

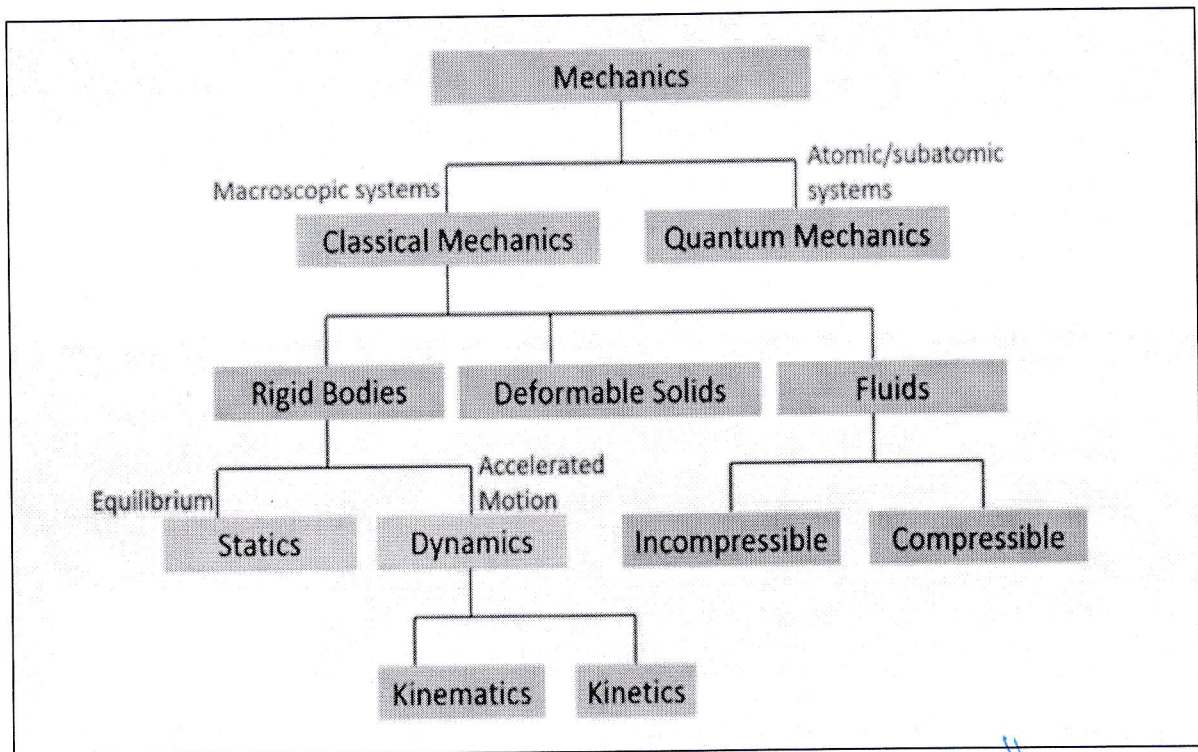
Engineering Mechanics Statics

Introduction to Mechanics

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies that are subjected to the action of forces. In general, this subject can be subdivided into three branches: Rigid-Body Mechanics, Deformable-Body Mechanics and Fluids Mechanics. We will study Rigid-Body Mechanics where this branch it is very essential for the design and analysis of many types of structural members, mechanical components, or electrical devices encountered in engineering.

Rigid-Body Mechanics is divided into two areas: Static and Dynamics. Statics deals with the equilibrium of bodies. Dynamics is concerned with the accelerated motion of bodies. We will study static because we can consider statics as a special case of dynamics in which the acceleration is zero; however, statics deserve separate treatment in engineering education since many objects are designed with intention that they remain in equilibrium.

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Forces

Since mechanics is primarily a study of the effects of forces, it is important to have a clear understanding of the concept of a force:

A force

- Action of one body on another which changes or tends to change the motion of the body.
- Required force can move a body in the direction of action, otherwise no effect.

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Forces Analysis 2D:

Forces Analysis for Two Dimensions: forces are in three shapes: vertical forces, horizontal forces and diagonal forces when the forces vertical and horizontal

they should not be analyzed but when the forces are diagonal, they will be analyzed into horizontal and vertical

forces. Any diagonal force must contain a slope for this force or slope angle of force. The axis near the angle or slope takes $\cos \theta$ and axis far the angle or slope takes $\sin \theta$.

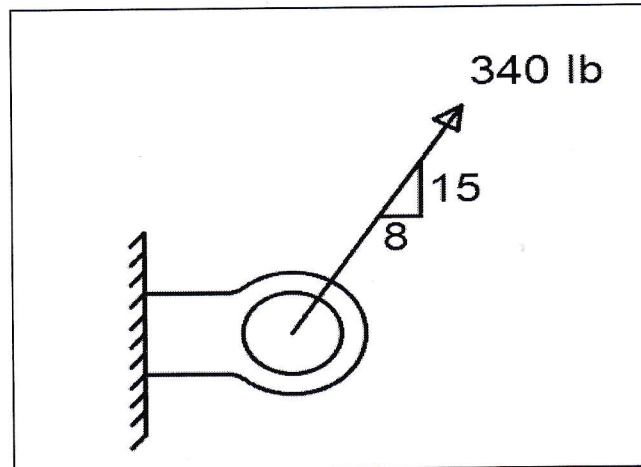
The Resultant of a Force System: is defined to be the simplest system that can replace the original system without changing its external effect on a rigid body.

When the resultant of a forces system is zero, the body on which the system act in equilibrium, and the force system is said to be balance.

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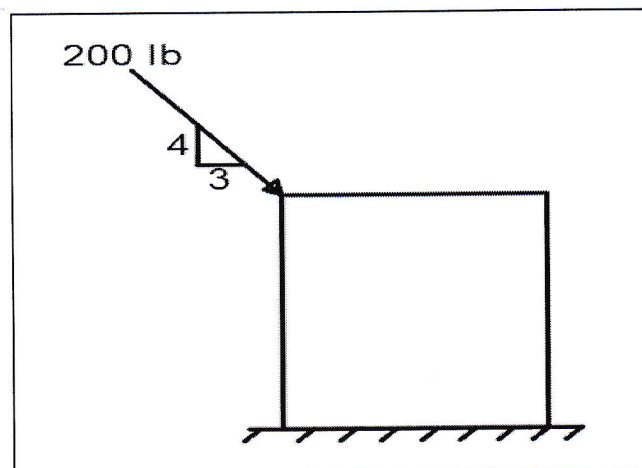
Examples:

1. Determine a pair of horizontal and vertical components of the **340 lb** force as shown in the figure below:



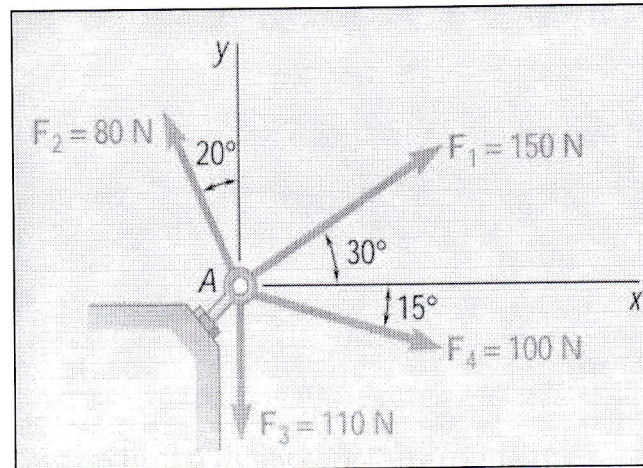
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2. Determine a set of horizontal and vertical components of the **200 lb** force as shown in the figure below:



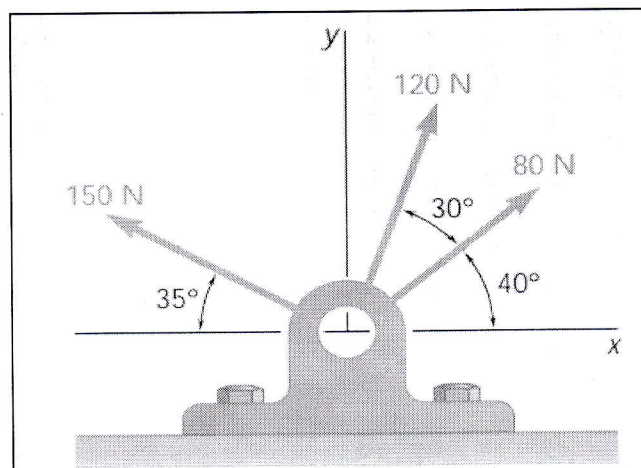
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3. Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt:



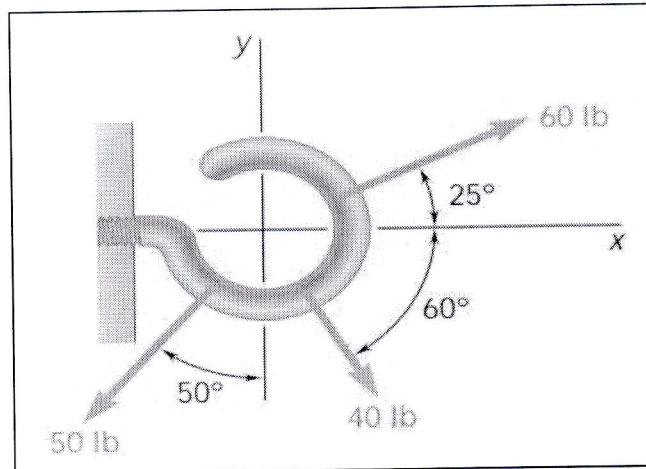
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4. Determine the x and y components of each of the forces shown. Then find the force vector.



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5. Determine the resultant and direction for each of the forces shown. Then find the total resultant.

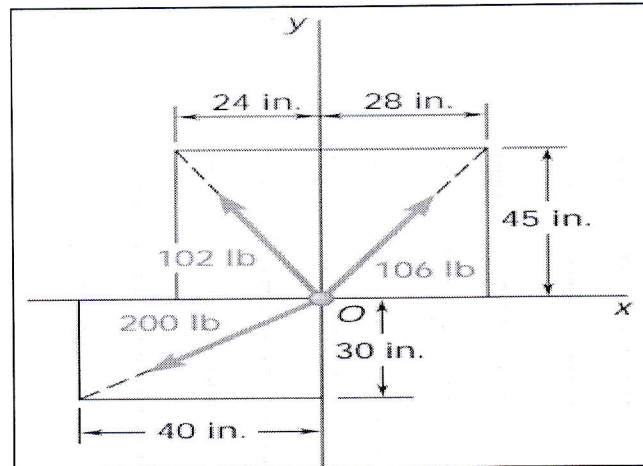


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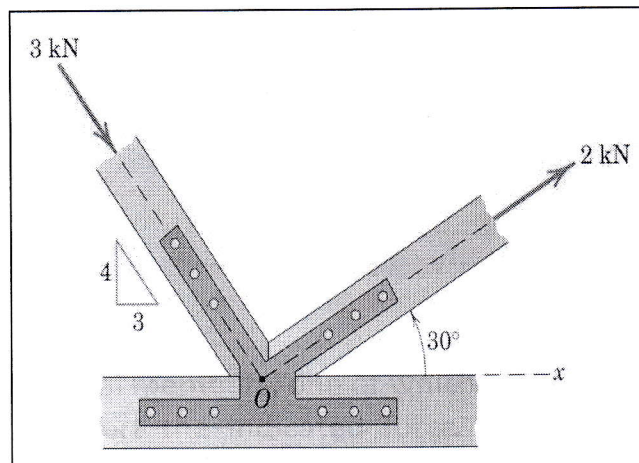
Homework

1. Determine the resultant and direction for each of the forces shown. Find the force vector.



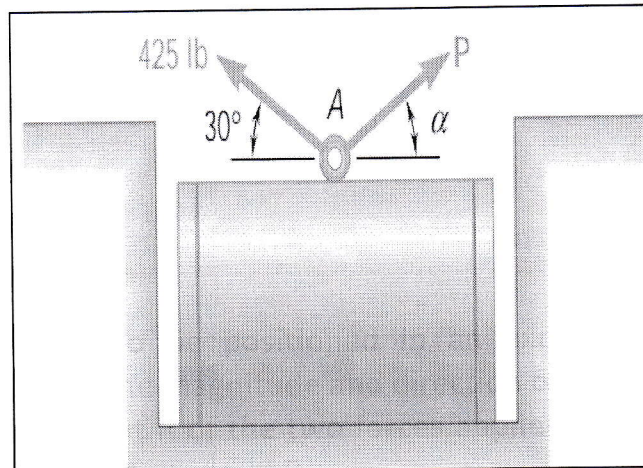
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2. The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint O. Determine the magnitude of the resultant R of the two forces and the angle θ which R makes with the positive x-axis.



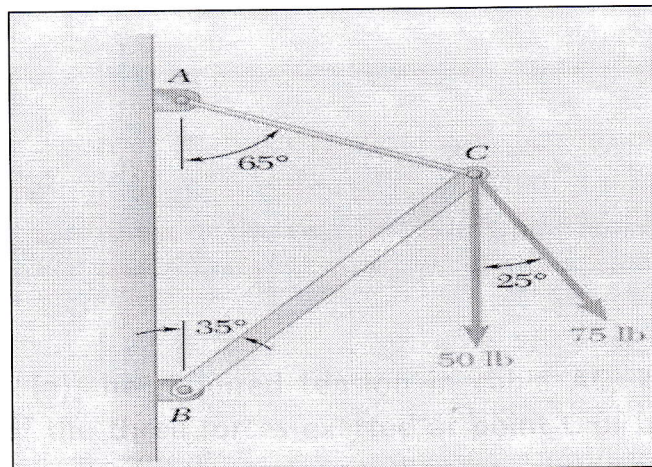
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3. A steel tank is to be positioned in an excavation. Determine by trigonometry (a) the magnitude and direction of the smallest force P for which the resultant R of the two forces applied at A is vertical, (b) the corresponding magnitude of R .



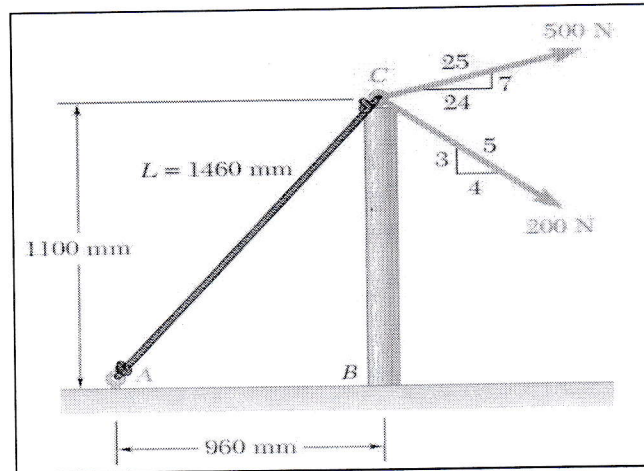
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4. Determine (a) the required tension in cable AC, knowing that the resultant of the three forces exerted at point C of boom BC must be directed along BC, (b) the corresponding magnitude of the resultant.



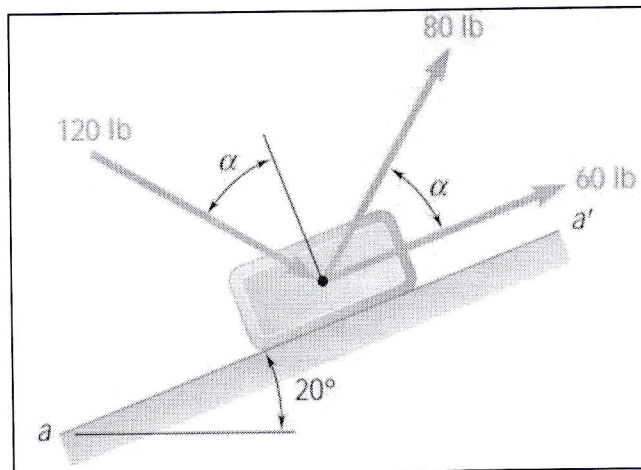
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5. For the post, determine (a) the required tension in rope AC if the resultant of the three forces exerted at point C is to be horizontal, (b) the corresponding magnitude of the resultant.



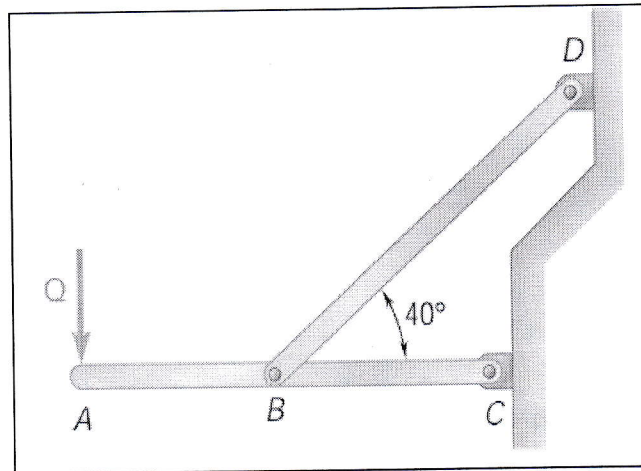
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6. Knowing that a $\alpha = 75^\circ$, determine the resultant of the three forces shown fig below:



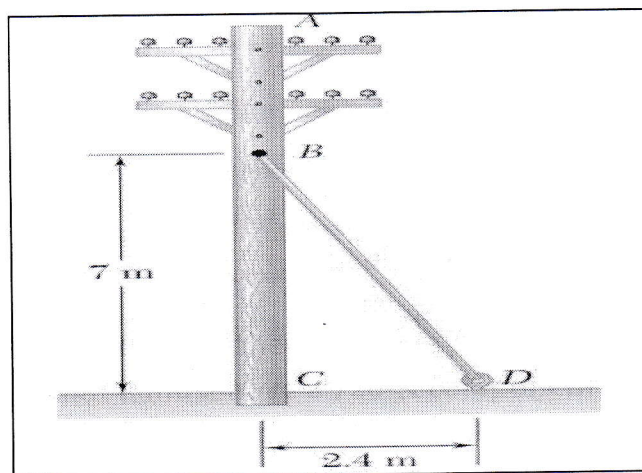
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7. Member BD exerts on member ABC a force P directed along line BD. Knowing that P must have a 240-lb vertical component, determine (a) the magnitude of the force P , (b) its horizontal component.



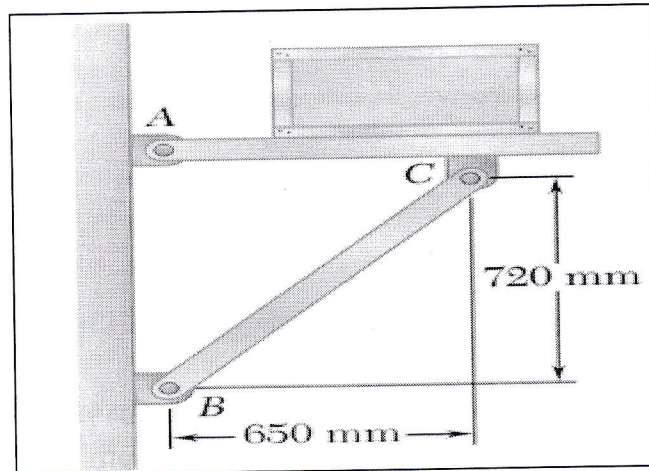
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8. The guy wire BD exerts on the telephone pole AC a force P directed along BD. Knowing that P must have a 720-N component perpendicular to the pole AC, determine (a) the magnitude of the force P , (b) its component along line AC.



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9. Member BC exerts on member AC a force P directed along line BC. Knowing that P must have a 325 N horizontal component, determine (a) the magnitude of the force P , (b) its vertical component.



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Forces Resultant 2 dimensions

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

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$$R_x = \sum F_x$$

مجموع القوى باتجاه محور x

$$R_y = \sum F_y$$

مجموع القوى باتجاه محور y

$$\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$$

R : Forces Resultant

مقدار القوة

θ : Forces Resultant angle

زاوية مقدار القوى

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ملاحظات

Horizontal
Resultant

(1) عندما يذكر في السؤال ان المحطة افقية (R)

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$$R_y = 0$$

$$R = \sqrt{(R_x)^2}$$

vertical
Resultant

(2) عندما يذكر في السؤال ان المحطة عمودية (R)

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$$R_x = 0$$

$$R = \sqrt{(R_y)^2}$$

(3) عندما يذكر في السؤال ان المحطة Couple

$$R_x = R_y = 0$$

$$R = 0$$

ع) عندما يذكر في السؤال عبارة (vertically down word) فهذا يعني

ان المحملة (R) عمودية وتكون اشارتها موجبة وفي هذه الحالة

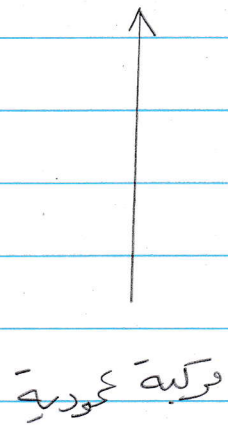
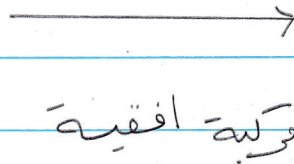
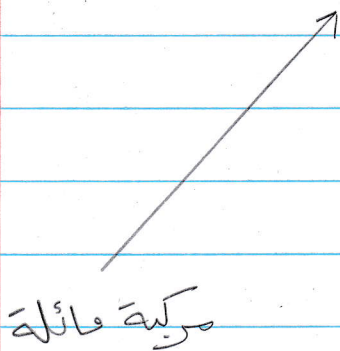
$$R_x = 0$$

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ه) عندما يذكر في السؤال عبارة (vertically up word) فهذا

يعني ان المحملة أفقية وتكون اشارتها سالبة وفي هذه الحالة

$$R_y = 0$$



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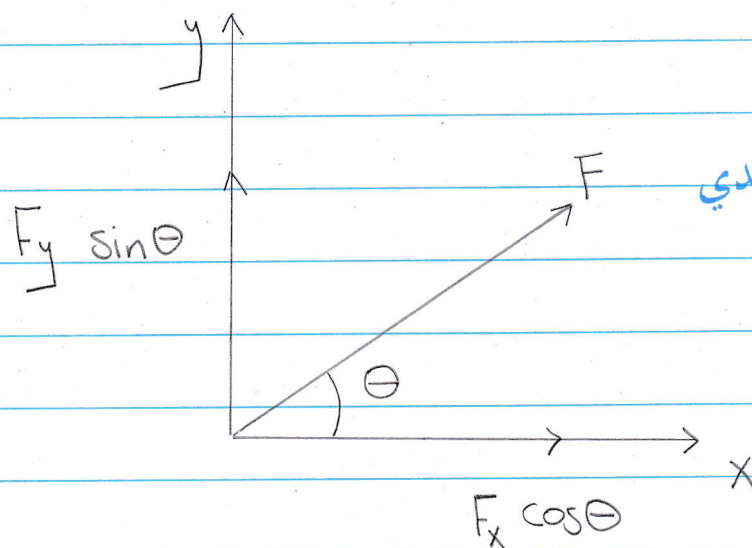
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(٦) حساب Resultant vector

$$\vec{R} = R_x \hat{i} + R_y \hat{j}$$

متجه
شعلة القوى

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$$R_x = \sum F_x$$

$$\sum F_x = F_x \cos \theta$$

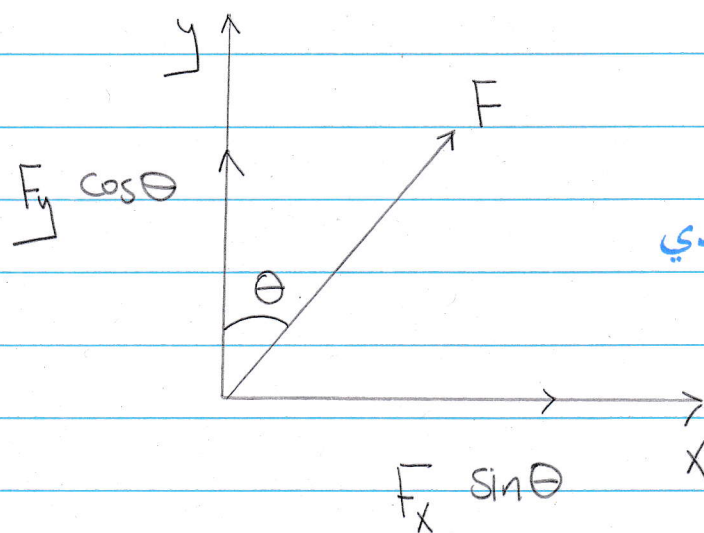
$$\therefore R_x = F_x \cos \theta$$

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$$R_y = \sum F_y$$

$$\sum F_y = F_y \sin \theta$$

$$\therefore R_y = F_y \sin \theta$$



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$$R_x = \sum F_x$$

$$\sum F_x = F_x \sin \theta$$

$$R_x = F_x \sin \theta$$

المهندسة
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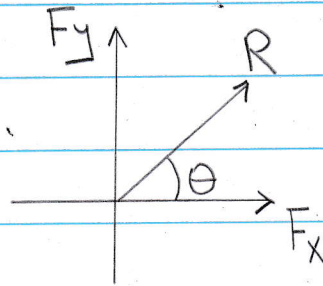
$$R_y = \sum F_y$$

$$\sum F_y = F_y \cos \theta$$

$$R_y = F_y \cos \theta$$

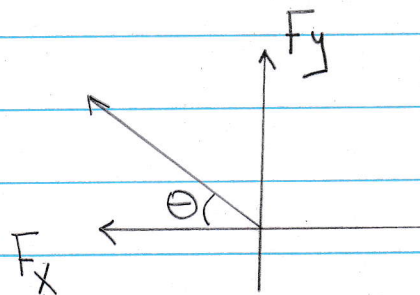
ملامحة : لرسم المحصلة واتجاهها =

Case 1 : F_x^+ , F_y^+



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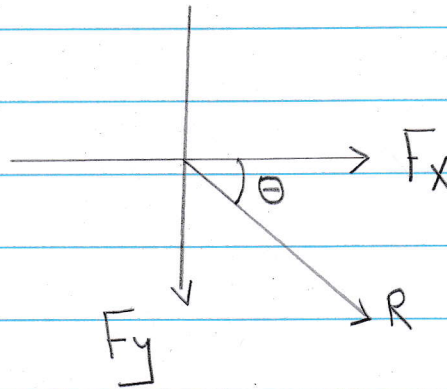
Case 2 : F_x^- , F_y^+



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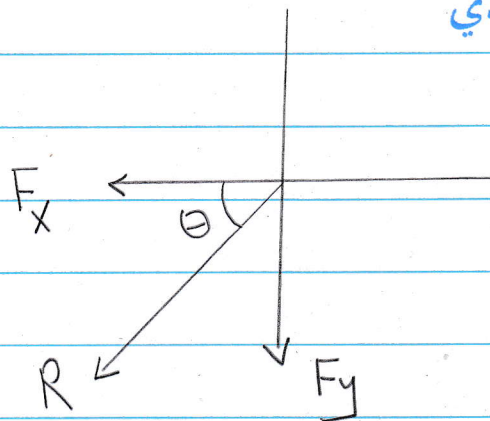
Case 3: F_x^+ , F_y^-

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case 4: F_x^- , F_y^-

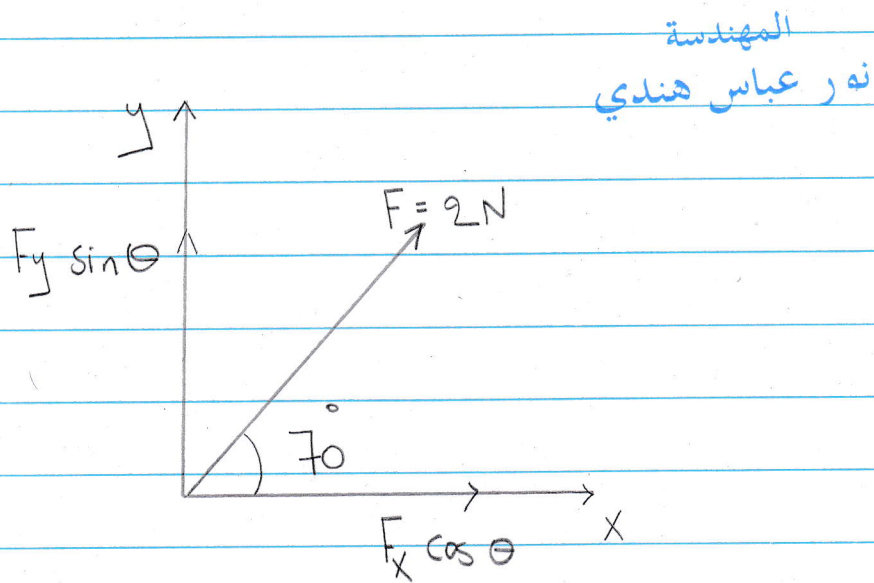
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θ تكون مع محور x دائماً

Ex:

Calculate the Resultant forces and its angle,
then find the force vector.



$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$R_x = \sum F_x$$

$$= F_x \cos \theta$$

$$= 2 \cos(70^\circ)$$

$$= 0.684 \text{ N}$$

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$$R_y = \sum F_y$$

$$= F_y \sin \theta$$

$$= 2 \sin(70^\circ)$$

(20)

$$R_y = 1.879 \text{ N}$$

$$\therefore R = \sqrt{(0.684)^2 + (1.879)^2}$$

$$R = 2 \text{ N}$$

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$$\therefore \Theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$$

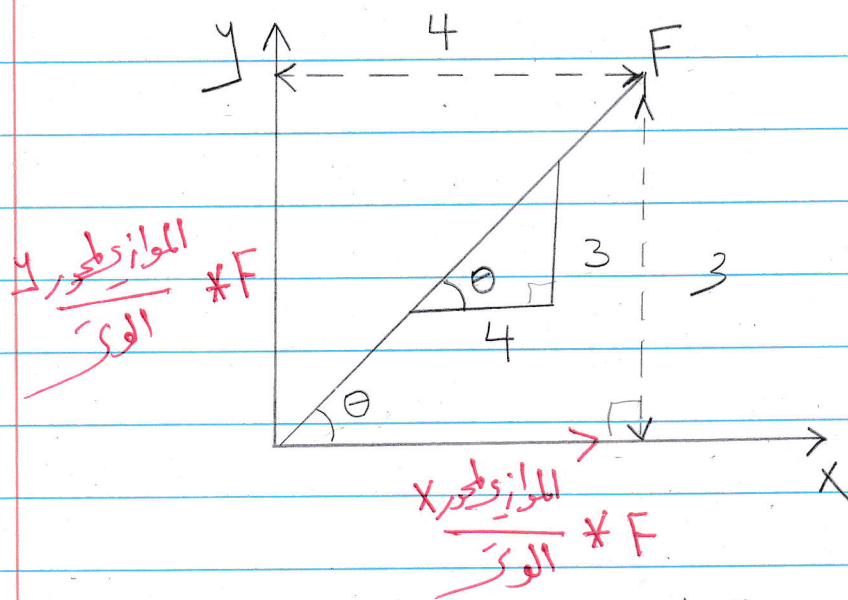
$$= \tan^{-1} \left(\frac{1.879}{0.684} \right)$$

$$\Theta = 70^\circ$$

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$$\therefore \vec{R} = R_x \hat{i} + R_y \hat{j}$$

$$\vec{R} = 0.684 \hat{i} + 1.879 \hat{j}$$



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$$\theta = \tan^{-1} \left(\frac{\text{مقابل}}{\text{جوار}} \right)$$

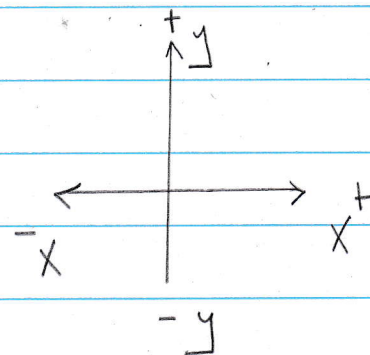
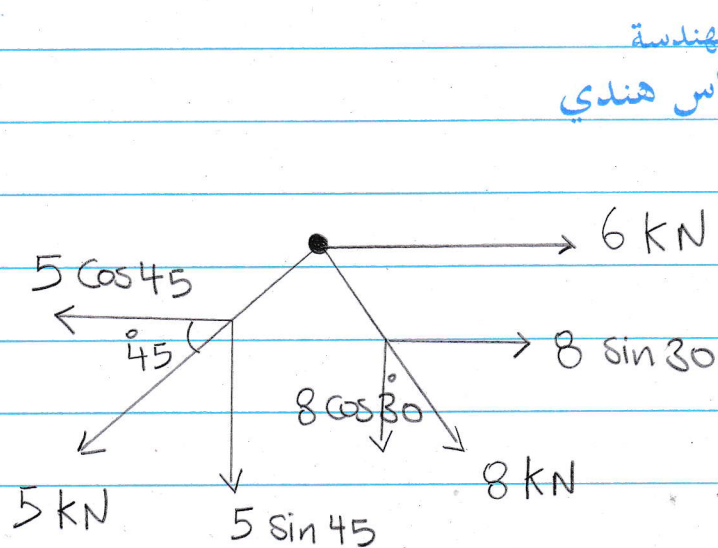
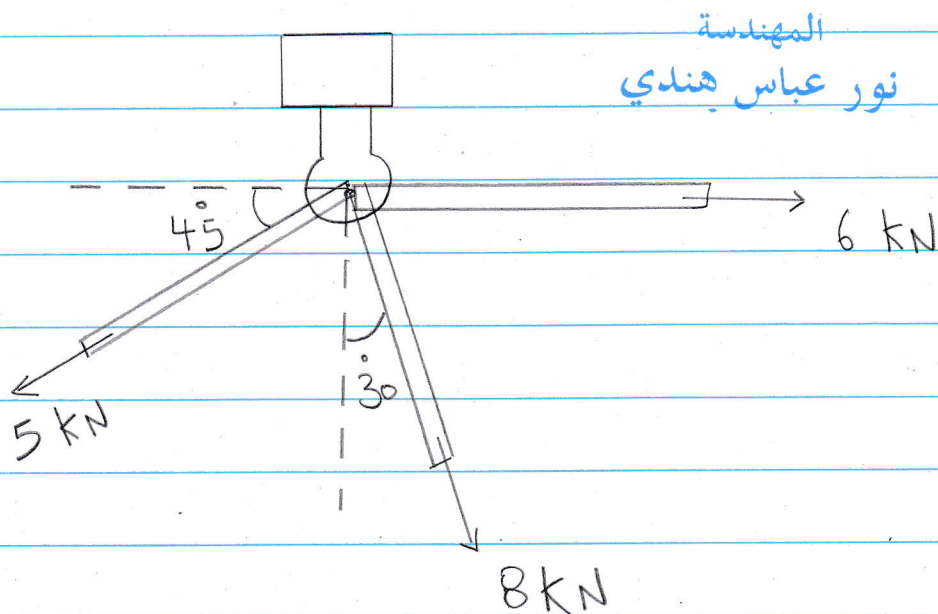
$$= \tan^{-1} \left(\frac{3}{4} \right)$$

$$\theta = 36.86$$

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Ex :

Determine the resultant forces



$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$R_x = \sum F_x$$

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$$= 6 + 8 \sin 30 - 5 \cos 45$$

$$= 6.464 \text{ kN} \longrightarrow R_x$$

لأن الإشارة
موجبة

$$R_y = \sum F_y$$

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$$= -5 \sin 45 - 8 \cos 30$$

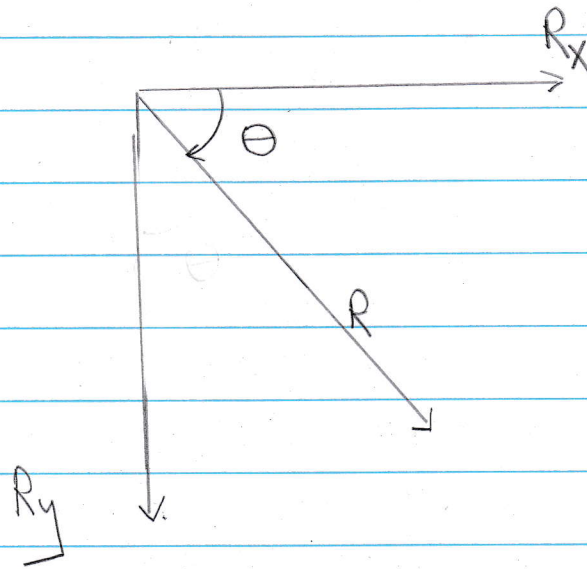
$$= -10.463 \text{ kN}$$

لأن الإشارة
سالبة
 $\downarrow R_y$

$$R = \sqrt{(6.464)^2 + (-10.463)^2}$$

$$= 12.3 \text{ kN}$$

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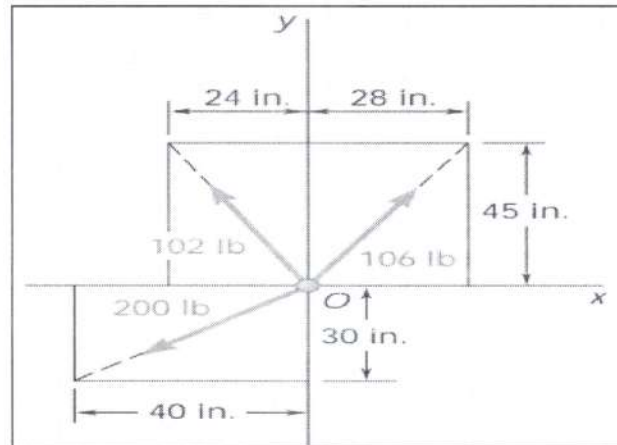
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$$\Theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$= \tan^{-1}\left(\frac{-10.463}{6.464}\right)$$

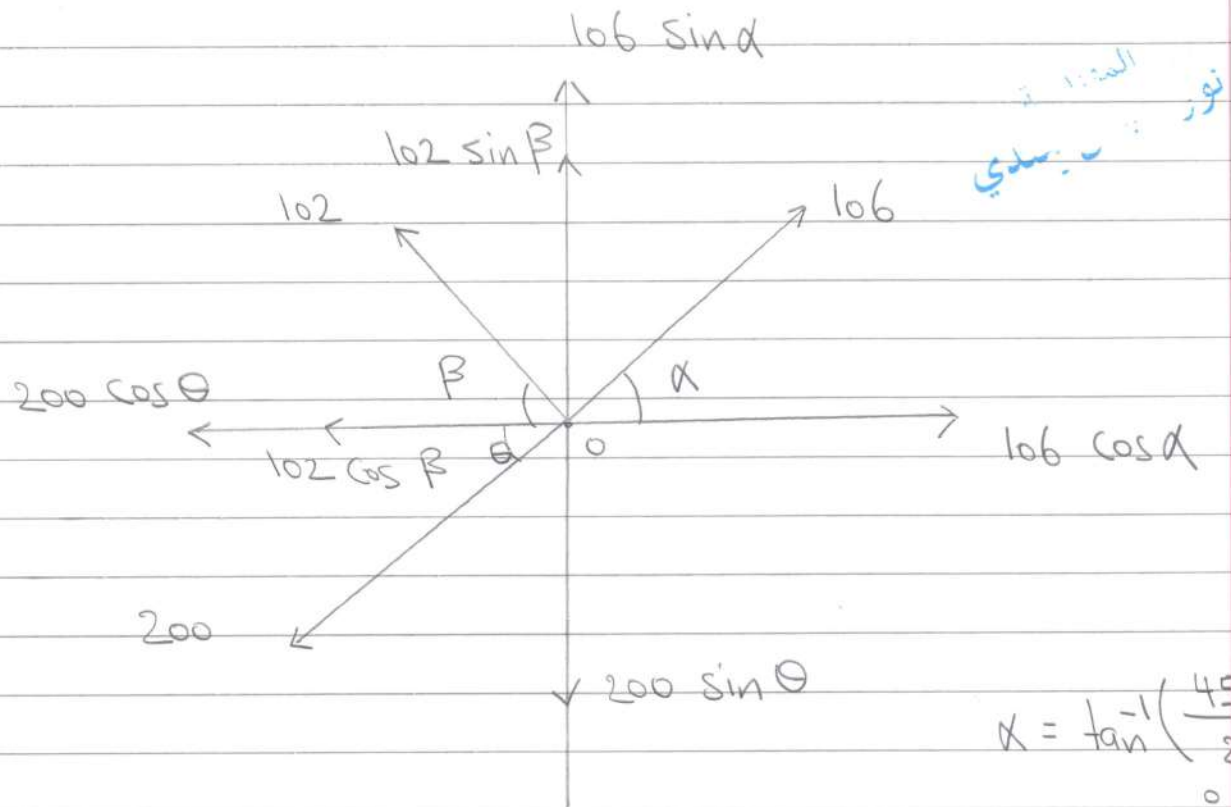
$$= -58.29$$

1. Determine the resultant and direction for each of the forces shown. Find the force vector.



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W. 1



$$\alpha = \tan^{-1}\left(\frac{45}{28}\right)$$

$$= 58.11^\circ$$

$$\beta = \tan^{-1}\left(\frac{45}{24}\right)$$

$$= 61.92^\circ$$

$$\theta = \tan^{-1}\left(\frac{30}{40}\right)$$

$$= 36.87^\circ$$

$$\sum F(x) = R(x)$$

$$= 106 \cos(58.11) - 102 \cos(61.92)$$

$$- 200 \cos(36.87)$$

$$= -152 \text{ Ib}$$

$$= 152 \text{ Ib} \leftarrow$$

$$\sum F_y = R_y$$

$$= 106 \sin(58.11) + 102 \sin(61.92) - 200 \sin(36.87)$$

$$= 60 \uparrow$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(-152)^2 + (60)^2}$$

$$= 163.41 \text{ Ib}$$

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$$\Theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$= \tan^{-1}\left(\frac{60}{-152}\right)$$

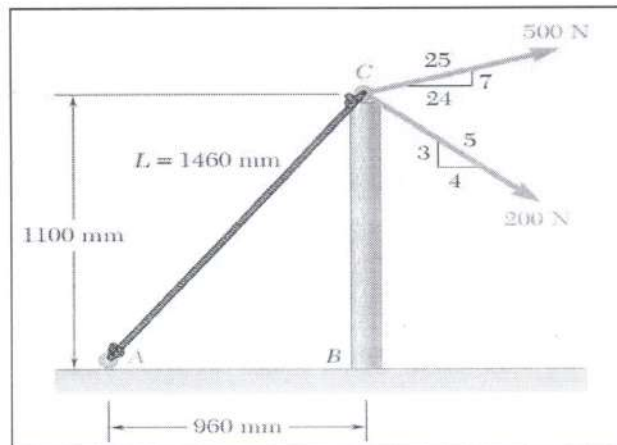
$$= -21.5^\circ$$

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$$\begin{aligned} \vec{R} &= R_x \mathbf{i} + R_y \mathbf{j} \\ &= -152 \mathbf{i} + 60 \mathbf{j} \end{aligned}$$

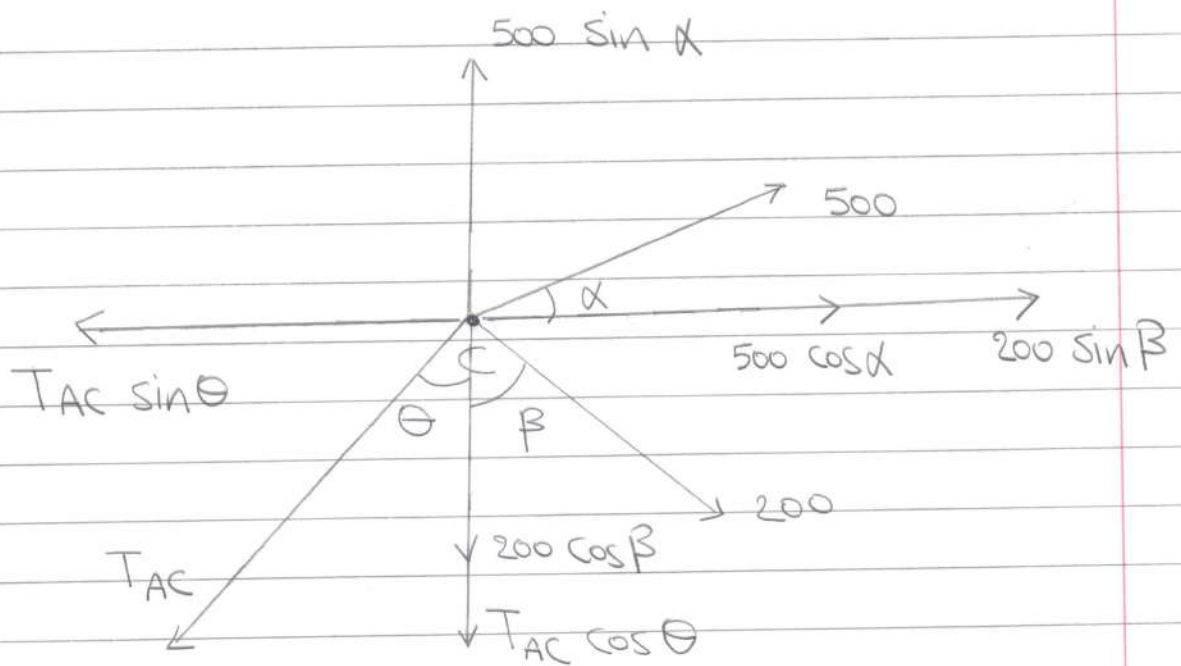
Resultant vector

5. For the post, determine (a) the required tension in rope AC if the resultant of the three forces exerted at point C is to be horizontal, (b) the corresponding magnitude of the resultant.



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a)



$$\alpha = \tan^{-1}\left(\frac{7}{24}\right)$$

$$= 16.26^\circ$$

$$\beta = \tan^{-1}\left(\frac{4}{3}\right)$$

$$= 53.13^\circ$$

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$$\theta = \tan^{-1}\left(\frac{960}{1100}\right)$$

$$= 41.11^\circ$$

$$\sum F_x = R_x$$

$$= 500 \cos(16.26) + 200 \sin(53.13) - T_{AC} \sin(41.11)$$

$$R_x = 640 - T_{AC} \sin(41.11) \dots \textcircled{1}$$

$$\sum F_y = R_y$$

$$= 500 \sin(16.26) - 200 \cos(53.13) - T_{AC} \cos(41.11)$$

$$R_y = 20 - T_{AC} \cos(41.11) \dots (2)$$

∴ the resultant of three forces exerted at point C is horizontal

$$R_y = 0 \quad \text{sub. in eq (2)}$$

$$0 = 20 - T_{AC} \cos(41.11)$$

$$T_{AC} = \frac{20}{\cos(41.11)}$$

$$= 26.54 \text{ N}$$

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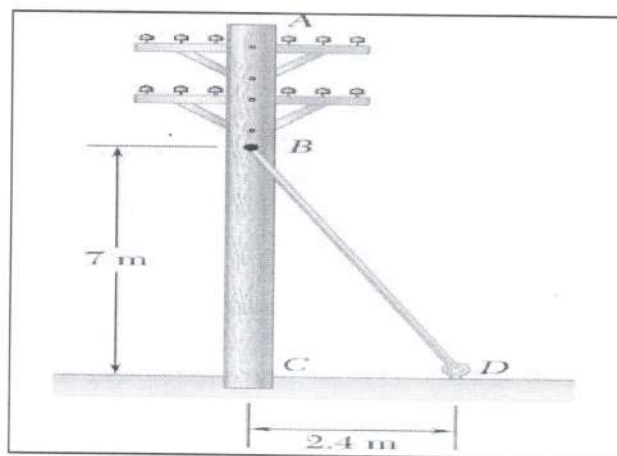
b)

$$R = \sqrt{(R_x)^2 + (R_y)^2} \rightarrow 0$$

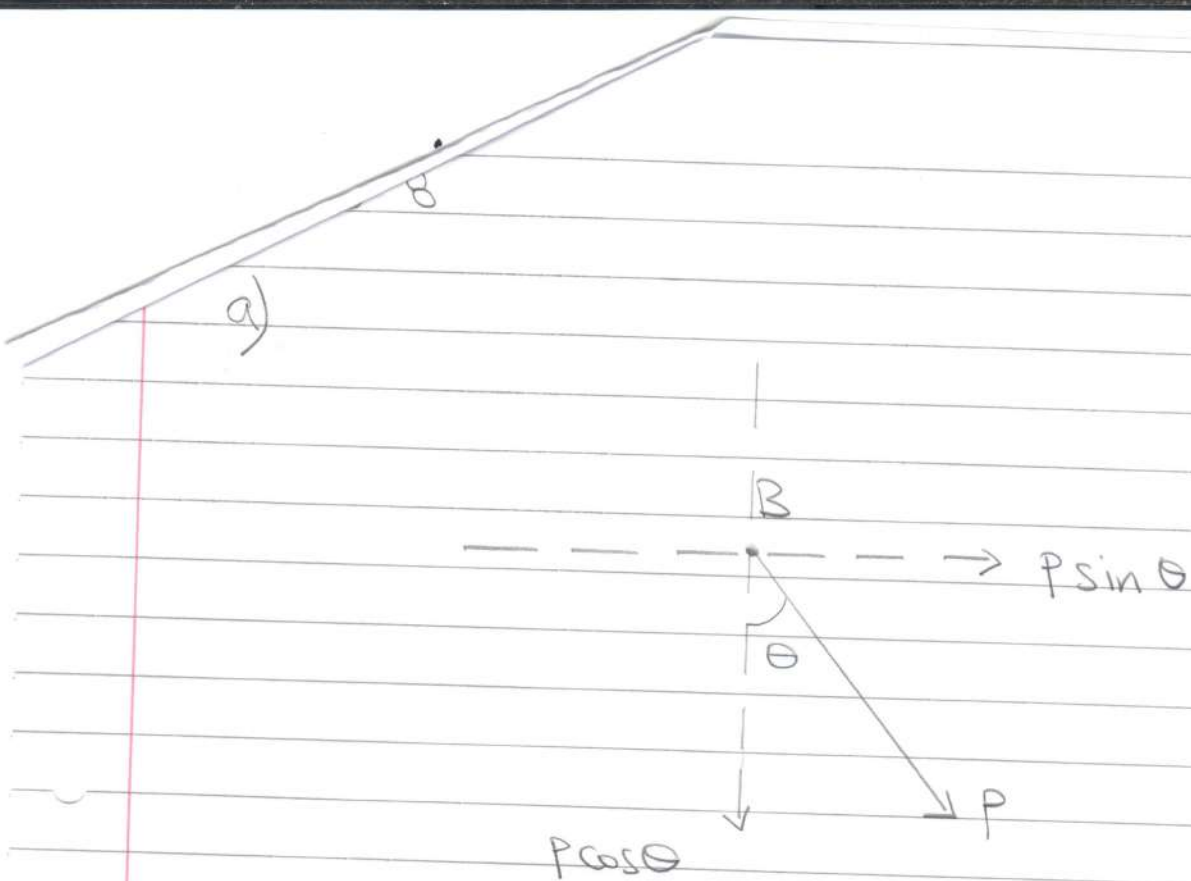
$$R = R_x = 640 - 26.54 \sin(41.11)$$

$$= 622.5 \text{ N}$$

8. The guy wire BD exerts on the telephone pole AC a force P directed along BD. Knowing that P must have a 720-N component perpendicular to the pole AC, determine (a) the magnitude of the force P , (b) its component along line AC.



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$$\theta = \tan^{-1} \left(\frac{2.4}{7} \right)$$

$$= 18.92^\circ$$

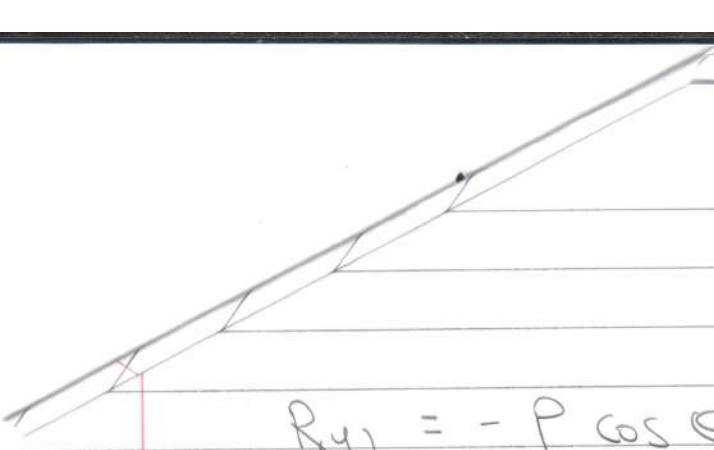
$$R_{(x)} = P \sin(18.92)$$

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$$720 = P \sin(18.92)$$

$$P = \frac{720}{\sin(18.92)}$$

$$= 2220.5 \text{ N}$$


$$R_y) = - P \cos \theta$$

$$= - 2220.5 \cos (18.92)$$

$$= - 2100.6 \text{ N}$$

$$= 2100.6 \text{ N} \downarrow$$

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Mechanics Engineering Statics

Equilibrium of a Particle

When the resultant of all the forces acting on a particle (body) is zero, the particle is in equilibrium.

To maintain equilibrium, it is necessary to satisfy Newton's first law of motion which requires the resultant force acting on a particle to be equal to zero.

$$\sum F = 0$$

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Free Body Diagram (F.B.D)

is a sketch showing the body (particle) and all the forces (and reaction) acting on it.

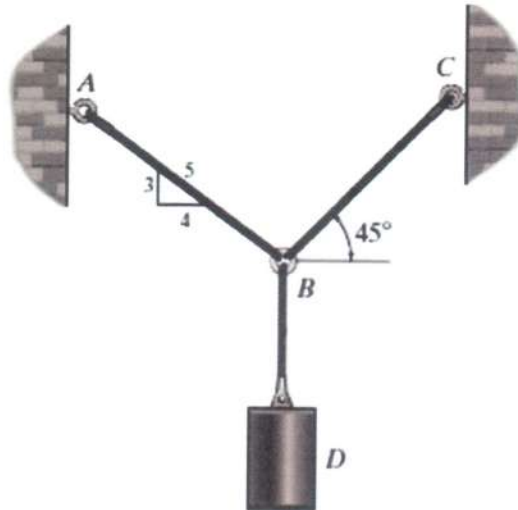
$$\sum F_x = 0$$

$$\sum F_y = 0$$

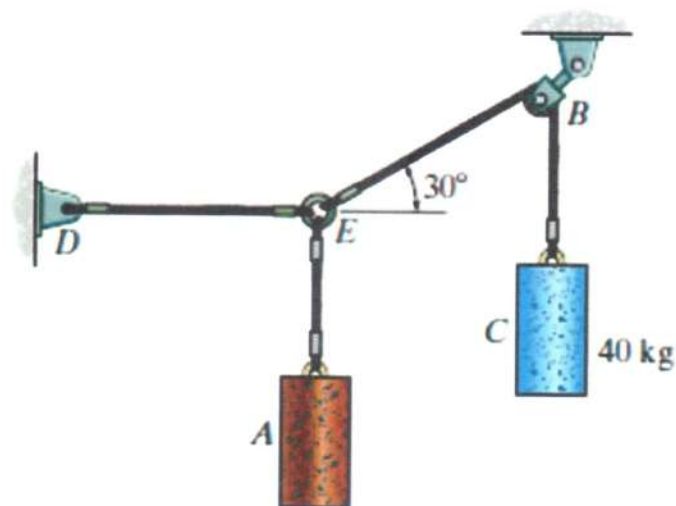
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يعوض عن الكيل بقوة الشر . واتجاه الوزن يكون دائماً للأسفل

Example -1: Determine the tension in cables BA and BC necessary to support the 60-kg cylinder.



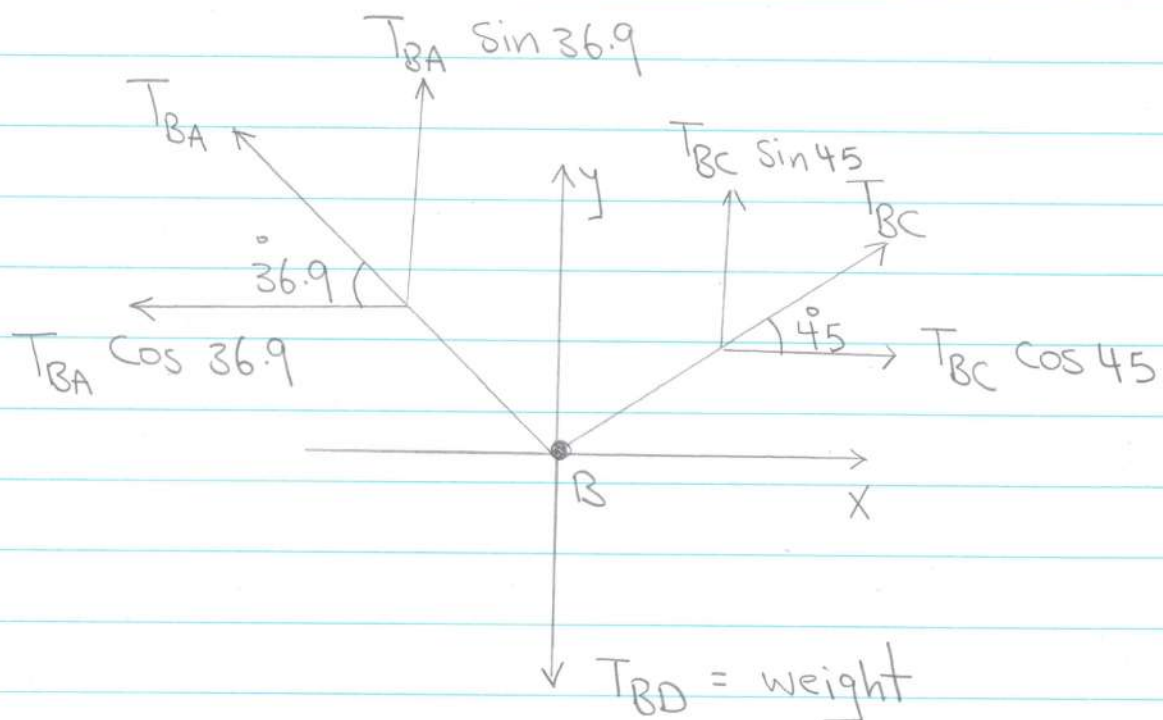
Example -2: If the mass of cylinder C is 40 kg, determine the mass of cylinder A in order to hold the assembly in the position shown.



Example 1 Solution of Q1

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F.B.D



$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$

$$= 36.9$$

$$T_{BD} = \text{weight}$$

$$= 60 \times 9.81$$

$$= 588.6 \text{ N}$$

المهندسة
نور عباس هندي

$$\sum F_x = 0$$

$$T_{BC} \cos 45 - T_{BA} \cos 36.9 = 0$$

$$0.707 T_{BC} = 0.8 T_{BA}$$

$$T_{BC} = \frac{0.8}{0.707} T_{BA}$$

(3)
4

$$T_{BC} = 1.132 T_{BA} \dots (1)$$

$$\sum F_y = 0$$

المهندسة
نور عباس هندي

$$T_{BA} \sin 36.9 + T_{BC} \sin 45 - 588.6 = 0$$

$$0.6 T_{BA} + 0.707 T_{BC} = 588.6$$

$$0.6 T_{BA} + 0.707 (1.132 T_{BA}) = 588.6$$

$$0.6 T_{BA} + 0.8 T_{BA} = 588.6$$

$$T_{BA} = \frac{588.6}{1.4}$$

$$= 420.43 \text{ N}$$

المهندسة
نور عباس هندي

sub. in eq (1)

$$T_{BC} = 1.132 (420.43)$$

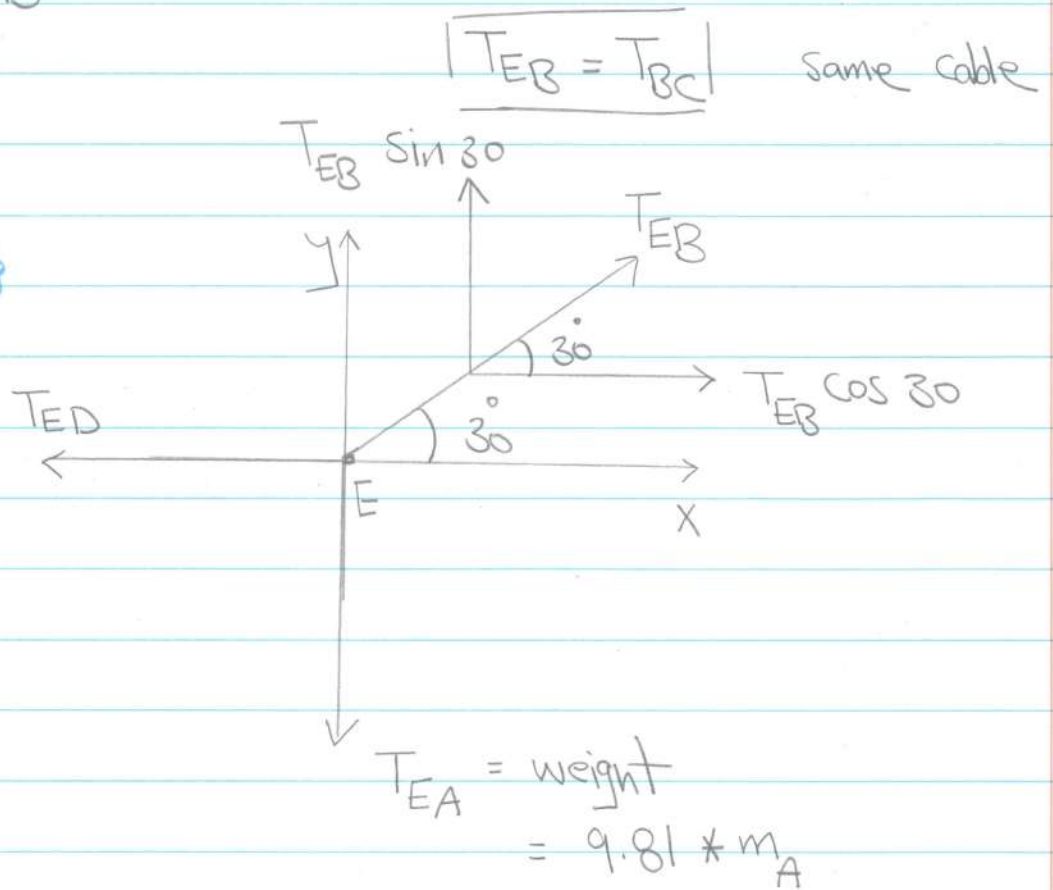
$$= 475.93 \text{ N}$$

(4)

Example 2
Solution of

F.B.D

المهندسة
نور عباس شندلي



(mass)

$$T_{EB} = 40 * 9.81$$
$$= 392.4 \text{ N} = T_{BC}$$

المهندسة
نور عباس شندلي

$$\sum F_x = 0$$

$$T_{EB} \cos 30 - T_{ED} = 0$$

$$T_{ED} = T_{EB} \cos 30$$

(7)
6

$$T_{ED} = 392.4 \cos 30$$

$$= 339.83 \text{ N}$$

$$\sum F_y = 0$$

المهندسة
نور عباس هندي

$$T_{EB} \sin 30 - 9.81 m_A = 0$$

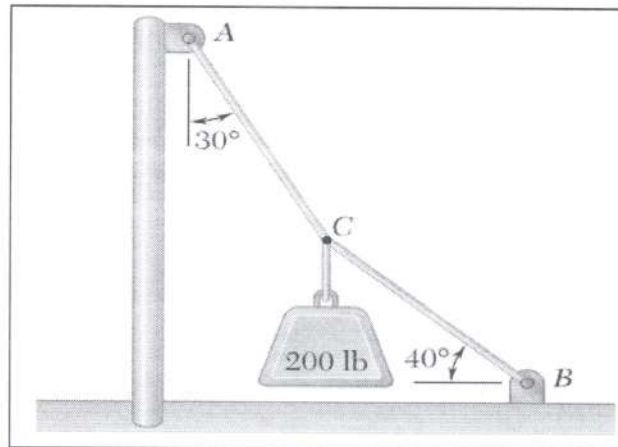
$$m_A = \frac{T_{EB} \sin 30}{9.81}$$

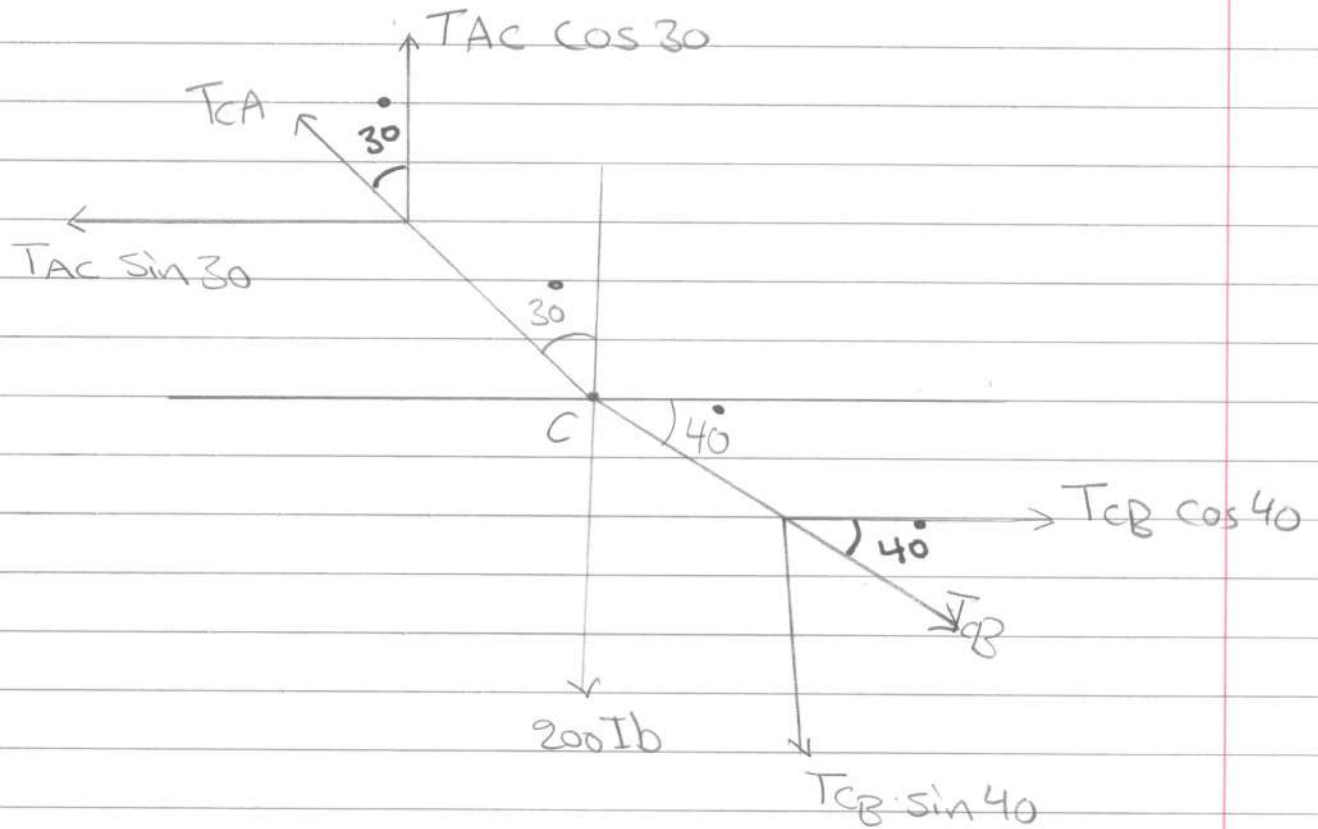
$$= \frac{392.4 \sin 30}{9.81}$$

$$m_A = 20 \text{ kg}$$

المهندسة
نور عباس هندي

H.W 1: Two cables are tied together at C and loaded as shown. Draw the free-body diagram needed to determine the tension in AC and BC



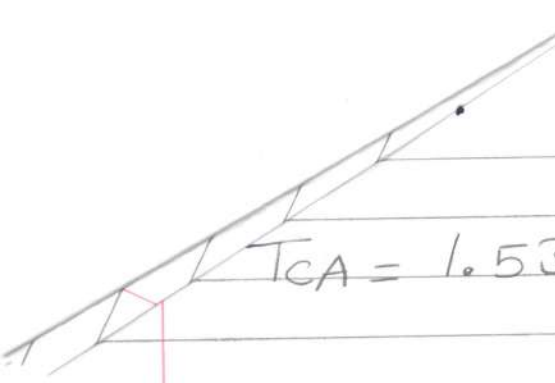


$$\sum F_x = 0$$

$$T_{CB} \cos 40 - T_{CA} \sin 30 = 0$$

$$T_{CB} \cos 40 = T_{CA} \sin 30$$

$$T_{CA} = T_{CB} \frac{\cos 40}{\sin 30}$$


$$T_{CA} = 1.53 T_{CB} \quad \dots (1)$$

$$\sum F_y = 0$$

$$T_{CA} \cos 30 - T_{CB} \sin 40 - 200 = 0 \quad \dots (2)$$

Sub eq. (1) in eq (2)

$$(1.53 T_{CB}) \cos 30 - T_{CB} \sin 40 - 200 = 0$$

$$1.326 T_{CB} - 0.642 T_{CB} = 200$$

$$0.683 T_{CB} = 200$$

$$T_{CB} = \frac{200}{0.683}$$

$$T_{CB} = 292.73 \text{ Ib.}$$

Sub in (1)

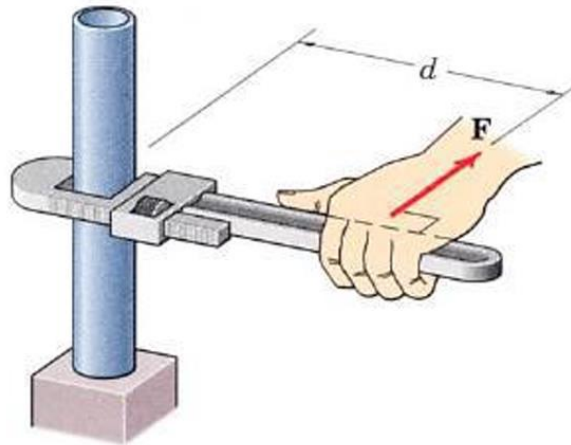
$$T_{CA} = 1.53 \times 292.73$$

$$= 447.8 \text{ Ib}$$



1. Moment of a Force

The moment of a force is the ability of the force to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force.



The magnitude of the moment is:

$$M = F \cdot d$$

Where:

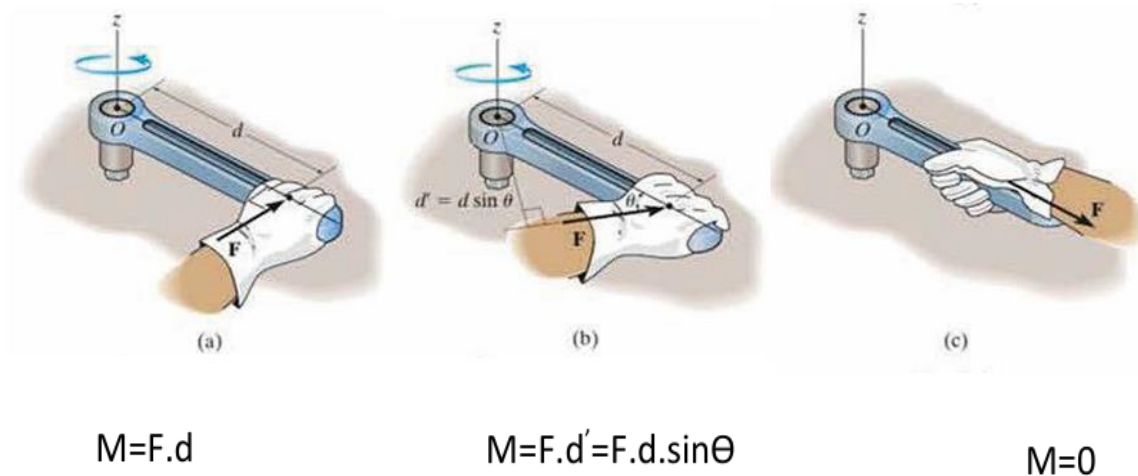
M: The moment of the force (N.m).

d: is the perpendicular distance from the axis moment to the line of action of the force.

Units: kN.m, N.m (1 kN = 1000 N).

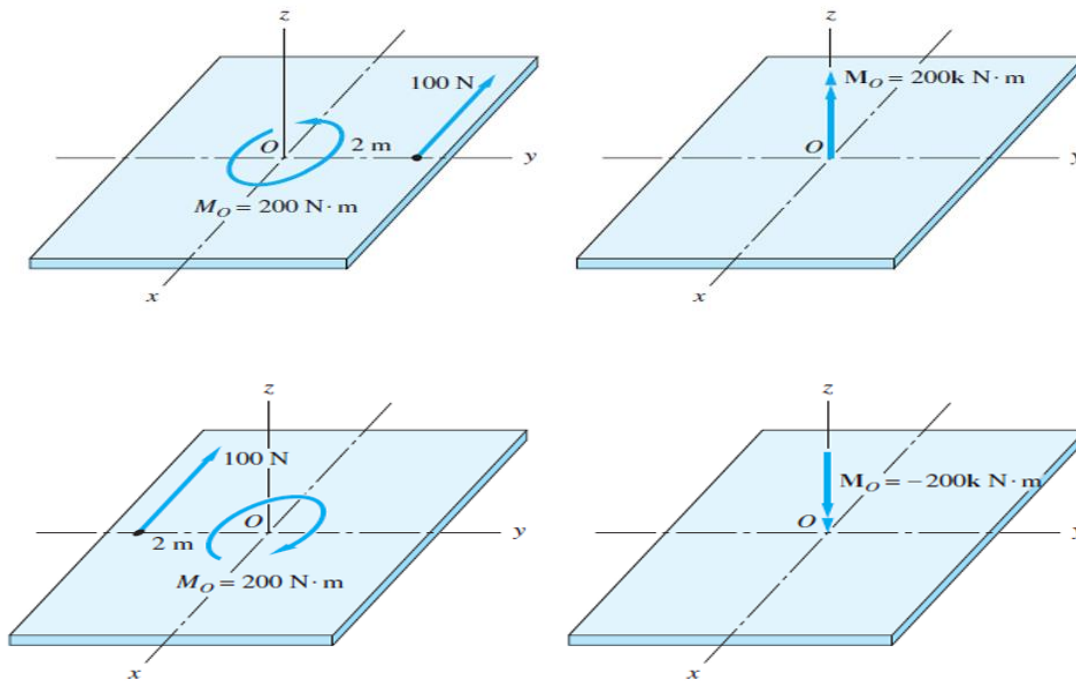
Note 1: If the force F is applied at an angle $\theta \neq 90^\circ$, Figure b, then it will be difficult to turn the bolt since the moment arm $d' = d \sin \theta$ will be smaller than d .

Note 2: If F is applied along the wrench, Figure c, its moment arm will be zero since the line of action of F will intersect point O (the z-axis). As a result, the moment of F about O is also zero and no turning can occur.

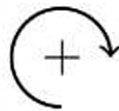


The moment M_o about O , or about an axis passing through O and perpendicular to the plane, is a **vector quantity** since it has a specified **magnitude** and **direction**.

1.2 Moment's Direction



As a convention, we will generally consider **positive moment as Clockwise**. **Counterclockwise moment will be negative**. Doing this, the directional sense of each moment can be represented by a **plus** or **minus** sign.



1.3 Principle of Moments (Resultant Moment)

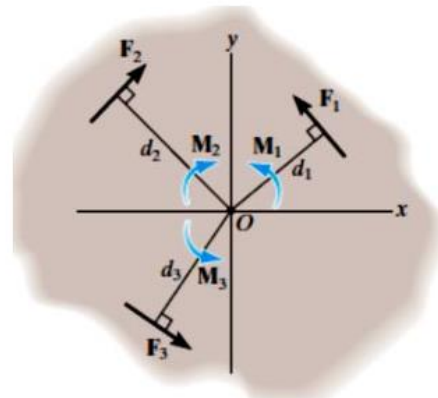
When determining the moment of a force about a point, it is often convenient to use the **principle of moments**, also known as **Varignon's theorem** which indicates that: **The moment of a force about a point is equal to the sum of the moments of its components about that point.**

$$M_o = \sum F \cdot d$$

The resultant moment for the figure below is:

$$(M_R)_O = \sum F \cdot d$$

$$(M_R)_O = -F_1 \cdot d_1 + F_2 \cdot d_2 - F_3 \cdot d_3$$



1.4 Examples

Example (1): For each case illustrated in the figure, determine the moment of the force about point O.

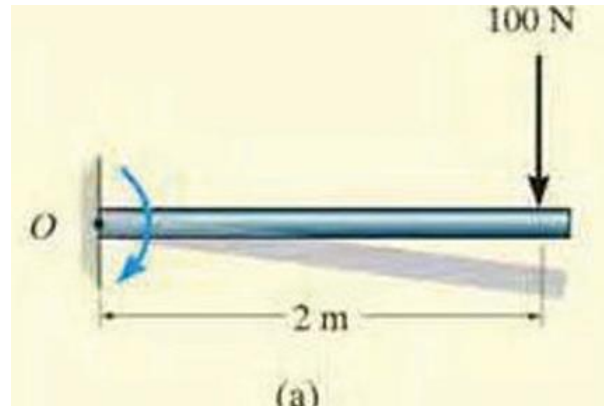


Figure (a):

القوة $F = 100 \text{ N}$ عمودية للأعلى.
المسافة من النقطة O إلى نقطة تأثير القوة = 2 m .

① قانون العزم: $M_o = F * d * \sin(\theta) \quad (+)$

كنى هذا الزاوية بين الذراع والقوة هي 90° لأن القوة عمودية على الذراع.

$$\sin(90^\circ) = 1$$

$$M_o = F * d * 1$$

$$= 100 * 2 \Rightarrow M_o = +200 \text{ N.m}$$

يتم تحديد الاتجاه: بما أن القوة تسبب للأعلى والذراع أفقية من O فماذا يكون،
عندئذ العزم يكون مع عقارب الساعة. ولأنه موجب فمبدأنا في بداية الحل سيكون
نتيجة العزم موجب.

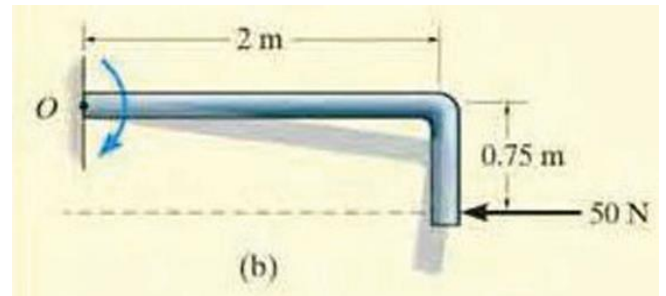


Figure (b):

لحساب العزم نحتاج إلى معرفة المسافة العمودية بين خط تأثير القوة ولبقعة O وفي هذه الحالة فإن القوة أفقية فإن المسافة العمودية هي $d = 0.75 \text{ m}$

$$+ \curvearrowleft M_o = F * d$$

$$= 50 * 0.75$$

$$= + 37.5 \text{ N.m} \quad + \curvearrowleft$$

الاتجاه موجب عن القوة تسبب ارتداد الجسم مع اتجاه دوران عقارب الساعة.

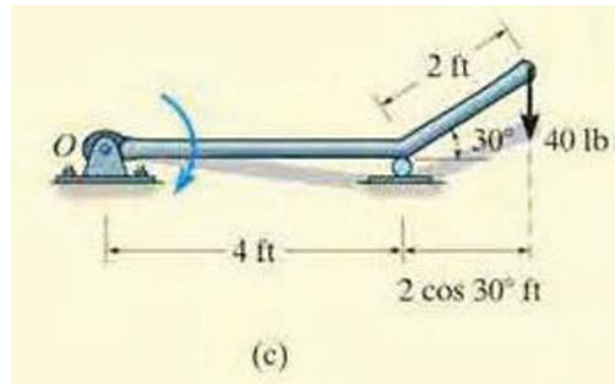
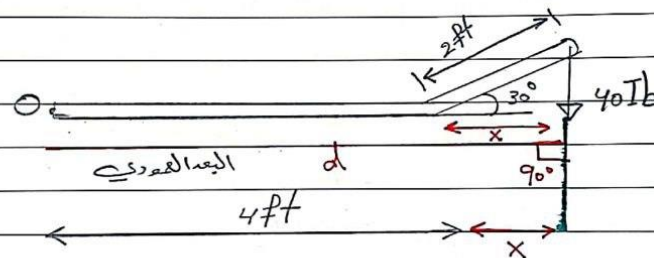


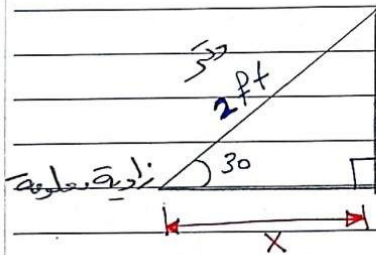
Figure (c):

① ندرس خط مستقيم على امتداد القوة $F = 40 \text{ lb}$.
 ② نقوم بتقسيم نقطة O على خط مستقيم على امتداد القوة F . وهذا يمثل كل البعد العمودي d .

$$M_o = F * d \quad (+)$$



لدينا $d = 4 \text{ ft} + x$ والبعد العمودي d والقياسات قائمة الزاوية.
 نستخدم قوانين المثلثات قائمة الزاوية.



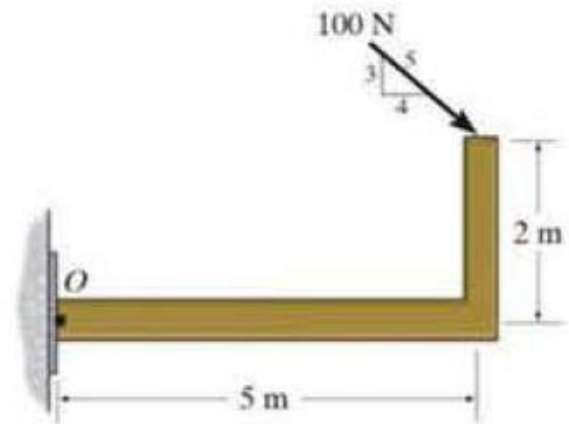
نطبق قانون $\cos$ من المثلثات قائمة الزاوية.

$$\cos \theta = \frac{\text{الجانب المجاور}}{\text{الوتر}} \Rightarrow \cos(30^\circ) = \frac{x}{2} \Rightarrow x = 2 \cos 30^\circ \text{ ft}$$

$$M_o = [40 * (4 + 2 \cos 30^\circ)]$$

$$M_o = +229 \text{ lb.ft} \quad (+)$$

Example (2): Determine the moment of the force about point O.



Solution :

$$M_o = F \cdot d \quad (+)$$

$$M_o = (F_x \cdot d) + (F_y \cdot d)$$

$$F_x = F \cos \theta$$

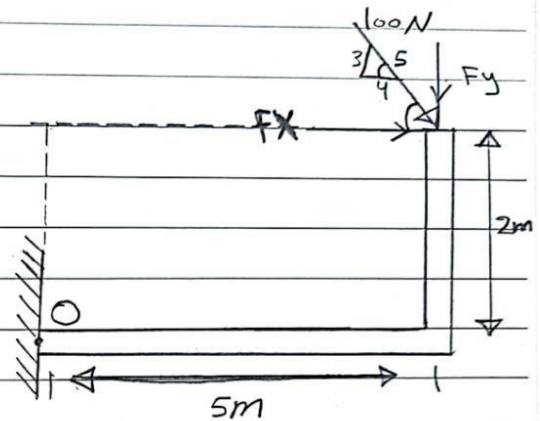
$$F_y = F \sin \theta$$

$$\theta = \tan^{-1} \left(\frac{\text{المقابل}}{\text{الجوار}} \right)$$

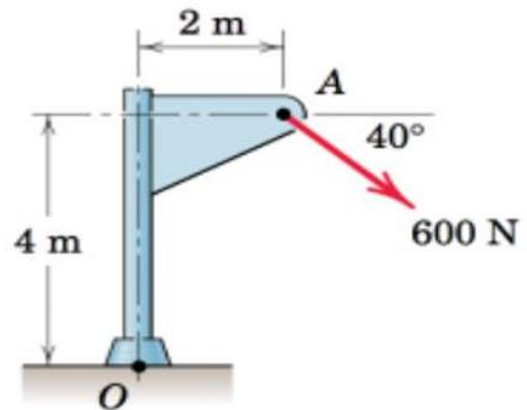
$$\theta = \tan^{-1} \left(\frac{3}{4} \right) \Rightarrow \theta = 36.86^\circ$$

$$M_o = (100 \cdot \cos 36.86 \cdot 2) + (100 \cdot \sin 36.86 \cdot 5)$$

$$M_o = 459.9 \approx +460 \text{ N.m} \quad (+)$$



Example (3): Calculate the moment about the base point O of the 600 N force.



Solution:

~~~~~

$$F_x = F \cos 40$$

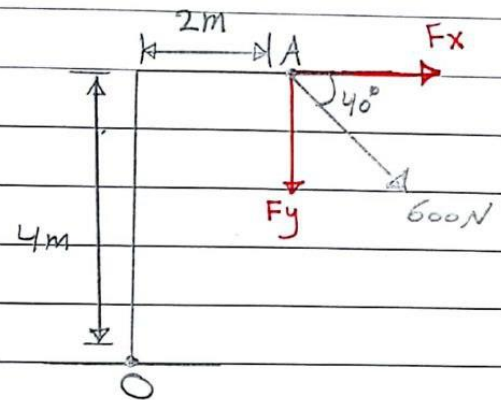
$$= 600 \cos 40$$

$$= 459.63 \text{ N} \rightarrow$$

$$F_y = F \sin 40$$

$$= 600 \sin 40$$

$$= 385.67 \text{ N} \downarrow$$



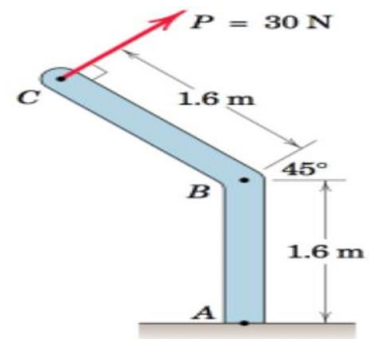
$$\sum M_o = F * d \quad (+)$$

$$\sum M_o = + F_x * d_1 + F_y * d_2$$

$$\sum M_o = +(459.63 * 4) + (385.67 * 2)$$

$$\sum M_o = 2609.86 \text{ N.m} \quad (+)$$

Example (4): The 30 N force P is applied perpendicular to the portion BC of the bent bar. Determine the moment of P about point A and point B?



Solution:

$$M_B = F \times d \quad (+)$$

$$M_B = +(30 \times 1.6)$$

$$M_B = 48 \text{ N}\cdot\text{m} \quad (+)$$

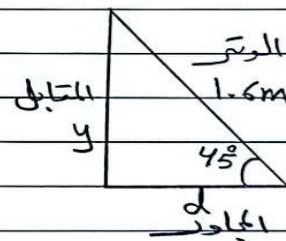
$$P_x = 30 \cos 45^\circ = 21.213 \text{ N}$$

$$P_y = 30 \sin 45^\circ = 21.213 \text{ N}$$

$$\theta = 180^\circ - 90^\circ - 45^\circ \Rightarrow \theta = 45^\circ$$

$$\cos 45^\circ = \frac{d}{1.6} \Rightarrow d = 1.6 \cos 45^\circ$$

$$\sin 45^\circ = \frac{y}{1.6} \Rightarrow y = 1.6 \sin 45^\circ$$

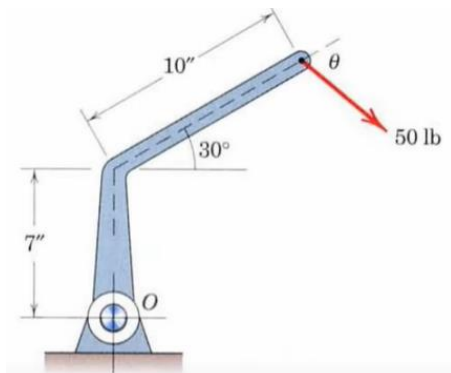


$$\sum M_A = + \left[ \overset{21.21}{\underbrace{(30 \sin 45^\circ \times 1.6 \cos 45^\circ)}} \right] + \left[ \overset{21.21}{\underbrace{(30 \cos 45^\circ \times (1.6 + 1.6 \sin 45^\circ))}} \right]$$

$$\sum M_A = +24 + 57.941$$

$$\sum M_A = 81.941 \text{ N}\cdot\text{m}$$

Example (5): Determine the angle  $\theta$  Which will maximize the moment  $M_o$  of the 50-lb force about the shaft axis at O. Also compute  $M_o$ .



Solution

~~~~~

$$M_o = F \times d$$

$$\cos 30^\circ = \frac{10}{X}$$

$$X = 10 \cos 30^\circ$$

$$\sin 30^\circ = \frac{9}{10}$$

$$9 = 10 \sin 30^\circ$$

$$d = \sqrt{(10 \cos 30^\circ)^2 + (7 + 10 \sin 30^\circ)^2}$$

$$\therefore d = 14.8$$

$$\frac{7}{\sin \alpha} = \frac{14.8}{\sin 120^\circ}$$

$$\sin \alpha = \frac{7 \times \sin 120^\circ}{14.8} \Rightarrow \sin \alpha = 0.409$$

$$\alpha = \sin^{-1}(0.409) \Rightarrow \alpha = 24.1^\circ$$

$$\therefore \theta = 180^\circ - \alpha - 90^\circ = 180^\circ - 24.1^\circ - 90^\circ$$

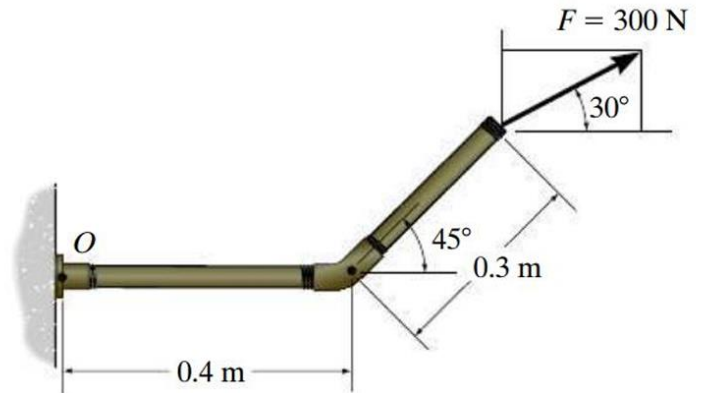
$$\therefore \theta = 65.9^\circ$$

$$M_o = F \times d \quad \curvearrowright$$

$$M_o = 50 \times 14.8 \Rightarrow M_o = 740 \text{ lb} \cdot \text{in}$$

Homework:

1. Determine the moment of the force about point O.



2. A crate of mass 80 kg is held in the position shown. Determine (a) the moment produced by the weight W of the crate about E, (b) the smallest force applied at B that creates a moment of equal magnitude and opposite sense about E.

