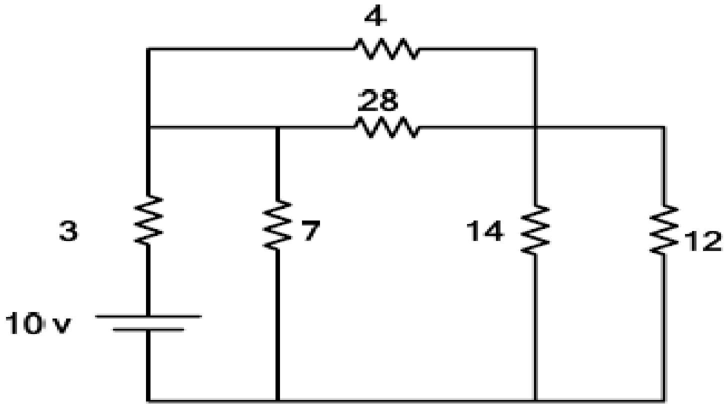
Fig. 1

<u>Convert star to delta</u>

$$R_1 = 4 + 8 + \frac{4 \times 8}{2} = 28\ \Omega$$

$$R_2 = 2 + 8 + \frac{2 \times 8}{4} = 14\ \Omega$$

$$R_3 = 2 + 4 + \frac{2 \times 4}{8} = 7\ \Omega$$



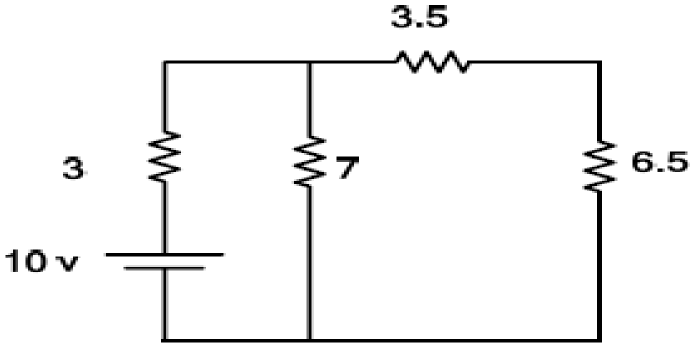
4 x 28

4 Ω // 28 Ω   ,   ------------ = 3.5 Ω

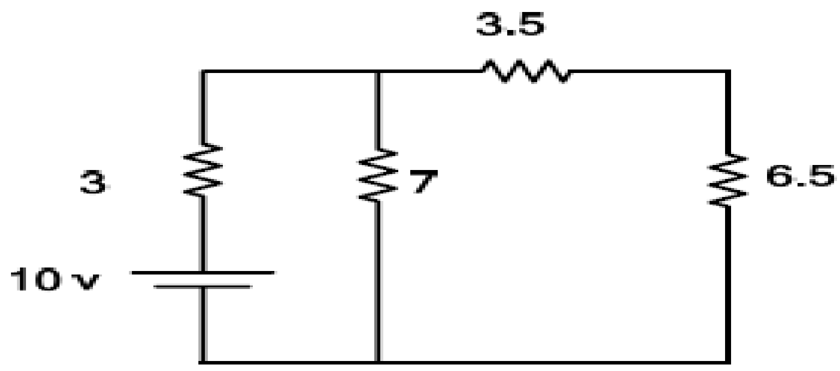
14 x 12

14 Ω // 12 Ω ,   ------------ = 6.5 Ω

14 + 12



3.5 + 6.5 = 10 Ω



7 Ω // 10 Ω

7 x 10
--------   = 4 Ω
7 + 10

$$R_t = 3 + 4 = 7\ \Omega$$

Nodal method

Nodal method :

In this method , every junction in the network where three or more branches meet is regarded as a node . One of these is regarded as the reference node (or zero potential node) .Consider the circuit in fig. 1 which has three nodes . Node 3 has been taken as the reference node . V_A represent the potential of node 1 with respect to node 3 . V_B represent the potential of node 2 with respect to node 3 .

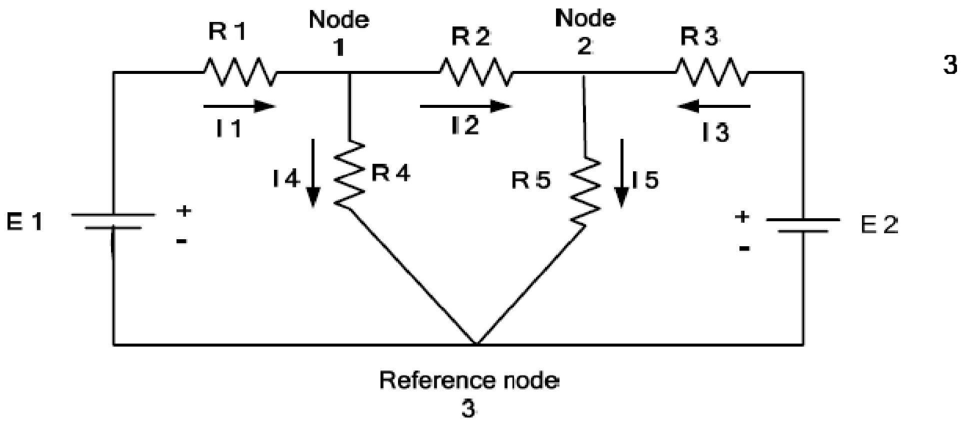


Fig. 1

Node 1 :

$$V_A \left\{ \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} \right\} - \frac{V_B}{R_2} - \frac{E_1}{R_1} = 0$$

Node 2 :

$$V_B \left\{ \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \right\} - \frac{V_A}{R_2} - \frac{E_2}{R_3} = 0$$

Example : Using nodal method , find all currents for the circuit shown in fig. 2 .

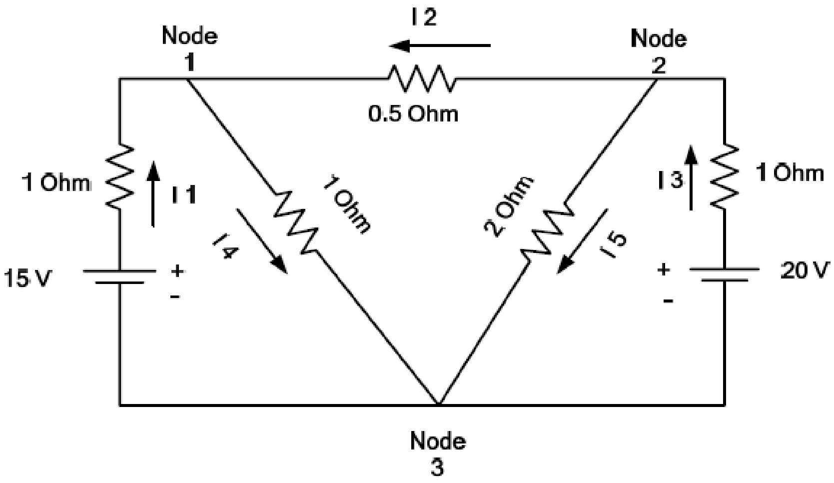


Fig. 2

Consider node 3 as reference node .

Node 1 :

$$V_1 \left\{ \frac{1}{1} + \frac{1}{1} + \frac{1}{0.5} \right\} - \frac{V_2}{0.5} - \frac{15}{1} = 0$$

$$4\,V_1 - 2\,V_2 = 15 \text{ ------- } (1)$$

Node 2 :

$$V_2 \left\{ \frac{1}{1} + \frac{1}{2} + \frac{1}{0.5} \right\} - \frac{V_1}{0.5} - \frac{20}{1} = 0$$

$$3.5\,V_2 - 2V_1 = 20 \text{ -------- } (2)$$

From Equations (1) and (2)

$V_1 = 9.25$ v , $V_2 = 11$v

$$I_1 = \frac{15 - 9.25}{1} = 5.75$$

$$I_2 = \frac{11 - 9.25}{0.5} = 3.5\ A$$

$$I_3 = \frac{20 - 11}{1} = 9\,A$$

$$I_4 = 5.75 + 3.5 = 9.25\,A$$

$$I_5 = 9 - 3.5 = 5.5\,A$$

Thevenins theorem :

The current flowing through a load resistance R_L connected across any two terminals A and B of a network as shown in fig. 1 is given by :

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

Where

V_{th} is the open circuit voltage across the two terminals A and B where R_L is removed.

R_{th} is the internal resistance of the network as viewed back into the network from terminals A and B with voltage source replaced by its internal resistance , while current source replaced by open circuit .

R_L load resistor.

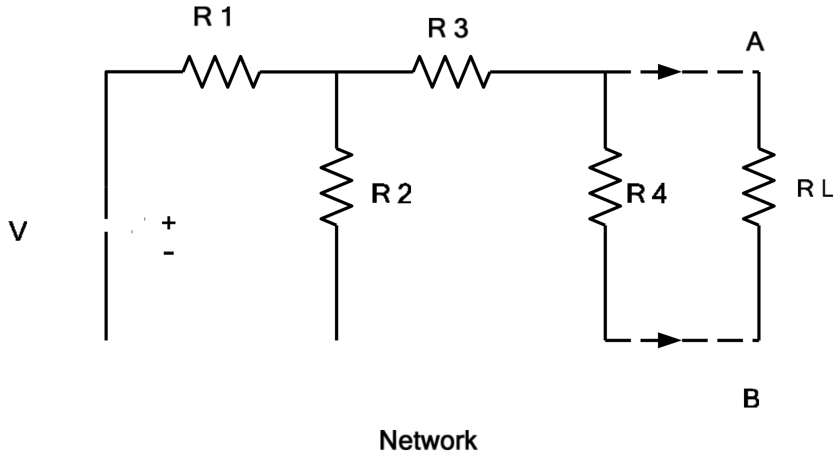
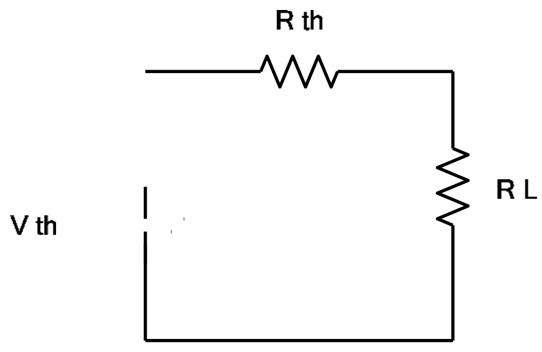


Fig. 1



Thevenins equivalent circuit

Now , R_{th} and V_{th} must be found . R_{th}

could be found as follows:

1. **Replace voltage source by short circuit (if there is no internal resistance) , while the current source replaced by open circuit .**

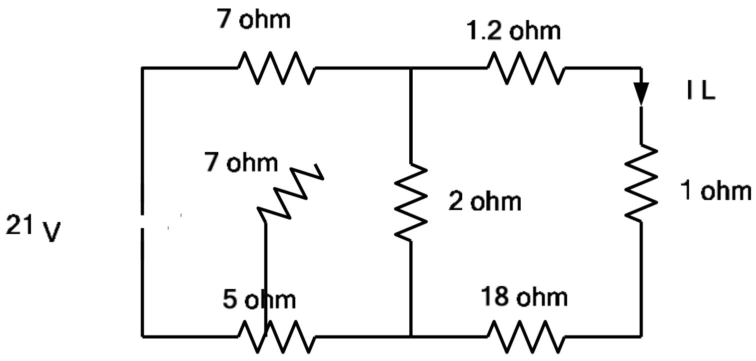
2. **Remove R_L from the circuit , then calculate R_{th} viewed from terminals A and B .**

V_{th} could be found as follows :

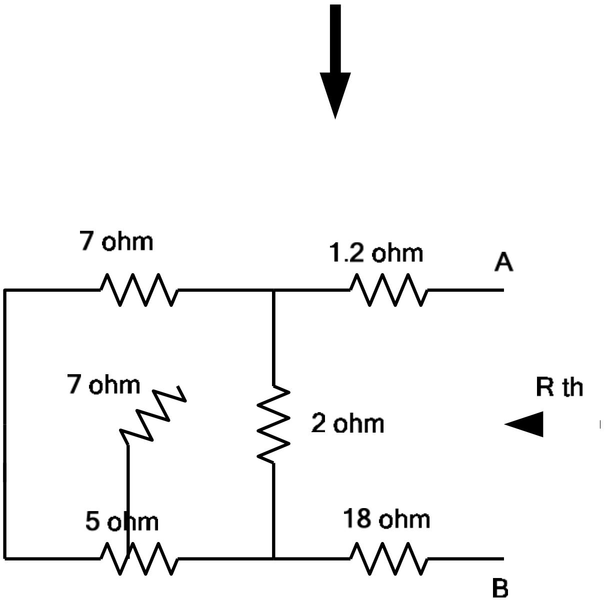
1. **Remove R_L and make sure that the voltage or current source is connected .**

2. **Calculate V_{th} between points A and B .**

<u>Example</u> : Using Thevenins theorem , find I_L in the circuit shown below .



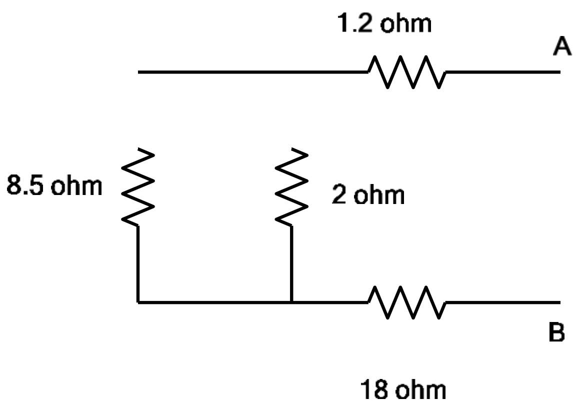
To find R_{th}



7 Ω // 7 Ω

$$\frac{7 \times 7}{7 + 7} = 3.5\ \Omega$$

3.5 + 5 = 8.5 Ω

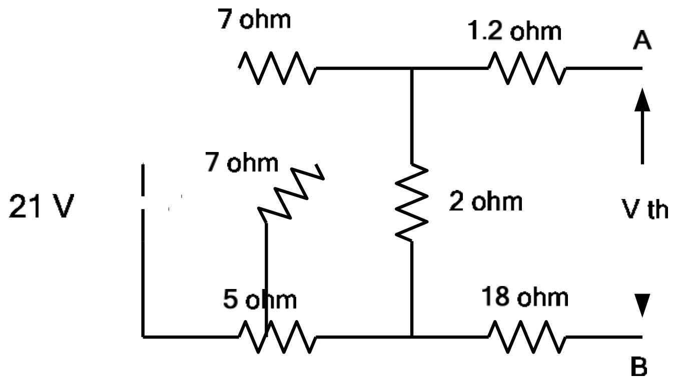


8.5 Ω // 2 Ω

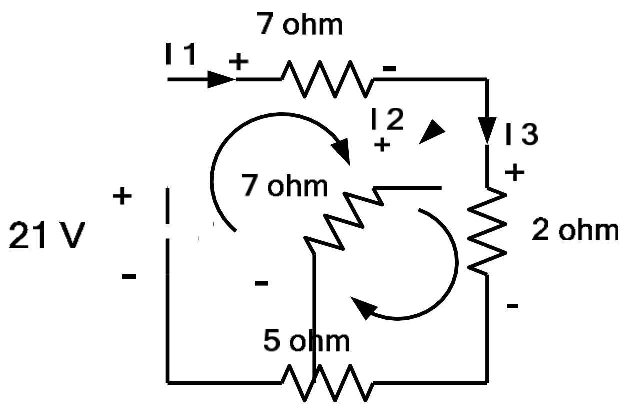
8.5 x 2
--------- = 1.6 Ω
8.5 +2

R_{th} = 1.2 + 1.6 + 18 = 20.8Ω

To find V_{th}



Since , there is no current flow through 1.2 Ω and 18 Ω , then the above circuit could be simplified to the following circuit .



$I_1 = I_2 + I_3$ -------- (1)

$-21 + 7I_1 + 7I_2 = 0$
$7(I_1 + I_2) = 21$
$I_1 + I_2 = 3$ ---------- (2)

$2I_3 + 5I_3 - 7I_2 = 0$
$7I_3 = 7I_2$
$I_2 = I_3$ ------------- (3)

From Equation (2)

$I_1 = 3 - I_2$ ----------- (4)

Sub. Equations (3) and (4) in (1)

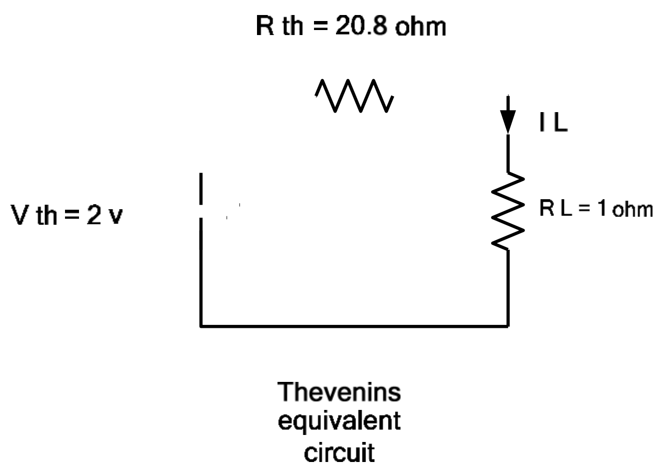
$3 - I_2 = I_2 + I_2$

$3 = 3I_2$, $I_2 = 1$ A , $I_3 = 1$ A

$I_1 = 1 + 1 = 2$ A

$V_{th} = V_{2\Omega}$

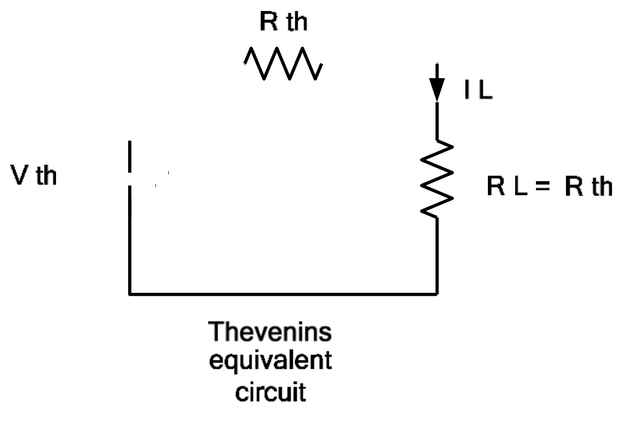
$V_{th} = 2 \times I_3 = 2 \times 1 = 2$ v



$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{2}{20.8 + 1} = 0.09\ A$$

<u>Maximum power transfer theorem</u> :

A resistor load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals with all voltage sources removed leaving behind their internal resistances and all current sources replaced by open circuit .



$$R_L = R_{th}$$

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

$$I_L = \frac{V_{th}}{2\,R_{th}}$$

$$P = (I_L)^2 \times R_{th}$$

$$P_{max} = \frac{(V_{th})^2}{4\,(R_{th})^2} \times R_{th}$$

$$P_{max} = \frac{(V_{th})^2}{4\,R_{th}}$$

Superposition theorem

In a network containing more than one source , the current which flows at any point is the sum of all currents which would flow through that point if each source was considered separately and all the other sources replaced for the time being by resistance equal to their internal resistances .

<u>Example</u> : For the circuit shown in fig. 1 , find the current in all branches , using Superposition theorem .

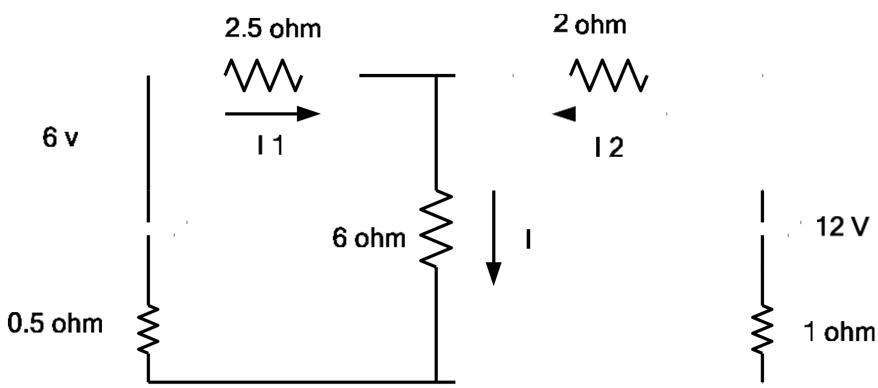
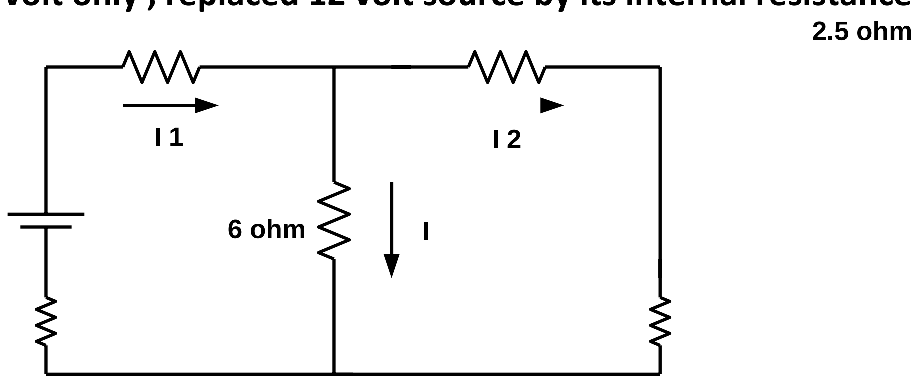


Fig. 1

1. Consider 6 volt only , replaced 12 volt source by its internal resistance



$$R_t = 2.5 + \frac{3 \times 6}{3+6} + 0.5 = 5\Omega$$

$$I_1' = \frac{V_t}{R_t} = \frac{6}{5} = 1.2\ A \rightarrow$$

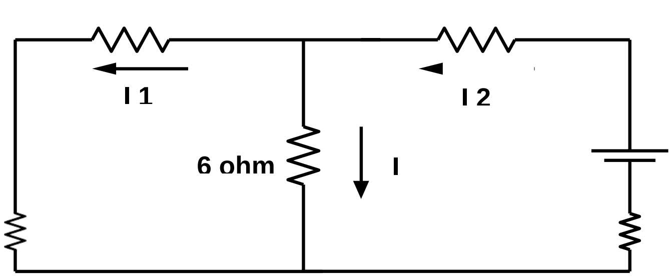
Using C.D.R :

$$I_2' = I_1' \times \frac{6}{6 + 3}$$

$$I_2' = 1.2 \times \frac{6}{9} = 0.8\ A \rightarrow$$

$$I' = I_1' - I_2' = 1.2 - 0.8 = 0.4\ A \downarrow$$

2. Consider 12 volt only , replaced 6 volt source by its internal resistance .



$$R_t = 2 + \frac{3 \times 6}{3 + 6} + 1 = 5\Omega$$

$$I_2 = \frac{V_t}{R_t} = \frac{12}{5} = 2.4 \text{ A} \leftarrow$$

Using C.D.R :

$$I_1 = I_2 \times \frac{6}{6 + 3}$$

$$I_1 = 2.4 \times \frac{6}{9} = 1.6 \text{ A} \leftarrow$$

$$I = I_2 - I_1 = 2.4 - 1.6 = 0.8 \text{ A} \downarrow$$

Now , take 6 volt and 12 volt sources in consideration : I

$$= I' + I'' = 0.4 + 0.8 = 1.2 \text{ A} \downarrow$$

$$I_1 = I_1'' - I_1' = 1.6 - 1.2 = 0.4 \text{ A} \leftarrow$$

$$I_2 = I_2'' - I_2' = 2.4 - 0.8 = 1.6 \text{ A} \leftarrow$$

<u>Example</u> : For the circuit shown in fig. 2 , find the current flows through 10 Ω resistor , using super position theorem .

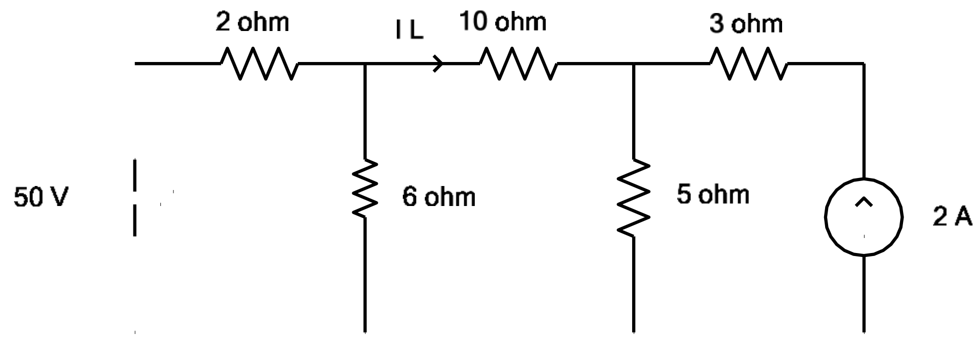
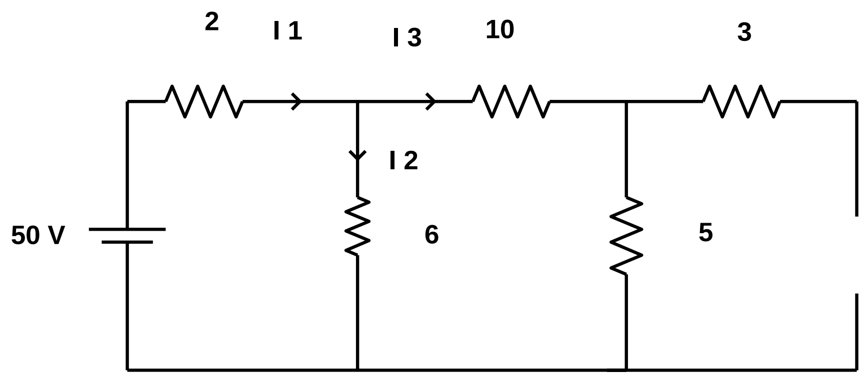


Fig. 2

1. **Consider 50 volt source only , replace 2 A current source by open circuit .**



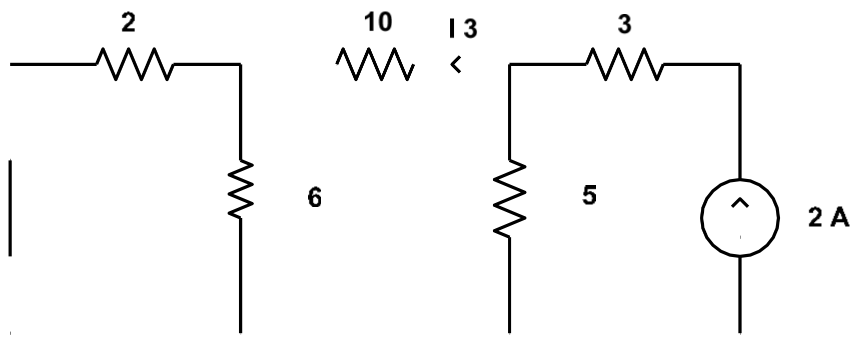
$$R_t = 2 + \frac{6 \times 15}{6 + 15} = 6.285\ \Omega$$

$$I_1 = \frac{V_t}{R_t} = \frac{50}{6.285} = 7.955\ A$$

Using C.D.R :

$$I_3 = I_1 \times \frac{6}{6+15} = 7.955 \times \frac{6}{21} = 2.272\ A \rightarrow$$

2. **Consider 2 A current source only , replaced 50 volt source by short circuit .**



$$\frac{2 \times 6}{2+6} + 10 = 11.5\ \Omega$$

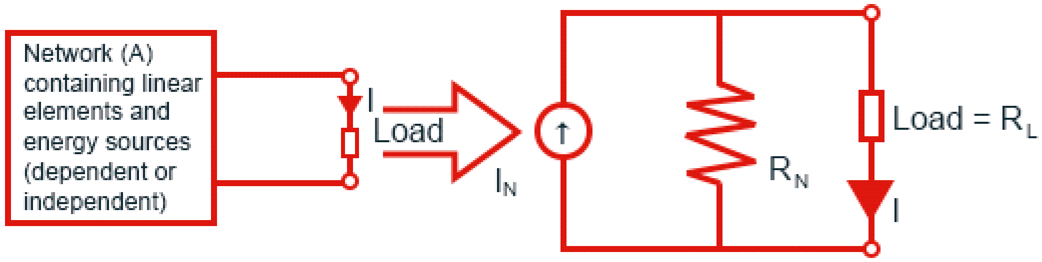
It is clear that 11.5 Ω ⁄⁄ 5 Ω

Using C.D.R :

$$I_3' = 2 \times \frac{5}{5+11.5} = 0.606\ A \leftarrow, \quad I_{10\Omega} = I_3 - I_3' = 2.272 - 0.606 = 1.66\ A \rightarrow$$

Norton's Theorem

We could see the Thevenin theorem that has two-terminal active network is converted into a voltage source and an equivalent series resistance across the load where current would be calculated. Here there is another method of analyzing a network called Norton's theorem. Here the two terminal network with current and voltage source is converted into a constant current source and a parallel resistance and is connected across the load through which the current is to be calculated.



Norton circuit

What is Norton's Theorem?

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Norton's theorem, as Thevenin's theorem is the way that is used to solve the complex circuits to represent control devices. It was developed by American scientist E.L. Norton, is generally used to reduce the complicated circuit network. It is the alternate to thevenin's theorem to analyze the network that has a simple current source and single parallel resistor. It states that "Any combination of linear bilateral circuit containing network elements and active sources, regardless of the connection to a given load Z_L, can be replaced by a simple network, that has a single current source of I_N amperes and a single impedance Z_{eq} in parallel with it, across the two terminals of the load Z_L. It is called Norton's current I_N and Z_{eq} is the equivalent impedance of given network as viewed through the load terminals, with Z_L removed and all the active sources are replaced by their internal impedances. If the internal impedances are unknown then the independent voltage sources must be replaced by short circuit while the independent current sources must be replaced by open circuit, while calculating Z_{eq}".

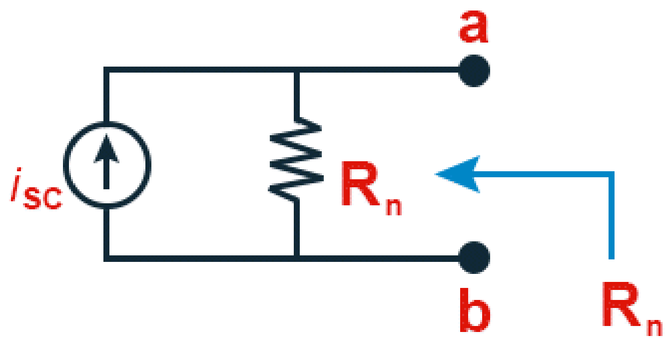
The steps used to convert the simple circuit into Norton's circuit using theorem are:

1. Short the branch, through which the current is to be calculated
2. Obtain the current through this short circuited branch, using any of the network simplification techniques. This current is Norton's current.
3. Draw the Norton's equivalent across the terminals, with current source I_N, with impedance Z_{eq} parallel with it. The current through the branch

$I = I_N$ ×× *ZeqZeq*+*ZL*ZeqZeq+ZL

Norton Equivalent Circuit

An American engineer, E.L. Norton at Bell telephone laboratories, proposed an equivalent circuit the current source and a equivalent resistance. This circuit is related to the Thevenin equivalent circuit by a source transformation. Hence a source transformation converts a Thevenin equivalent circuit into a Norton's equivalent circuit or vice versa. Norton published this method in 1926, 43 years after Thevenin.



The Norton's equivalent circuit replaces the simple circuit by a parallel combination of an ideal current source isc and a conductance G_n, where i_{sc} is the short circuit at the two terminals and Gn is the ratio of the short circuit current to the open-circuit voltage at the terminal pair.

Difference between Thevenin Theorem and Norton's Theorem

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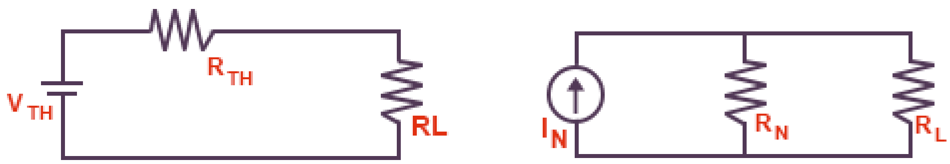
Even though Thevenin's and Norton's theorem can be derived from each other and their resistance are equal in magnitude. There are some differences that rule them out:

	Thevenin's theorem	Norton's theorem
1	The Thevenin's theorem is derived without referring any theorem	It is the converse of Thevenin's theorem derived by referring it
2	It is the theorem where we get the simple circuit from the complicated circuit that has voltage source V_{TH}, resistance R_{TH} and load R_L.	It is the theorem where we just follow the similar steps like Thevenin's but some parameters to determine are different (resistance R_N, current I_N)
3	Here Voltage source V_{TH} is used in the circuit	Here current source I_N is used in the circuit instead of voltage source
4	The equivalent resistance R_{TH} is in series with the source	The equivalent resistance R_N is in parallel with the source



Thevenin and norton equivalent circuit

Norton's Theorem Examples

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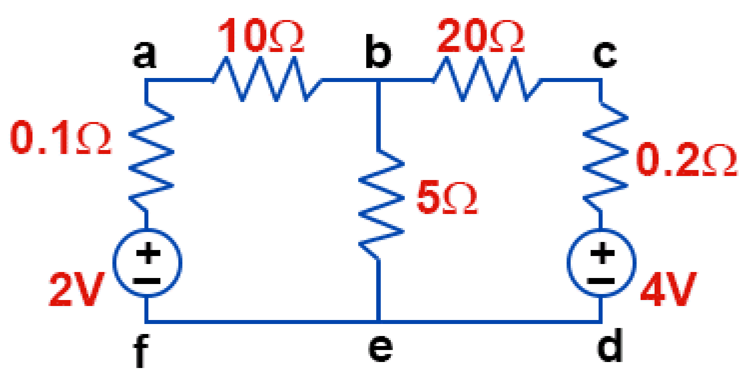
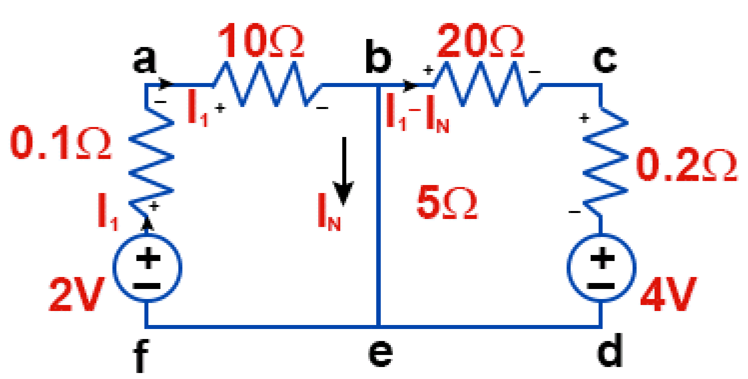
Lets see some examples on Norton's theorem:

Example 1:

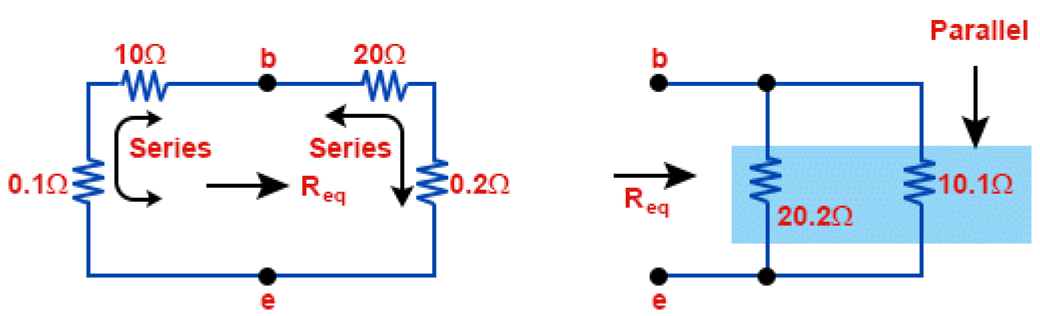
Calculate the current through the branch be using Norton's theorem



Step 1 : Short the branch be



Step 2: Calculate the short circuit current using Kirrchhoff's laws



Apply KVL to two loops,

$- 10 I_1 + 2 - 0.1 I_1 = 0$

$\therefore 10.1 I_1 = 2$

$\therefore I_1 = 0.198$ A

$-20 (I_1 - I_N) - 0.2 (I_1 - I_N) - 4 = 0$

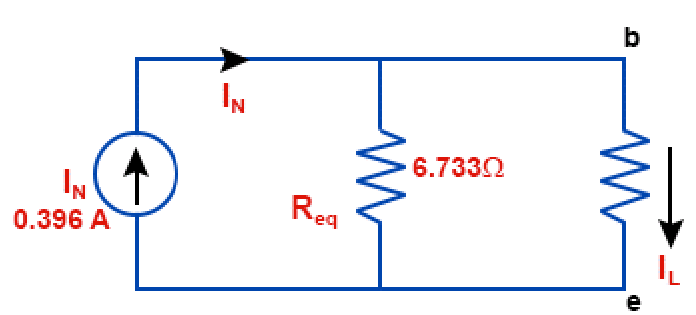
$\therefore - 20.2 I_1 + 20.2 I_N = 4$

$\therefore I_N = 0.396$ A

Step 3: Calculate R_{eq} shorting voltage sources by simplifying the circuit, we get

Req = 20.2 || 10.1 = 6.733 ΩΩ

Step 4: Norton equivalent is shown in the figure gives the idea that flow of load current I_L is in downward direction while the flow of Norton current I_N is in upward direction



Step 5 : $I_L = I_N$ ×× *ReqReq+RL*ReqReq+RL

= 0.396 ×× 6.7336.733+56.7336.733+5

= 0.2272 A

Example 2:

In a Norton's circuit there would be a flow of the Norton's current of 2 A having a equivalent resistance of 100 ΩΩ and carries load resistance of 10 ΩΩ. What would be the load current in the circuit?

Given:

Norton current I_N = 2 A, equivalent resistance R_{eq} = 100 ΩΩ, Load resistance R_L= 10 ΩΩ

Hence the load current is

I_L = I_N ×× *ReqReq*+*RL*ReqReq+RL

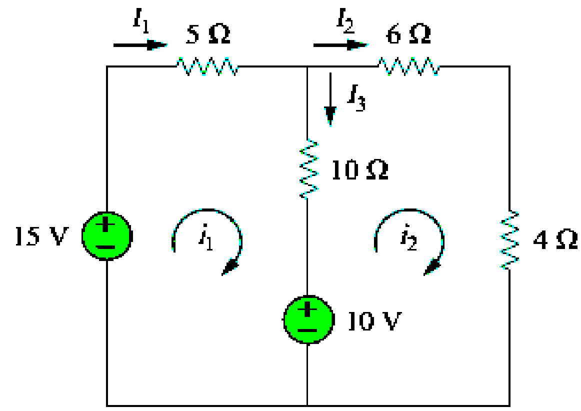
= 2 A ×× 100Ω100Ω+10Ω100Ω100Ω+10Ω

= 1.818 A .

Loop Current Method (Mesh Analysis)

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously.

Example: For the circuit below, find the branch currents I_1, I_2, and I_3 using mesh analysis.



$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1$$

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1$$

$$6i_2 - 3 - 2i_2 = 1 \quad \Longrightarrow \quad i_2 = 1 \text{ A}$$

$i_1 = 2i_2 - 1 = 2 - 1 = 1$ A. Thus,

$$I_1 = i_1 = 1 \text{ A}, \qquad I_2 = i_2 = 1 \text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

Second method:

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We obtain the determinants

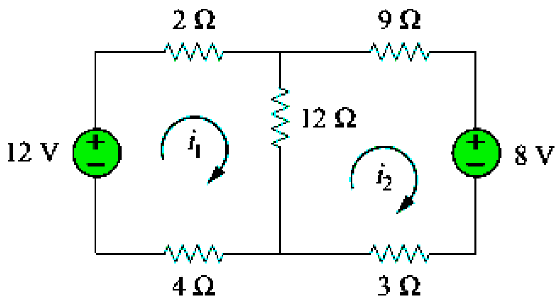
$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \qquad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

Thus,

$$i_1 = \frac{\Delta_1}{\Delta} = 1 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = 1 \text{ A}$$

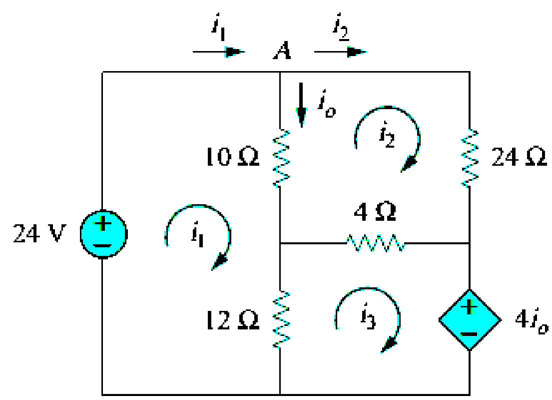
as before.



Calculate the mesh currents i_1 and i_2 in the circuit

Answer: $i_1 = \frac{2}{3}$ A, $i_2 = 0$ A.

Example: Use mesh analysis to find the current i_0 :



We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or

$$-5i_1 + 19i_2 - 2i_3 = 0$$

For mesh 3,

$$4i_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

But at node A, $i_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or

$$-i_1 - i_2 + 2i_3 = 0$$

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \\ 11 & -5 & -6 \\ -5 & 19 & -2 \end{vmatrix}$$

$$= 418 - 30 - 10 - 114 - 22 - 50 = 192$$

$$\Delta_1 = \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \\ 12 & -5 & -6 \\ 0 & 19 & -2 \end{vmatrix} = 456 - 24 = 432$$

$$\Delta_2 = \begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \\ 11 & 12 & -6 \\ -5 & 0 & -2 \end{vmatrix} = 24 + 120 = 144$$

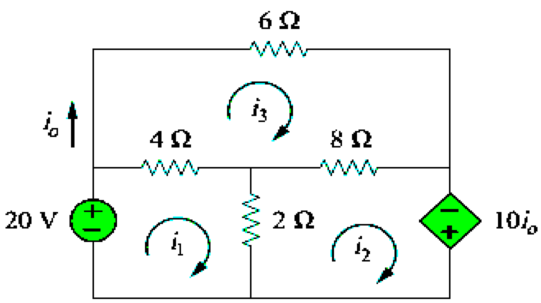
$$\Delta_3 = \begin{vmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \\ 11 & -5 & 12 \\ -5 & 19 & 0 \end{vmatrix} = 60 + 228 = 288$$

We calculate the mesh currents using Cramer's rule as

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A}$$

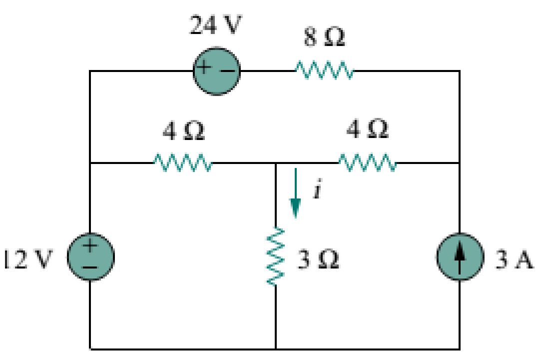
$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

Thus, $i_o = i_1 - i_2 = 1.5$ A.



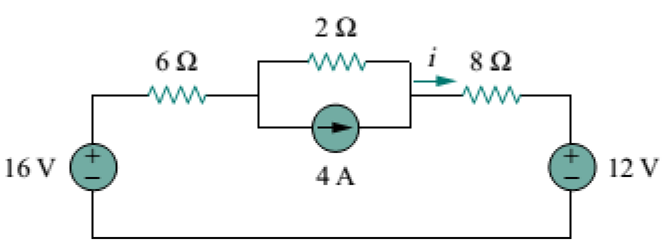
Using mesh analysis, find i_o in the circuit

Answer: −5 A.



Find i.

Ans.: i=2A



Find i

Ans.: i=0.75A

Nonlinear direct current circuit:

There are, components of electrical circuits which do not obey Ohm's law; that is, their relationship between current and voltage (their I–V curve) is *nonlinear*. An example is the p-n junction diode (curve at right). As seen in the Fig.1, the current does not increase linearly with applied voltage for a diode.

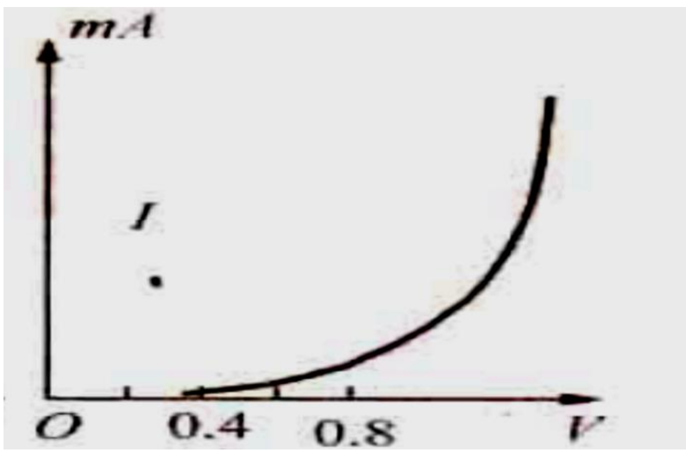


Fig. 1

One can determine a value of current (I) for a given value of applied voltage (V) from the curve, but not from Ohm's law, since the value of "resistance" is not constant as a function of applied voltage.

The source-free RC circuit:

Consider a series combination of a resistor and an initially charged capacitor, as shown in Fig. 2.

Our objective is to determine the circuit response, we assume to be the voltage *v(t)* across the capacitor. Since the capacitor is initially charged, we can assume that at time *t* = 0, the initial voltage is $v(0) = V_0$

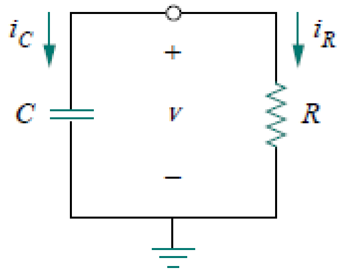


Fig. 2

Applying KCL at the top node of the circuit in Fig. 2,

$$i_c + i_R = 0$$

By definition, $i_c = C\frac{dv}{dt}$ *and* $i_R = \frac{v}{R}$. Thus,

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$\frac{dv}{v} = -\frac{1}{RC}dt$$

Integrating both sides, we get

$$\ln v = -\frac{t}{RC} + \ln A$$

where $\ln A$ is the integration constant. Thus,

$$\ln\frac{v}{A} = -\frac{t}{RC}$$

Taking powers of e produces

$$v(t) = Ae^{-t/RC}$$

But from the initial conditions, $v(0) = A = V_0$. Hence,

$$v(t) = V_0e^{-t/RC}$$

Since the response is due to the initial energy stored and the physical characteristics of the circuit and not due to some external voltage or current source, it is called the *natural response* of the circuit.

The natural response is illustrated graphically in Fig. 3. Note that at t = 0, we have the correct initial condition. As t increases, the voltage decreases toward zero. The rapidity with which the voltage decreases is expressed in terms of the *time constant*, denoted by the lower case Greek letter tau, τ.

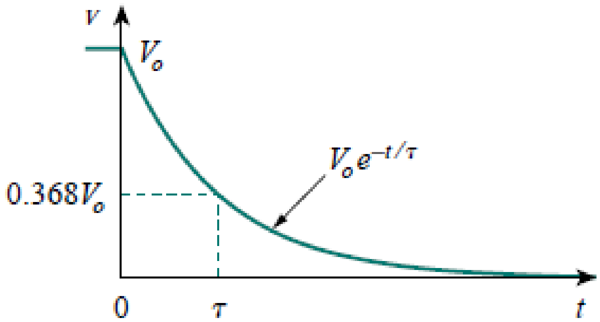


Fig. 3

This implies that at $t = \tau$,

$$V_0 e^{-\tau/RC} = V_0 e^{-1} = 0.368\, V_0$$

or

$$\tau = RC$$

At any rate, whether the time constant is small or large, the circuit reaches steady state in five time constants. We can find the current $i_R(t)$,

$$i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau}$$

The power dissipated in the resistor is

$$p(t) = v i_R = \frac{V_0^2}{R} e^{-2t/\tau}$$

Example 15: In Fig. 4, let $v_C(0)$ = 15 V. Find v_C, v_x , and i_x for t > 0.

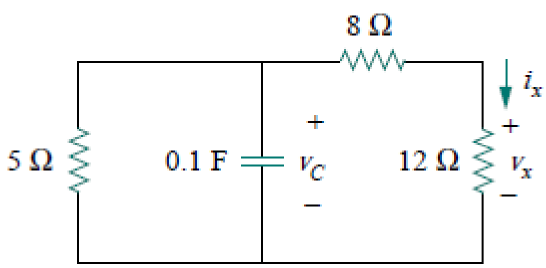


Fig.4

We find the equivalent resistance .

$$R_{eq} = (8+12)\|5 = \frac{20\times 5}{20+5} = 4\Omega$$

$$\tau = R_{eq}C = 4\times 0.1 = 0.4\ s$$

Thus,

$$v = v(0)e^{-t/\tau} = 15e^{-t/0.4}\ \text{V}, \qquad v_C = v = 15e^{-2.5t}\ \text{V}$$

From Fig. 4, we can use voltage division to get v_x; so

$$v_x = \frac{12}{12+8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t}\ \text{V}$$

Finally,

$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t}\,\text{A}$$

Practice problem:

Refer to the circuit in Fig. 5. Let $v_C(0) = 30$ V. Determine v_C, v_x , and i_0 for $t \geq 0$.

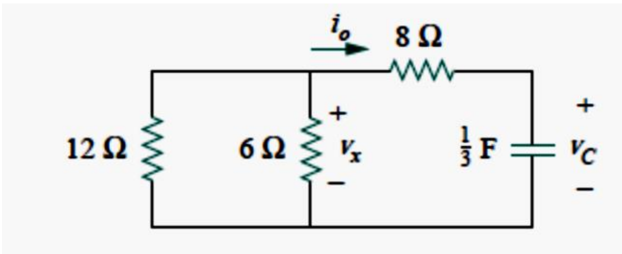


Fig. 5

Answer: $30e^{-0.25t}$ V, $10e^{-0.25t}$ V, $-2.5e^{-0.25t}$ A.

Example:

The switch in the circuit in Fig. 6 has been closed for a long time, and it is opened at $t = 0$. Find $v(t)$ for $t \geq 0$. Calculate the initial energy stored in the capacitor.

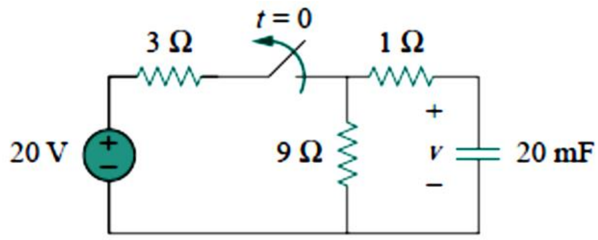


Fig.6

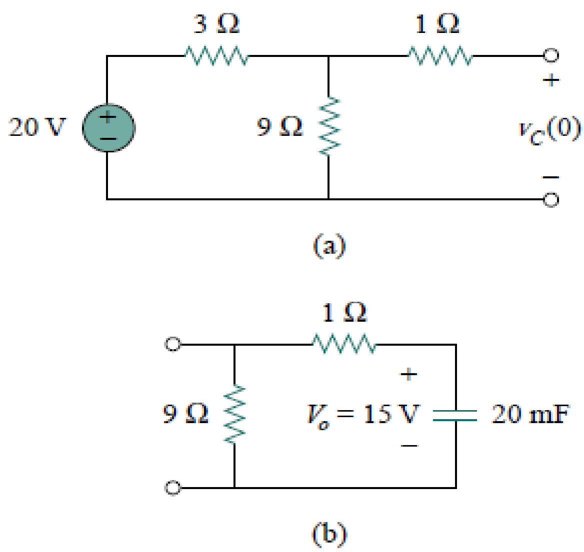


Fig. 7

For $t < 0$, and using voltage division for fig. 27 (a):

$$v_c(t) = \frac{9}{3+9} \times 20 = 15\ V \qquad t < 0$$

Hence $$vc(0) = V_0 = 15\ V$$

For $t > 0$, the switch is opened, and we have the *RC* circuit shown in Fig.7 (b)

$$R_{eq} = 1 + 9 = 10\ \Omega$$

The time constant is

$$\tau = R_{eq}C = 10 \times 20 \times 10^{-3} = 0.2 \text{ s}$$

Thus, the voltage across the capacitor for $t \geq 0$ is

$$v(t) = vC(0)e^{-t/\tau} = 15e^{-t/0.2}\ V$$

or

$$v(t) = 15e^{-5t}$$

The initial energy stored in the capacitor is

$$w_C(0) = \frac{1}{2}Cv_C^2(0) = \frac{1}{2} \times 20 \times 10^{-3} \times 15^2 = 2.25 \text{ J}$$

The source-free RL circuit:

Consider the series connection of a resistor and an inductor, as shown in Fig. 8.

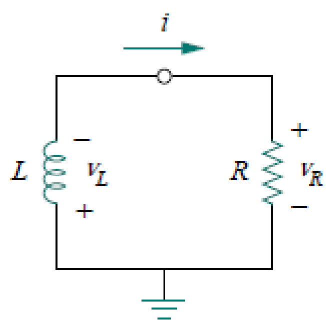


Fig. 8

Our goal is to determine the circuit response, which we will assume to be the current *i(t)* through the inductor. At $t = 0$, we assume that the inductor has an initial current I_0 , or

$i(0) = I_0$

Applying KVL around the loop in Fig. 28,

$$v_L + v_R = 0$$

But $v_L = L\,di/dt$ and $v_R = iR$. Thus,

$$L\frac{di}{dt} + Ri = 0$$

or

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

Rearranging terms and integrating gives

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L}dt$$

$$\ln i \Big|_{I_0}^{i(t)} = -\frac{Rt}{L}\Big|_0^t \qquad \Longrightarrow \qquad \ln i(t) - \ln I_0 = -\frac{Rt}{L} + 0$$

or

$$\ln\frac{i(t)}{I_0} = -\frac{Rt}{L}$$

Taking the powers of e, we have

$$i(t) = I_0 e^{-Rt/L}$$

This shows that the natural response of the *RL* circuit is an exponential decay of the initial current as shown in fig.9.

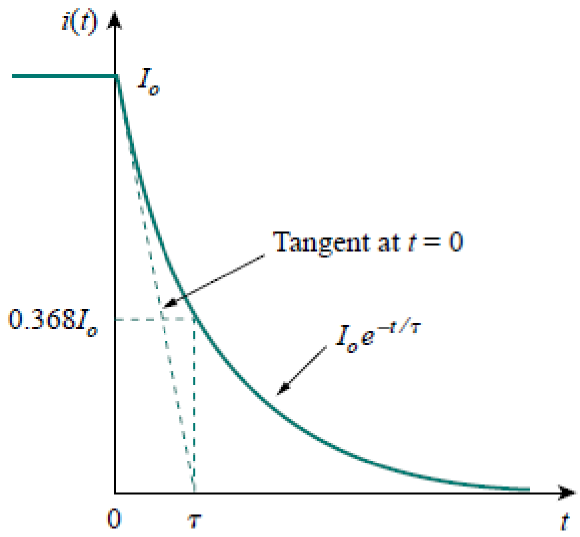


Fig. 9

The time constant of RL circuit is,

$$\tau = \frac{L}{R}$$

Hence $$i(t) = I_0 e^{-t/\tau}$$

So we can find the voltage across the resistor as

$$v_R(t) = i_R = I_0 e^{-t/\tau}$$

The power dissipated in the resistor is

$$p = v_R i = I_0^2 R e^{-2t/\tau}$$

The energy absorbed by the resistor is

$$w_R(t) = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau})$$

Note that as $t \to \infty$, $w_R(\infty) \to \frac{1}{2} L I_0^2$, which is the same as $w_L(0)$, the initial energy stored in the inductor as. Again, the energy initially stored in the inductor is eventually dissipated in the resistor.

Example:

The switch in the circuit of Fig. 10 has been closed for a long time. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$.

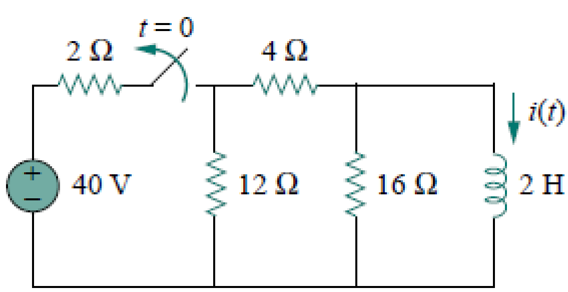


Fig.10

When $t < 0$, resulting circuit is shown in Fig. 11(a).

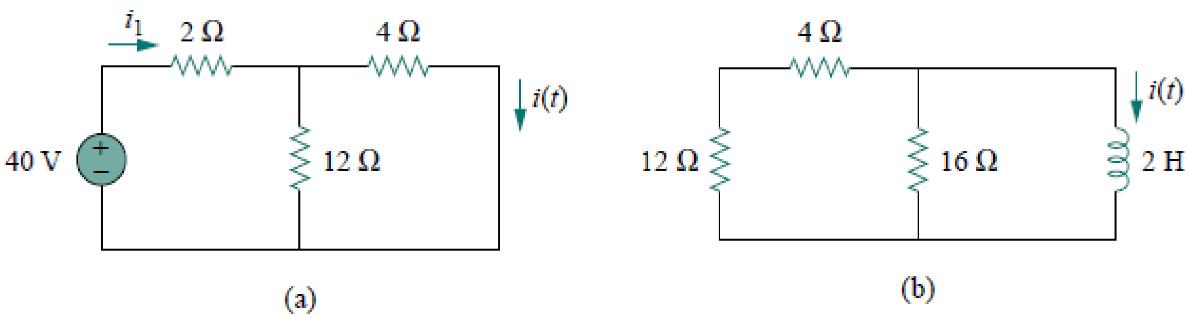


Fig. 11

we combine the 4Ω and 12Ω resistors in parallel to get

$$\frac{4 \times 12}{4 + 12} = 3\Omega$$

Hence
$$i_1 = \frac{40}{2+3} = 8\ A$$

$$i(t) = \frac{8 \times 12}{12 + 4} = 6\text{ A} \qquad \text{t} < 0$$

When $t > 0$, the switch is open and the voltage source is disconnected. We now have the *RL* circuit in Fig. 11 (b). Combining the resistors, we have

$$R_{eq} = (4 + 12) \| 16 = 8\ \Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{8} = \frac{1}{4}\ s$$

Thus
$$i(t) = i(0)e^{-t/\tau} = 6e^{-4t} A$$

The current after 1/8 s is

$$i(1/8) = 6e^{-4\times\frac{1}{8}} = 3.64\ A$$

Practice problem:

For the circuit in Fig. 12, find *i(t)* for *t* > 0.

Answer: $2e^{-2t}$ A, $t > 0$.

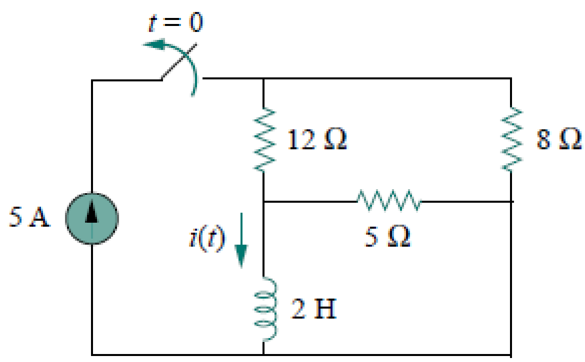


Fig.12

Chapter Two

Magnetism and Electromagnetism

1-Magnetic Field:

Magnetism refers to the force that acts between magnets and magnetic materials. We know, for example, that magnets attract pieces of iron. This force acts at a distance and without the need for direct physical contact. The region where the force is felt is called the "field of the magnet" or simply, its **magnetic field.** Thus, *a magnetic field is a force field.*

Using Faraday's representation, magnetic fields are shown as lines in space. These lines, called **flux lines** or **lines of force,** figure (1) show the direction and intensity of the field at all points. As indicated, the field is strongest at the **poles** of the magnet (where flux lines are most dense), its direction is from north (N) to south (S) external to the magnet, and flux lines never cross. The symbol for magnetic flux is the Greek letter φ (phi), the unit of flux is the weber (Wb).

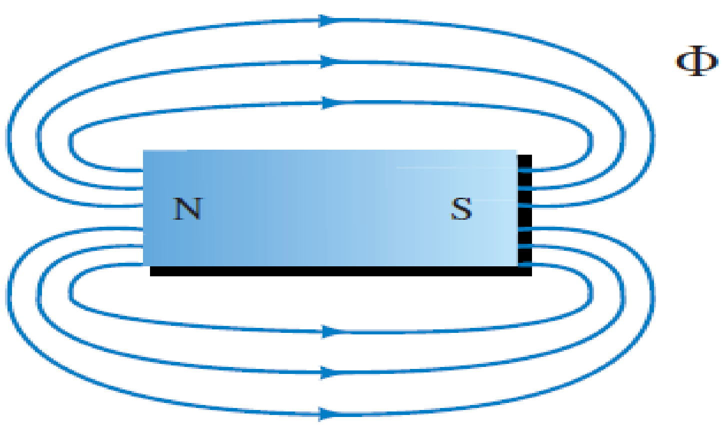
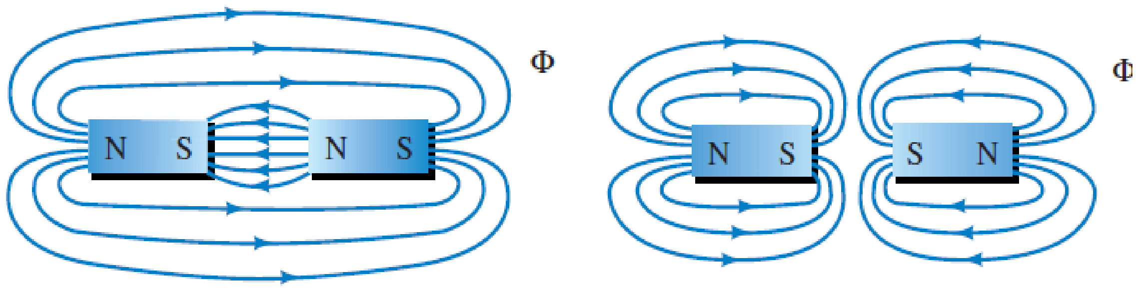


Figure 1

Figure 2 shows what happens when two magnets are brought close together. In (a), unlike poles attract, and flux lines pass from one magnet to the other. In (b), like poles repel, and the flux lines are pushed back as indicated by the flattening of the field between the two magnets.

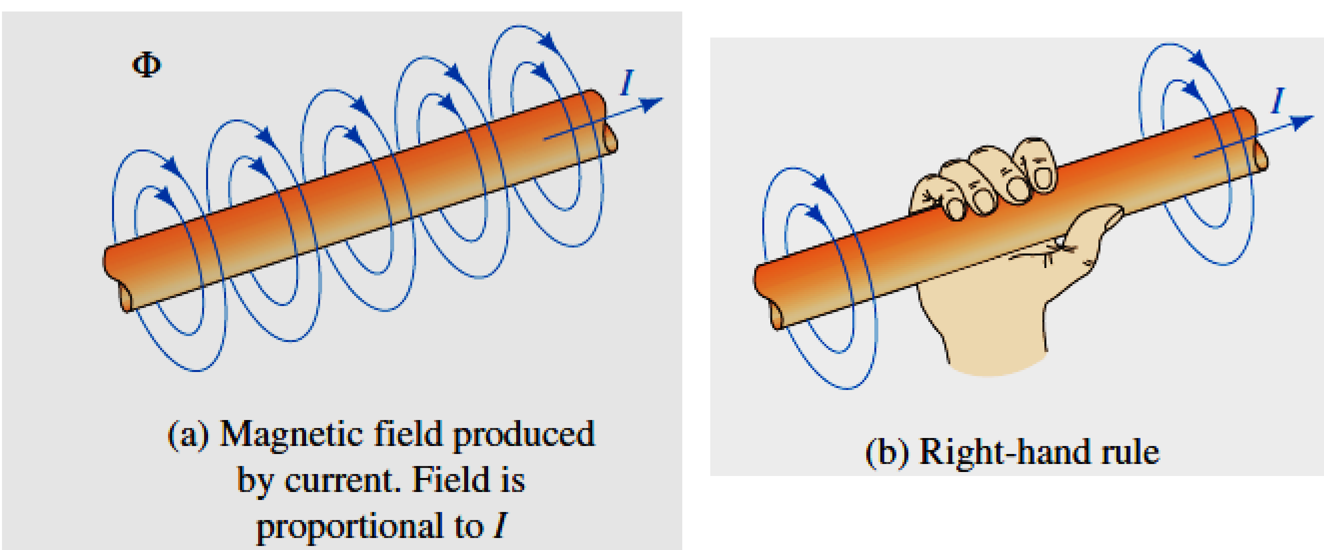


(a) Attraction (b) Repulsion

Figure 2

2- Electromagnetism:

Most applications of magnetism involve magnetic effects due to electric currents. Consider Figure 3. The current, *I*, creates a magnetic field that is concentric about the conductor, uniform along its length, and whose strength is directly proportional to *I*. Note the direction of the field. It may be remembered with the aid of the **right-hand rule.**

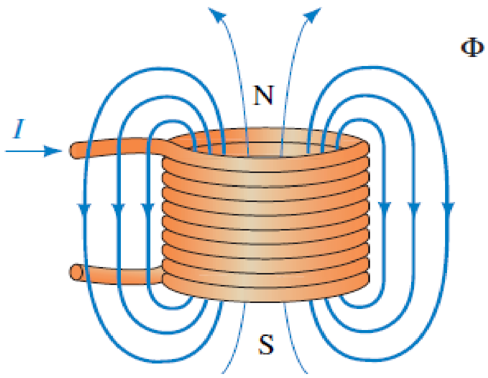


(a) Magnetic field produced by current. Field is proportional to *I*

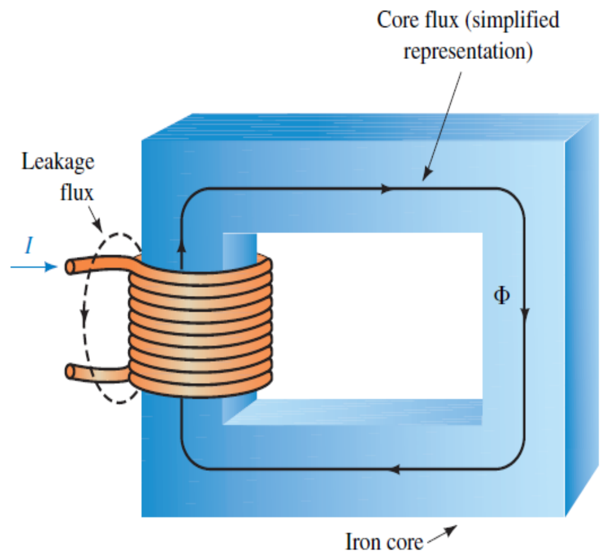
(b) Right-hand rule

Figure 3

If the conductor is wound into a coil, the fields of its individual turns combine, producing a resultant field as in Figure 4-a. If the coil is wound on a ferromagnetic core as in Figure 5-b (transformers are built this way), almost all flux is confined to the core, although a small amount (called stray or leakage flux) passes through the surrounding air.



(a)

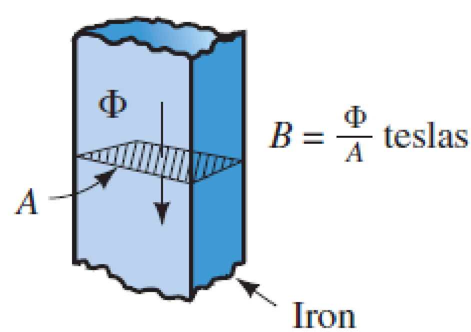


(b)

Figure 4

Flux density **B** is found by dividing the total flux passing perpendicularly through an area by the size of the area, Figure 5. That is, Since flux is measured in Wb and area *A* in m^2, flux density is measured as Wb/m^2. The unit of flux density is called the **tesla** (T) where 1 T = 1 Wb/m^2.

$$B = \frac{\varphi}{A} \quad \text{(tesla, T)}$$



Example 1:

Refer to the core of Figure 5:

If *A* is 2 cm = 2.5 cm and *B* = 0.4 T,

compute φ in webers.

Figure 5

Solution:

$A = 2 \times 10^{-2} \times 2.5 \times 10^{-2} = 5 \times 10^{-4} m^2$

$\varphi = A \times B = 5 \times 10^{-4} \times 0.4 = 2 \times 5 \times 10^{-4} Wb$

Example 2 For the magnetic core of Figure 6, the flux density at cross section 1 is $B_1 = 0.4$ T. Determine B_2 .

Solution:

$\varphi = B_1 \times A_1 = (0.4T) \times 2 \times 10^{-2}$

$= 0.8 \times 10^{-2}\ Wb$

Since all flux is confined to the core, the flux at cross section 2 is the same as at cross-section

Therefore,

$$B_2 = \frac{\varphi}{A_2} = \frac{0.8 \times 10^{-2}}{1 \times 10^{-2}} = 0.8T$$

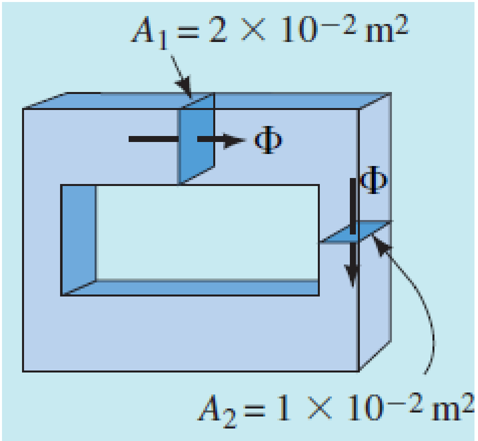


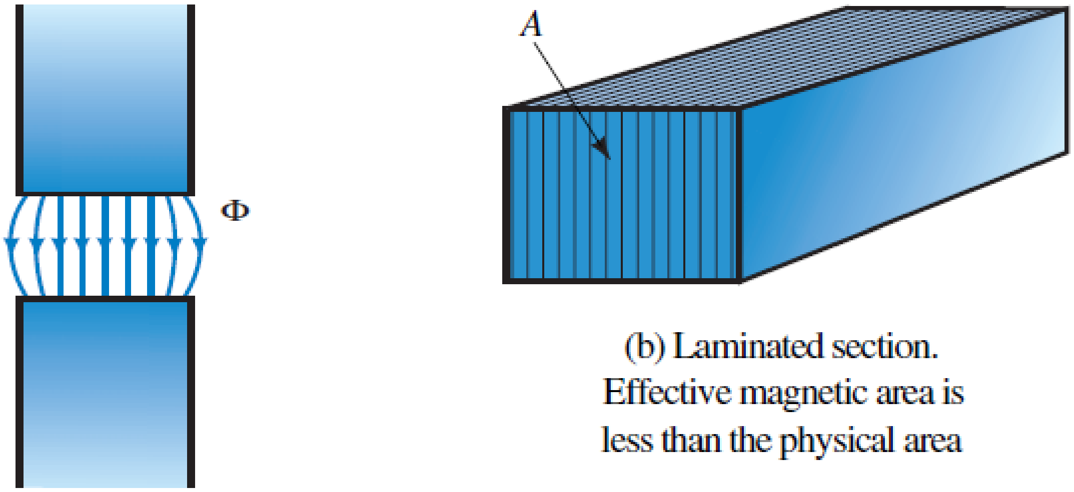
Figure 6

Practice problem:

For Figure 6, if A_1 is 2 cm x 2.5 cm, B_1 is 0.5 T, and $B2$ = 0.25 T, what is A_2?

3-Air Gaps, Fringing, and Laminated Cores:

For magnetic circuits with air gaps, **fringing** occurs, causing a decrease in flux density in the gap as in Figure 7(a). For short gaps, fringing can usually be neglected. Alternatively, correction can be made by increasing each cross-sectional dimension of the gap by the size of the gap to approximate the decrease in flux density.



(a) Fringing at gap

(b) Laminated section. Effective magnetic area is less than the physical area

Figure 7

Many practical magnetic circuits (such as transformers) use thin sheets of stacked iron or steel as in Figure 7(b). Since the core is not a solid block, its effective cross-sectional area (i.e., the actual area of iron) is less than its physical area. A **stacking factor,** defined as the ratio of the actual area of ferrous material to the physical area of the core, permits you to determine the core's effective area.

Practice problem: A laminated section of core has cross-sectional dimensions of 0.03 m by 0.05 m and a stacking factor of 0.9.

a. What is the effective area of the core?

b. Given $\varphi = 1.4 \times 10^{-3}$ Wb, what is the flux density, *B?*

Answers: a. 1.35×10^{-3} m^2 b. 1.04 T

4- Magnetic Circuits with DC Excitation:

Current through a coil creates magnetic flux. The greater the current or the greater the number of turns, the greater will be the flux. This flux-producing ability of a coil is called its **magnetomotive force** (mmf). Magnetomotive force is given the symbol F and is defined as

$$F = NI \quad (amper - turn, At)$$

Thus, a coil with 100 turns and 2.5 amps will have an mmf of 250 ampere-turns, while a coil with 500 turns and 4 amps will have an mmf of 2000 ampere-turns.

Flux in a magnetic circuit also depends on the opposition that the circuit presents to it. Termed **reluctance,** ($\mathcal{R}$) this opposition depends on the dimensions of the core and the material of which it is made. Like the resistance of a wire, reluctance is directly proportional to length and inversely proportional to cross-sectional area. In equation form,

$$R = \frac{l}{\mu A} \; At/Wb$$

where μ is a property of the core material called its **permeability**.

The relationship between flux, mmf, and reluctance is similar to Ohm's low:

$\varphi = \mathrm{F}/R \qquad Wb$

And depicted symbolically in Figure 8.

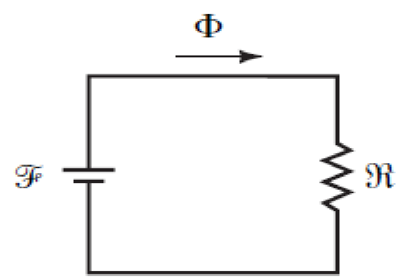
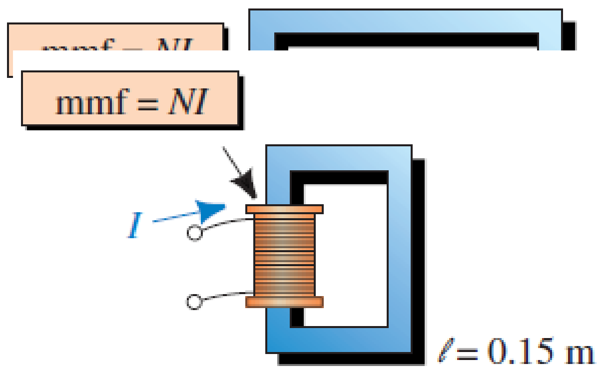


Figure 8

We require a quantity called **magnetic field intensity,** *H* (also known as **magnetizing force**). It is the ratio of applied mmf to the length of path that it acts over. Thus,

$$H = \frac{F}{l} = \frac{NI}{l} \quad \mathrm{At/m}$$



(b) A short path

(a) A long path

Figure 9

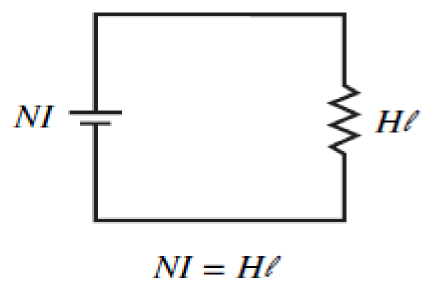
For the circuit of Figure 9(a), *H* =600 At/0.6 m = 1000 At/m, while for the circuit of (b), *H* =600 At/0.15 =4000 At/m.

From the above equation we can write

$$NI = Hl \quad \text{At}$$

In an analogy with electric circuits (Figure 10), the *NI* product is an **mmf source,** while the *Hl* product is an **mmf drop.**

Figure 10 ➡



5-The Relationship between *B* and *H*:

The relationship is:

$$B = \mu H$$

From this, it follows that the larger the permeability, the more flux you get for a given magnetizing current The permeability of free space is $\mu_0 = 4\pi \times 10^{-7}$. For all practical purposes, the permeability of air and other nonmagnetic materials is the same as for a vacuum. Thus, in air gaps,

$$B_g = \mu_0 H_g = 4\pi \times 10^{-7} \times H_g$$

$$\text{Then} \qquad H_g = \frac{B_g}{4\pi\times10^{-7}} = 7.96\times10^5 B_g \quad \text{(At/m)}$$

For ferromagnetic materials, m is not constant but varies with flux density. A set of curves, called *B-H* or *magnetization* curves, provides this information. (These curves are obtained experimentally and are available in handbooks. A separate curve is required for each material.) Figure 11 shows typical curves for cast iron, cast steel, and sheet steel.

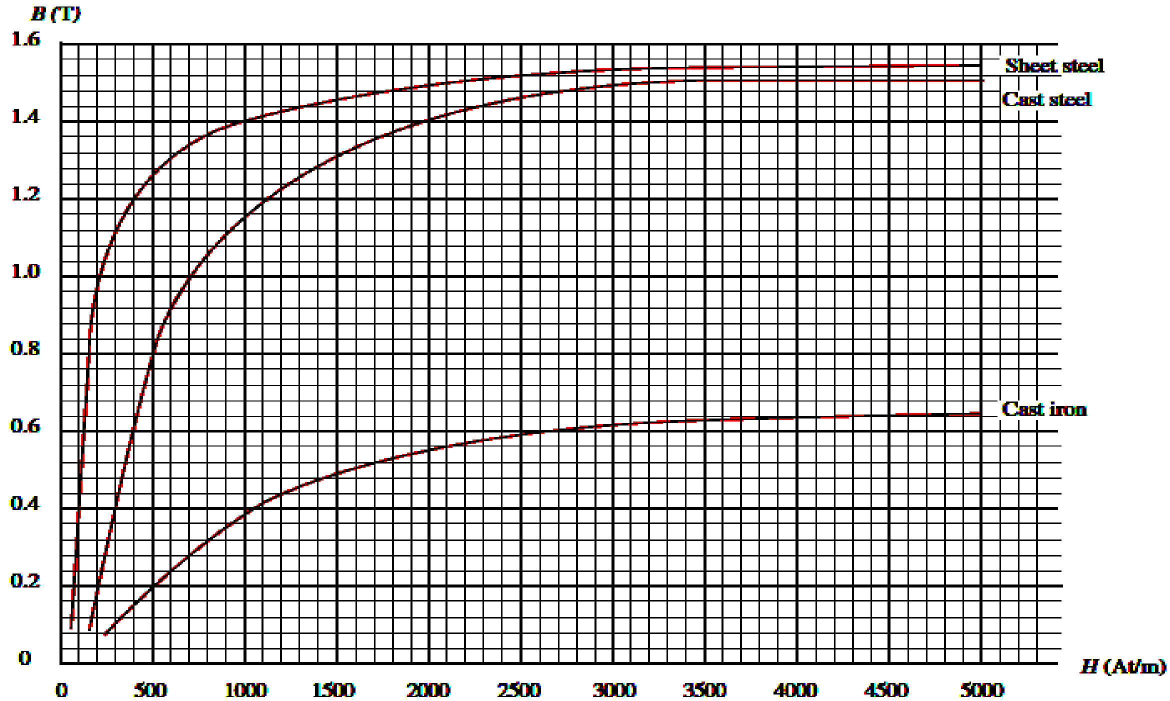


Figure 11

From figure 11 if *B* = 1.4 T for sheet, then *H* =1000 At/m.

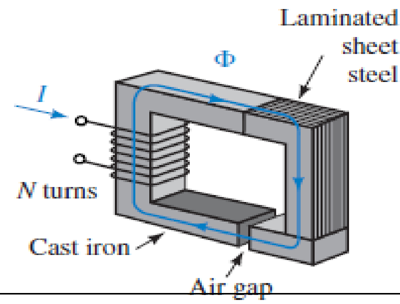
One of the key relationships in magnetic circuit theory is **Ampere's circuital law.** Ampere showed that the algebraic sum of mmfs around a closed loop in a magnetic circuit is zero, regardless of the number of sections or coils. That is,

$$\sum\nolimits_{O} NI = \sum\nolimits_{O} Hl \qquad At$$

Consider Figure 12:

$NI - H_{iron}l_{iron} - H_{steel}l_{steel} - H_g l_g = 0$

which states that the applied mmf *NI* is equal to



the sum of the *Hl* drops around the loop.

Figure 12

EXAMPLE 1: If the core of Figure 13 is cast iron and $\varphi = 0.1 \times 10^{-3}$Wb, what is the coil current?

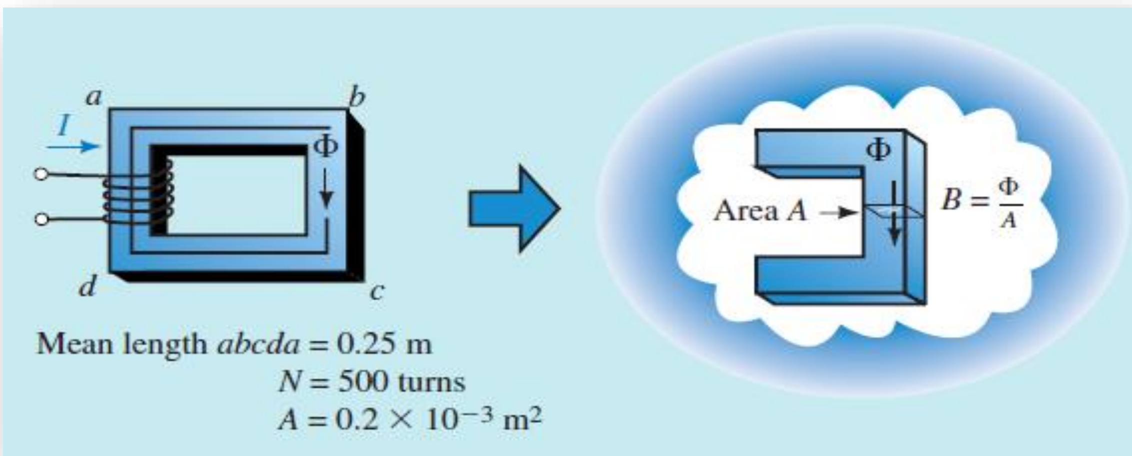


Figure 13

Solution:

Problems of this type can be solved using four basic steps:

1. The flux density is

$$B = \frac{\varphi}{A} = \frac{0.1 \times 10^{-3}}{0.2 \times 10^{-3}} = 0.5\ T$$

2. From the *B-H* curve (cast iron), Figure 11 , *H* =1550 At/m.

3- Apply Ampere's law. There is only one coil and one core section.

Length = 0.25 m. Thus,

$$NI = Hl = 1550 \times 0.25 = 388 \quad At$$

4- divide by N

$$I = \frac{388}{500} = 0.78\, amps$$

H.W : If $NI = 250\ At \quad l = 0.2\, m\, and\, cross - sectional\, area$

$A = 0.01\, m^2$**.** Determine φ**.**

EXAMPLE 2: The core of Figure 13 has a 0.008-m gap cut as shown in figure 14. Determine how much the current must increase to maintain the original core flux. Neglect fringing.

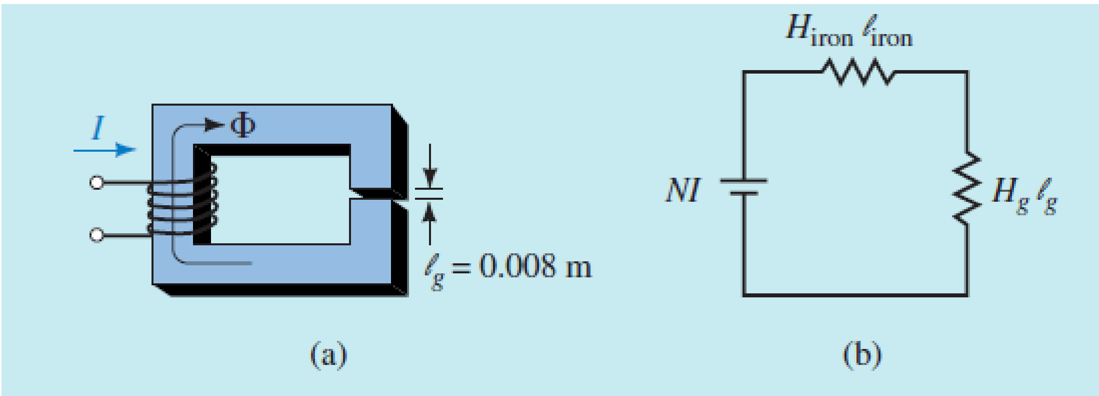


Figure 14

Solution

Iron

$l_{iron} = 0.25 - 0.008 = 0.242\, m$. Since φ does not change, B and H will be the same as before. Thus, $B_{iron} = 0.5T\ and\ H_{iron} = 1550\ At/m$.

Air Gap

B_g is the same as B_{iron}. Thus, $B_g = 0.5T$ and

$H_g = 7.96 \times 10^5 \times B_g = 3.98 \times 10^5$ At/m.

Ampere's Law

$$NI = H_{iron} l_{iron} + H_g l_g = 1550 \times 0.242 + 3.98 \times 10^5 \times 0.008 = 375 + 3184$$

$$= 3559\ At$$

$$Thus \qquad I = \frac{3559}{500} = 7.1 \ amps$$

EXAMPLE 3: The laminated sheet steel section of Figure 3 has a stacking factor of 0.9. Compute the current required to establish a flux of 1.4×10^{-4} Wb. Neglect fringing.

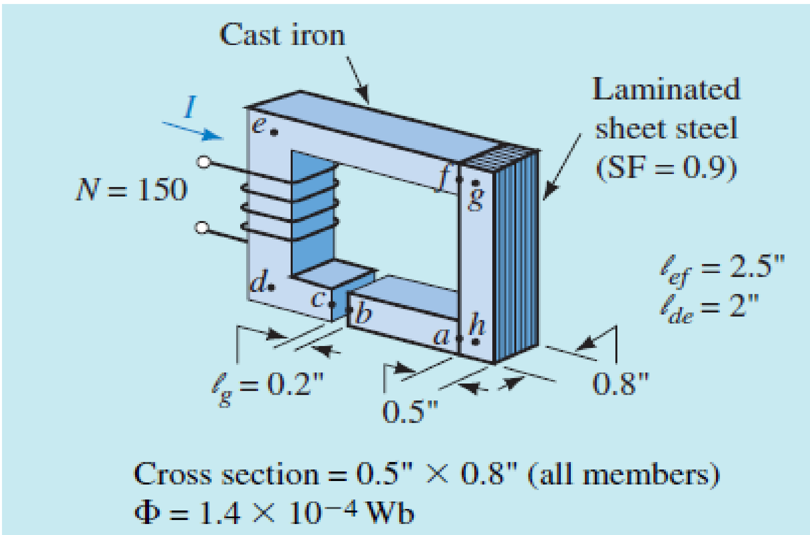


Figure 15

Cast Iron

$$l_{iron} = l_{ab} + l_{cdef} = 2.5 + 2 + 2.5 - 0.2 = 6.8\ in = 0.173m$$

$$A_{iron} = 0.5 \times 0.8 = 0.4\ in^2 = 0.258 \times 10^{-3} m^2$$

$$B_{iron} = \frac{\varphi}{A_{iron}} = \frac{1.4 \times 10^{-4}}{0.258 \times 10^{-3}} = 0.54\ T$$

$$H_{iron} = 1850 \ \ At \ \ (from\ figure\ 11)$$

Sheet Steel

$$l_{steel} = l_{fg} + l_{gh} + l_{ha} = 0.25 + 2 + 0.25 = 2.5\ in = 6.35 \times 10^{-2} m$$

$$A_{steel} = 0.9 \times 0.258 \times 10^{-3} = 0.232 \times 10^{-3} m^2$$

$$B_{steel} = \frac{\varphi}{A_{steel}} = \frac{1.4 \times 10^{-4}}{0.232 \times 10^{-3}} = 0.6\,T$$

$$H_{steel} = 125\ \ At\ \ (from\ figure\ 11)$$

Air Gap

$$B_g = B_{iron} = 0.54T$$

$$H_g = 7.96 \times 10^5 \times 0.54 = 4.3 \times 10^5\ \ At/m$$

Ampere's Law

$$NI = H_{iron}l_{iron} + H_{steel}l_{steel} + H_g l_g$$

$$= 1850 \times 0.173 + 125 \times 6.35 \times 10^{-2} + 4.3 \times 10^5 \times 5.08 \times 10^{-3}$$

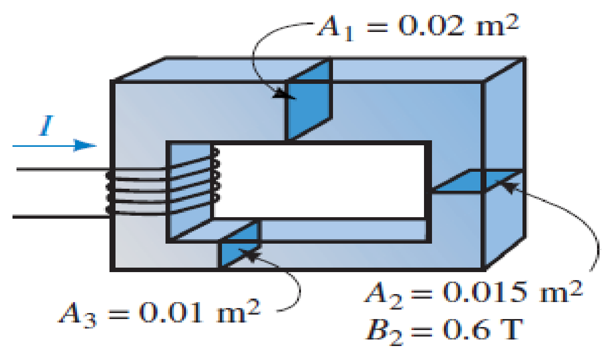
$$= 320 + 7.9 + 2184 = 2512\,At$$

$$I = \frac{2512}{N} = \frac{2512}{150} = 16.7\,amps$$

H.W:

1- For the section of iron core of Figure 16, compute B_1 and B_3?

Figure 16



2- For the circuit of Figure 17, φ = 25,000 lines. The stacking factor for the sheet steel portion is 0.95. Find current *I*. A second coil of 450 turns with I_2 = 4 amps is wound on the cast steel portion. Its flux is in opposition to the flux produced by the original coil. The resulting flux is 35 000 lines in the counterclockwise direction. Find the current *I*1. ***Hint: 1Wb=10^8 lins***.

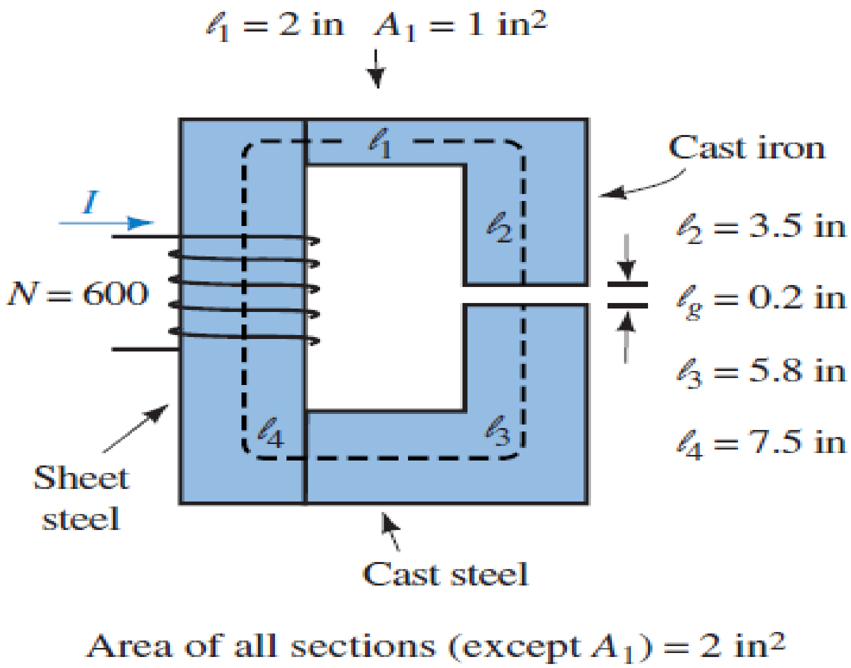


Figure 17

EXAMPLE 12–10 The core of Figure 12–29 is cast steel. Determine the current to establish an air-gap flux $\Phi_g = 6 \times 10^{-3}$ Wb. Neglect fringing.

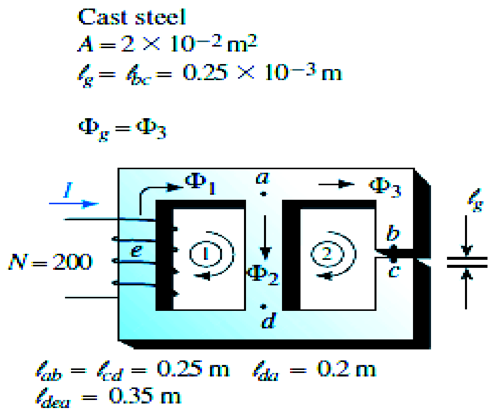


FIGURE 12–29

Solution Consider each section in turn.

Air Gap

$$B_g = \Phi_g/A_g = (6 \times 10^{-3})/(2 \times 10^{-2}) = 0.3\text{ T}$$
$$H_g = (7.96 \times 10^5)(0.3) = 2.388 \times 10^5\text{ At/m}$$

Sections ab and cd

$$B_{ab} = B_{cd} = B_g = 0.3\text{ T}$$
$$H_{ab} = H_{cd} = 250\text{ At/m} \quad \text{(from Figure 12–19)}$$

Ampere's Law (Loop 2)

$\sum_{\bigcirc} NI = \sum_{\bigcirc} H\ell$. Since you are going opposite to flux in leg *da*, the corresponding term (i.e., $H_{da}\ell_{da}$) will be subtractive. Also, $NI = 0$ for loop 2. Thus,

$$0 = \sum\nolimits_{\bigcirc\ \text{loop2}} H\ell$$
$$0 = H_{ab}\ell_{ab} + H_g\ell_g + H_{cd}\ell_{cd} - H_{da}\ell_{da}$$
$$= (250)(0.25) + (2.388 \times 10^5)(0.25 \times 10^{-3}) + (250)(0.25) - 0.2H_{da}$$
$$= 62.5 + 59.7 + 62.5 - 0.2H_{da} = 184.7 - 0.2H_{da}$$

Thus, $0.2H_{da} = 184.7$ and $H_{da} = 925$ At/m. From Figure 12–19, $B_{da} = 1.12$ T.

$$\Phi_2 = B_{da}A = 1.12 \times 0.02 = 2.24 \times 10^{-2}\text{ Wb}$$
$$\Phi_1 = \Phi_2 + \Phi_3 = 2.84 \times 10^{-2}\text{ Wb.}$$
$$B_{dea} = \Phi_1/A = (2.84 \times 10^{-2})/0.02 = 1.42\text{ T}$$
$$H_{dea} = 2125\text{ At/m} \quad \text{(from Figure 12–19)}$$

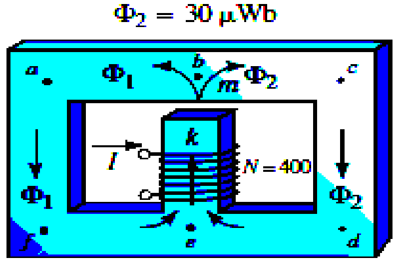
Ampere's Law (Loop 1)

$$NI = H_{dea}\ell_{dea} + H_{ad}\ell_{ad} = (2125)(0.35) + 184.7 = 929\text{ At}$$
$$I = 929/200 = 4.65\text{ A}$$

H.W1

The cast-iron core of Figure 12–30 is symmetrical. Determine current *I*. Hint: To find *NI*, you can write Ampere's law around either loop. Be sure to make use of symmetry.

FIGURE 12–30

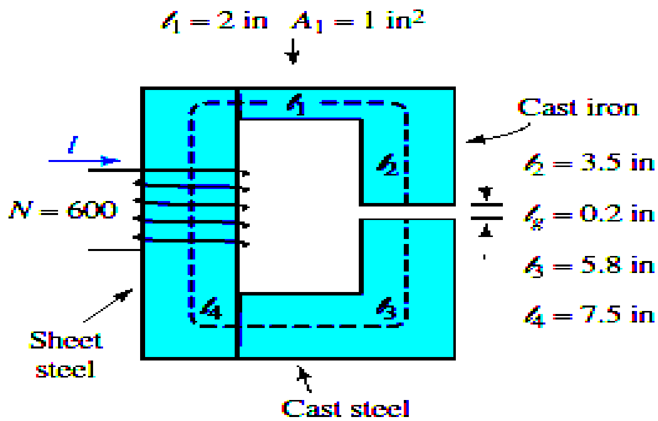


$\ell_{ab} = \ell_{bc} = \ell_{cd} = 4$ cm
Gap: $\ell_g = 0.5$ cm
$\ell_{ek} = 3$ cm

Core dimensions: 1 cm × 1 cm

Answer: 6.5 A

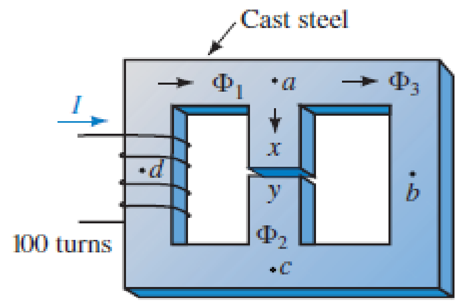
HW 2: For the circuit of Fig. below $\emptyset = 25{,}000\ lines$. The stacking factor for the sheet steel portion is 0.95. find current I.



Area of all sections (except A_1) = 2 in^2

HW3: For the circuit below if $\emptyset_g = 80\mu Wb$, find I.

If the circuit has no gap $\emptyset_3 = 0.2mWb$, find I



$\ell_g = \ell_{xy} = 0.001$ m
$\ell_{abc} = 0.14$ m
$\ell_{cda} = 0.16$ m

HW3:

Solution Consider each section in turn.

Air Gap

$$B_g = \Phi_g/A_g = (6 \times 10^{-3})/(2 \times 10^{-2}) = 0.3\text{ T}$$
$$H_g = (7.96 \times 10^5)(0.3) = 2.388 \times 10^5\text{ At/m}$$

Sections ab and cd

$$B_{ab} = B_{cd} = B_g = 0.3\text{ T}$$
$$H_{ab} = H_{cd} = 250\text{ At/m} \quad \text{(from Figure 12–19)}$$

Ampere's Law (Loop 2)

$\sum_{\bigcirc} NI = \sum_{\bigcirc} H\ell$. Since you are going opposite to flux in leg *da*, the corresponding term (i.e., $H_{da}\ell_{da}$) will be subtractive. Also, $NI = 0$ for loop 2. Thus,

$$0 = \sum_{\bigcirc\,\text{loop2}} H\ell$$
$$0 = H_{ab}\ell_{ab} + H_g\ell_g + H_{cd}\ell_{cd} - H_{da}\ell_{da}$$
$$= (250)(0.25) + (2.388 \times 10^5)(0.25 \times 10^{-3}) + (250)(0.25) - 0.2H_{da}$$
$$= 62.5 + 59.7 + 62.5 - 0.2H_{da} = 184.7 - 0.2H_{da}$$

Thus, $0.2H_{da} = 184.7$ and $H_{da} = 925$ At/m. From Figure 12–19, $B_{da} = 1.12$ T.

$$\Phi_2 = B_{da}A = 1.12 \times 0.02 = 2.24 \times 10^{-2}\text{ Wb}$$
$$\Phi_1 = \Phi_2 + \Phi_3 = 2.84 \times 10^{-2}\text{ Wb.}$$
$$B_{dea} = \Phi_1/A = (2.84 \times 10^{-2})/0.02 = 1.42\text{ T}$$
$$H_{dea} = 2125\text{ At/m} \quad \text{(from Figure 12–19)}$$

Ampere's Law (Loop 1)

$$NI = H_{dea}\ell_{dea} + H_{ad}\ell_{ad} = (2125)(0.35) + 184.7 = 929\text{ At}$$
$$I = 929/200 = 4.65\text{ A}$$