Power Mechanics Technical Engineering Department

Second Stage

Fluid Mechanics - Static

Chapter one: Fluid Properties

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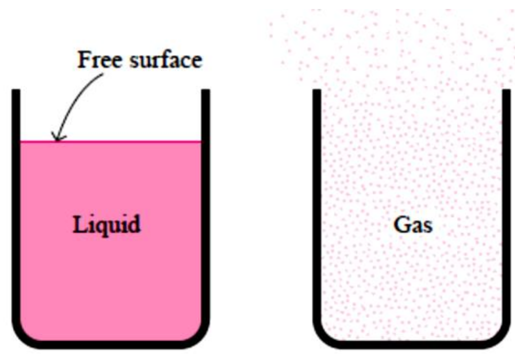
1. Introduction

Fluid mechanics is defined as the science that deals with the behavior of fluids at rest (*fluid statics*) or in motion (*fluid dynamics*), and the interaction of fluids with solids or other fluids at the boundaries.

From the point of view of fluid mechanics, all matter consists of only two states, fluid and solid. The difference between fluid and solid is the reaction of the two to an applied shear stress. *A solid can resist a shear stress; a fluid cannot*. Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as the shear stress is applied. We can say that a fluid at rest must be in a state of zero shear stress.

Given the definition of a fluid above, there are two classes of fluids, ***liquids*** and ***gases***. The distinction between *liquids* and *gases* concerning with the effect of cohesive forces. A liquid, being composed of relatively close-packed molecules with strong cohesive forces, tends to retain its volume and will form a free surface in a gravitational field if unconfined from above.



Since gas molecules are widely spaced with negligible cohesive forces, a gas is free to expand, has no definite volume, and Gases cannot form a free surface.

2. Fluid Power System

Fluid Power is the technology that deals with the generation, control, and transmission of power, using pressurized fluids. Fluid power is called ***hydraulics*** when the fluid is a liquid and is called ***pneumatics*** when the fluid is a gas.

Hydraulic systems use liquids such as petroleum oils, synthetic oils, and water. Pneumatic systems use air as the gas medium because air is very abundant and can be readily exhausted into the atmosphere after completing its assigned task.

3. Dimensions and Units

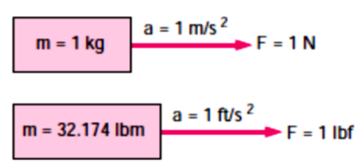
Any physical quantity can be characterized by **dimensions.** The magnitudes assigned to the dimensions are called **units.** Some basic dimensions such as mass *m*, length *L*, time *t*, and temperature *T* are selected as **primary dimensions,** while others such as velocity *V*, energy *E*, and volume *V* are expressed in terms of the primary dimensions and are called **secondary dimensions.**

A number of unit systems have been developed over the years. Two sets of units are still in common use today: the **English system,** which is also known as the *United States System*, and the metric **SI** which is known as the *International System.* SI was based on six fundamental quantities, and their units; *meter* (m) for length, *kilogram* (kg) for mass, *second* (s) for time, *ampere* (A) for electric current, *degree Kelvin* (°K) for temperature, and *candela* (cd) for luminous intensity (amount of light). The prefixes used to express the multiples of the various units are listed in Table. They are standard for all units.

Standard prefixes in SI units

Multiple	Prefix
10^{12}	tera, T
10^{9}	giga, G
10^{6}	mega, M
10^{3}	kilo, k
10^{2}	hecto, h
10^{1}	deka, da
10^{-1}	deci, d
10^{-2}	centi, c
10^{-3}	milli, m
10^{-6}	micro, μ
10^{-9}	nano, n
10^{-12}	pico, p

In SI, the units of **mass**, **length**, and time are the kilogram (**kg**), meter (**m**), and second (s), respectively. The respective units in the English system are the pound-mass (**lbm**), foot (**ft**), and second (s). The mass and length units in the two systems are related to each other by



$$\mathbf{1\ lbm = 0.45359\ kg}$$

$$\mathbf{1\ ft = 0.3048\ m\ ,1\ m = 3.281\ ft}$$

Force is a secondary dimension whose unit is derived from Newton's second law,

$$Force = (Mass)(Acceleration) \quad \rightarrow \quad F = ma$$

In SI, the force unit is the newton (**N**), and it is defined as the *force required to accelerate a mass of 1 kg at a rate of 1 m/s^2*. In the English system, the force unit is the pound-force (**lbf**) and is defined as the *force required to accelerate a mass of 32.174 lbm (1 slug) at a rate of 1 ft/s^2*. That is,

$$\mathbf{1\ N = 1\ kg.m/s^2}$$
$$\mathbf{1\ lbf = 32.174\ lbm.ft/s^2}$$

The term **weight** is often incorrectly used to express mass, weight *W* is a *force*. It is the gravitational force applied to a body, and its magnitude is determined from Newton's second law,

$$W = m\,g \qquad (N)$$

where *m* is the mass of the body, and *g* is the local gravitational acceleration (*g* is 9.807 m/s^2 or 32.174 ft/s^2 at sea level and 45° latitude).

Work, which is a form of energy, can simply be defined as force times distance; therefore, it has the unit "newton-meter (N. m)," which is called a joule (**J**). A more common unit for energy in SI is the kilojoule (1 kJ = 10^3 J). In the English system, the energy unit is the **Btu** (British thermal unit). The magnitudes of the kilojoule and Btu are almost identical (1 Btu = 1.0551 kJ).

The English system, however, has no apparent systematic numerical base, and various units in this system are related to each other rather arbitrarily (**12 in = 1 ft**, 1 mile = 5280 ft, etc.)

Quantity	Dimension	SI	Units	English	Units
Length l	L	meter	m	foot	ft
Mass m	M	kilogram	kg	slug	slug
Time t	T	second	s	second	sec
Temperature T	Θ	kelvin	K	Rankine	°R
Plane angle		radian	rad	radian	rad

Basic Dimensions and Their Units

Unity Conversion Ratios

All ***secondary unit*** *can be formed by combinations of primary units*. Force units, for example, can be expressed as

$$N = kg.\frac{m}{s^2} \qquad \text{and} \qquad lbf = 32.174\ lbm\,.\frac{ft}{s^2}$$

They can also be expressed more conveniently as **unity conversion ratios** as

$$\frac{N}{kg.m/s^2} = 1 \qquad and \qquad \frac{lbm}{32.174\ lbm.ft/s^2} = 1$$

Unity conversion ratios are identically equal to 1 and are unitless, and thus such ratios (or their inverses) can be inserted conveniently into any calculation to properly convert units.

Quantity	Dimension	SI units	English units
Area A	L^2	m^2	ft^2
Volume $\forall$	L^3	m^3 or L (liter)	ft^3
Velocity V	L/T	m/s	ft/sec
Acceleration a	L/T^2	m/s^2	ft/sec^2
Angular velocity Ω	T^{-1}	s^{-1}	sec^{-1}
Force F	ML/T^2	$kg{\cdot}m/s^2$ or N (newton)	$slug{\text -}ft/sec^2$ or lb
Density ρ	M/L^3	kg/m^3	$slug/ft^3$
Specific weight γ	M/L^2T^2	N/m^3	lb/ft^3
Frequency f	T^{-1}	s^{-1}	sec^{-1}
Pressure p	M/LT^2	N/m^2 or Pa (pascal)	lb/ft^2
Stress τ	M/LT^2	N/m^2 or Pa (pascal)	lb/ft^2
Surface tension σ	M/T^2	N/m	lb/ft
Work W	ML^2/T^2	N·m or J (joule)	ft-lb
Energy E	ML^2/T^2	N·m or J (joule)	ft-lb
Heat rate $\dot{Q}$	ML^2/T^3	J/s	Btu/sec

Derived Dimensions and Their Units

4. Shear Stress in Moving Fluids

Shear stress arise in a fluid due to its motion and therefore is consider as one of the flow characteristics similar to pressure and velocity. This means that shear stress doesn't exist when the fluid is at rest or moving with uniform velocity. Thus, shear stress τ can be defined as the force **F** acting on a surface area **A**:

$$\tau = F/A$$

Shear stress have is a vector quantity having value and direction with units (**N/m^2**).

5. Properties of Fluids

Any characteristic of a system is called a **property.** Properties are considered to be either *intensive* or *extensive*. **Intensive properties** are those that are independent of the mass of a system, such as temperature, pressure, and density. **Extensive properties** are those whose values depend on the size or extent of the system. Total mass, total volume *V*, and total momentum are some examples of extensive properties.

5.1 Density and Specific Gravity

Density is defined as *mass per unit volume*. The density of a substance, in general, depends on temperature and pressure.

$$\rho = \frac{m}{V} \qquad (kg/m^3)$$

The reciprocal of density is the **specific volume** v, which is defined as *volume per unit mass*.

$$v = \frac{V}{m} = \frac{1}{\rho} \qquad (m^3/kg)$$

Specific gravity or **relative density,** is defined as *the ratio of the density of a substance to the density of some standard substance at a specified temperature* (usually water at 4°C, for which ρ_{water} =1000 kg/m^3). That is,

$$SG = \frac{\rho_{fluid}}{\rho_{standard\,fluid}}$$

Note that the specific gravity of a substance is a dimensionless quantity.
The weight of a unit volume of a substance is called **specific weight** and is expressed as

$$\gamma = \frac{W}{V} = \frac{m\,g}{V} = \rho g \qquad (N/m^3)$$

where g is the gravitational acceleration.

5.2 Viscosity

Viscosity is a property that represents the internal resistance of a fluid to motion or the “fluidity”.

5.2.1 Dynamic Viscosity (μ) is the measurement of a fluid's internal resistance to flow due to cohesion and interaction between molecules, which offers resistance to sheer deformation.

Units: N.s/m^2 (Pa.s), or kg/m.s. The poise (p) and centipoise (cp) are also used where:

1 N.s/m^2 (pa.s) = 10 p =1000 cp

5.2.2 Kinematic viscosity (ν) is expressed as

$$\nu = \frac{\mu}{\rho}$$

Two common units of kinematic viscosity are m^2/s and **stoke** (1 stoke = 1 cm^2/s, 1 stoke = 0.0001 m^2/s).

5.2.3 Newton's Law of Viscosity

It states that shear stress **(τ)** on a fluid element layer is directly proportional to the rate of shear strain. Fluids for which the rate of deformation is proportional to the shear stress are called **Newtonian fluids** after Sir Isaac Newton, who expressed it first in 1687. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids. Blood and liquid plastics are examples of non-Newtonian fluids.

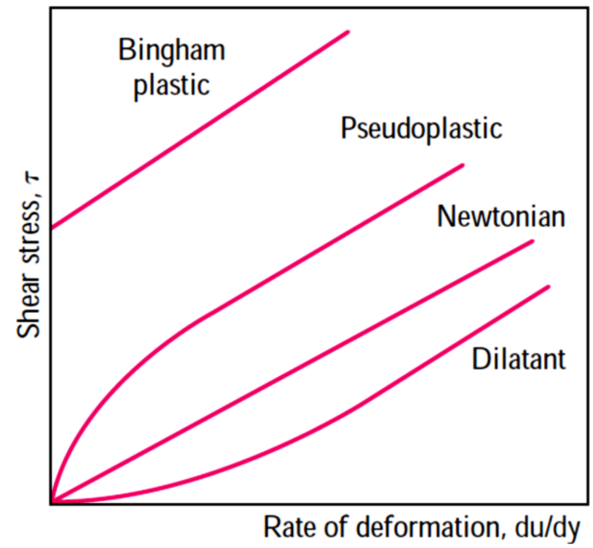


Fig. 1. Variation of shear stress with the rate of deformation for Newtonian and non-Newtonian fluids (the slope of a curve at a point is the viscosity).

In one-dimensional shear flow of Newtonian fluids, shear stress can be expressed by the linear relationship

$$\boldsymbol{\tau = \mu \frac{du}{dy}} \qquad (\boldsymbol{N/m^2})$$

5.2.4 Variation of Viscosity with Temperature

Temperature effected with viscosity. The viscosity of liquid decrease with increase the temperature while the viscosity of gases increase with increase the temperature. This is due to reason that the viscous forces in a fluid are due to cohesive forces and molecular momentum transfer in liquids the cohesive forces predominates the molecular momentum transfer. Due to closely packed molecules and with the increase in temperature. The cohesive forces decreases with the result of decreasing viscosity. But in case of gases the cohesive force are small and molecular momentum transfer predominates. With the increase in temperature, molecular momentum transfer increases and hence viscosity increases. The relation between viscosity and temperature for liquids and gases are:

For liquid:

$$\mu = \mu_0 \left(\frac{1}{1 + \alpha T + \beta T^2} \right)$$

For gases:

$$\mu = \mu_0 + \alpha T - \beta T^2$$

Where: μ = viscosity of fluid at T °C in poise, μ_0 = viscosity of fluid at 0°C in poise, (α, β) = constant of fluid.

Example(1): *Calculate the specific weight, density and weight of 2 L of an oil which specific gravity = 0.9.*

Solution:

$$SG = \frac{\rho_{oil}}{\rho_{water}} \quad , \quad 0.9 = \frac{\rho_{oil}}{1000} \quad \rightarrow \quad \rho_{oil} = 900\ kg/m^3$$

$$\gamma = \rho g = 900 \times 9.81 = 8829\ N/m^3$$

$$\gamma = \frac{W}{V} \quad , \quad W = \gamma\, V$$

$$V = 2L = \frac{2}{1000} m^3 = 0.002 m^3 ,$$

$$W = 8829 * 0.002 = 17.658\ N$$

Example (2): *a 35cm by 55cm block slides on oil (μ= 0.81 Pa. s) over a large plane surface. What force is required to drag the block at 3m/s, if the separating oil film is 0.6mm thick?*

Solution:

$$\tau = \mu \frac{du}{dy} = 0.81 \times (\frac{3}{0.6 * 10^{-3}}) = 4050\ N/m^2$$

$A = L \times W = 0.35 \times 0.55 = 0.1925\ m^2$

$$\tau = F/A \quad , \quad F = \tau \times A = 4050 \times 0.1925 = 780\ N$$

Example (3): *Two large plate surfaces 2.4cm apart, the space between the surface if filled with glycerin is. What force is required to drag a very thin plate of surface area 0.5m^2 between the two large plane surfaces speed of 0.6m/s, if: a) The thin plate is in the middle of the two plate surfaces. b) The thin plate is at a distance of 0.8cm from one of the plane surfaces. Take the dynamic viscosity of glycerin = 8.1×10^{-1} N s/m^2.*

Solution:

a) When the thin plate is in the middle of the two plane surfaces;

Let F_1= Shear force on the upper side of the thin plate

F_2= Shear force on the lower side of the thin plate

F = Total force required to drag the plate

Then, $F = F_1 + F_2$

$$\tau_1 = \mu \frac{du}{dy_1} = 8.1 \times 10^{-1} \times \frac{0.6}{0.012} = 40.5\ N/\ m^2$$

$$F_1 = \tau_1 \times A = 40.5 \times 0.5 = 20.25\ N$$

∵ The same distance and same velocity; then $\tau_2 = 40.5\ N/\ m^2\ and\ F_2 = 20.25\ N$

$$F = 20.25 + 20.25 = 40.5\ N$$

b) When the thin plate is at a distance of 0.8 cm from one of the plane surfaces.

Let the thin plate is at a distance 0.8 cm from the lower plane surface. Then distance of the plate from the upper plane surface,

$$2.4 - 0.8 = 1.6\ cm = 0.016m$$

$$\tau_1 = \mu \frac{du}{dy_1} = 8.1 \times 10^{-1} \times \frac{0.6}{0.016} = 30.37\ N/\ m^2$$

$$F_1 = \tau_1 \times A = 30.37 \times 0.5 = 15.18\ N$$

$$\tau_2 = \mu \frac{du}{dy_2} = 8.1 \times 10^{-1} \times \frac{0.6}{0.008} = 60.75\ N/\ m^2$$

$$F_2 = \tau_2 \times A = 60.75 \times 0.5 = 30.36\ N$$

$$F = 15.18 + 30.36 = 45.54\ N$$

5.3 Vapor Pressure

The vapor pressure of a liquid is the point at which equilibrium pressure is reached, in a closed container, between molecules leaving the liquid and going into the gaseous phase (evaporation) and molecules leaving the gaseous phase and entering the liquid phase (condensation).

Note the mention of a "closed container", in an open container the molecules in the gaseous phase will just fly off and an equilibrium would not be reached. Also note that at equilibrium the movement of molecules between liquid and gas does not stop, but the number of molecules in the gaseous phase stays the same—there is always movement between phases. So, at equilibrium there is a certain concentration of molecules in the gaseous phase; the pressure of the gas is exerting is the **vapor pressure.**

Fig. 2. Vapor pressure is constant when there is an equilibrium of water molecules, in a closed container.

As the temperature of a liquid increases, the kinetic energy of its molecules also increases and as the kinetic energy of the molecules increases, the number of molecules transitioning into a vapor also increases, thereby increasing the vapor pressure.

The reason for our interest in vapor pressure is the possibility of the liquid pressure in liquid-flow systems dropping below the vapor pressure at some locations, and the resulting unplanned vaporization. The vapor bubbles collapse as they are swept away from the low pressure regions, this phenomenon is called **cavitation.** Cavitation must be avoided in flow systems since it reduces performance, generates vibrations and noise, and causes damage to equipment.

5.4 Compressibility and Bulk Modulus of Elasticity

Compressibility a parameter describing the relationship between the pressure and change in volume (or density) of a fluid at constant temperature. A compressible fluid is which changes its volume appreciably under application of pressure. Therefore, the liquids are virtually incompressible whereas gases are easily compressed. The compressibility of fluid is expressed by **bulk modules of elasticity *(K)*,** which is the ratio of the change in unit pressure to the corresponding volume change per unit volume at constant temperature.

$$K = \frac{change\ of\ pressure}{change\ of\ volume\ (or\ change\ of\ denity)} \quad , \quad where\ T = constant$$

$$\boldsymbol{K = \frac{dP}{-\frac{dV}{V}} = \frac{dP}{\frac{d\rho}{\rho}}}$$

Noting that $\frac{dV}{V}$ or $\frac{d\rho}{\rho}$ is dimensionless, K must have the dimension of pressure (Pa or psi). Negative sign means the volume decreases with increase of pressure. The inverse of the bulk modules of elasticity is called the **compressibility ($\boldsymbol{\alpha}$)**

$$\boldsymbol{\alpha = \frac{1}{K}}$$

5.5 Coefficient of Volume Expansion

The density of a fluid, in general, depends more strongly on temperature than it does on pressure, and the variation of density with temperature is responsible for numerous natural phenomena such as winds, currents in oceans, the operation of hot-air balloons, and heat transfer by natural convection. To quantify these effects, we need a property. The property that represents the *variation of the density of a fluid with temperature at constant pressure* is the **coefficient of volume expansion** (or volume expansivity) β, defined as

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

5.6 Surface Tension

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension. The

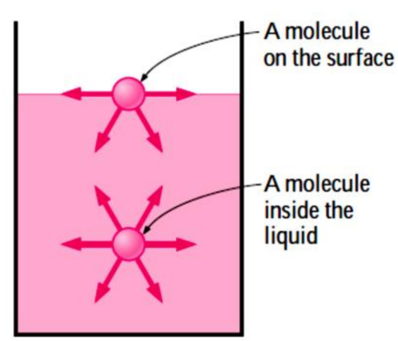


Fig. 3. Attractive forces acting on a liquid molecule at the surface and deep inside the liquid.

magnitude of this force per unit length of the free surface, it is denoted by **(σ)** (called sigma) in **N/m** units.

The surface tension varies greatly from substance to substance, and with temperature for a given substance. The surface tension of a liquid decreases with temperature and becomes zero at the critical point. The effect of pressure on surface tension is usually negligible. Surface tension determines the size of the liquid droplets that form. A droplet that keeps growing by the addition of more mass will break down when the surface tension can no longer hold it together.

A curved interface indicates a pressure difference across the interface with pressure being higher on the concave side. The excess pressure *&P* inside a droplet or bubble above the atmospheric pressure, for example, can be determined by considering the free-body diagram of half a droplet or bubble (Fig. 4). Noting that surface tension acts along the circumference and the pressure acts on the area, horizontal force balances for the droplet and the bubble give

$$\Delta P_{droplet} = \frac{2\sigma}{R} = \frac{4\sigma}{D}$$

$$\Delta P_{bubble} = \frac{4\sigma}{R} = \frac{8\sigma}{D}$$

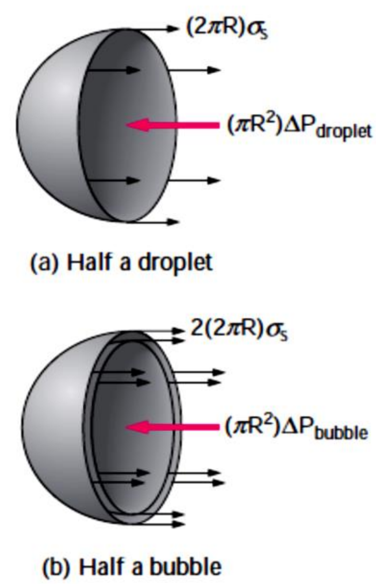
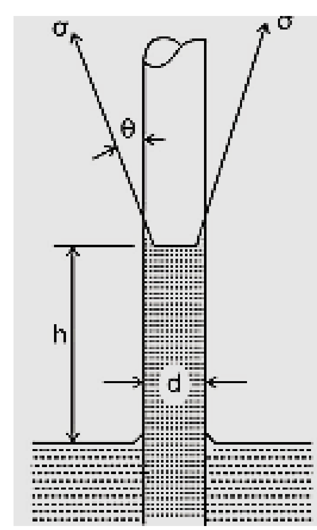


Fig. 4. The free-body diagram of half a droplet and half a bubble.

5.7 Capillary Effect

Another interesting consequence of surface tension is the **capillary effect,** which is the rise or fall of a liquid in a small-diameter tube inserted into the liquid. Such narrow tubes or confined flow channels are called **capillaries.** The thinner the tube is, the greater the rise (or fall) of the liquid in the tube. In practice, the capillary effect is usually negligible in tubes whose diameter is greater than 1 cm.



Consider a glass tube of small diameter **(d)** opened at both ends and is interested in a liquid, say water. The liquid will rise in the tube above the level of the liquid. Let h = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height h is balanced by the force at the surface of the liquid in the tube. However, the force at the surface of the liquid in the tube is due to surface tension.

$$\boldsymbol{h = \frac{4\sigma \cos\theta}{\rho g d}}$$

The water molecules are more strongly attracted to the glass molecules than they are to other water molecules, and thus water tends to rise along the glass surface. The opposite occurs for mercury, which causes the liquid surface near the glass wall to be suppressed.

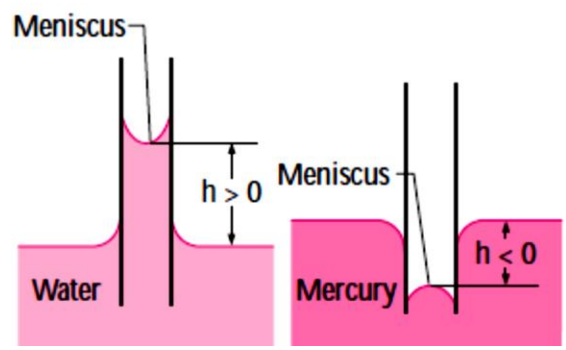


Fig. 5. The capillary rise of water and the capillary fall of mercury in a small-diameter glass tube.

The value of θ between water and clean glass tube is approximately equal to zero and hence $\cos\theta$ is equal 1 unity.

<u>Example (5):</u> *Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water (b) mercury. Take surface tension σ =0.0725 N/m for water and σ =0.52N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact =130°*

<u>Solution:</u> **(a) Capillary rise for water (θ =0)**

$$h = \frac{4\sigma}{\rho g d} = \frac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}} = 0.0118m = 1.18cm$$

(b) Capillary rise for *mercury* (θ =130°)

$$\rho_{mercury} = SG \times \rho_{water} = 13.6 \times 1000 = 13600\ kg/m^3$$

$$h = \frac{4\sigma \cos\theta}{\rho g d} = \frac{4 \times 0.52 \times cos130}{13600 \times 9.81 \times 2.5 \times 10^{-3}} = -0.004m = -0.4cm$$

Note: The negative sign indicates **the capillary depression.**

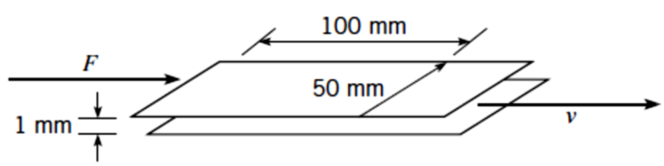
Example (6): *Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m^2 above atmospheric pressure.*

Solution:

$$\Delta P_{bubble} = \frac{8\sigma}{D} \quad \rightarrow \quad 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}} \quad \rightarrow \quad \sigma = 0.0125\ N/m$$

Homework

1. A reservoir of glycerin has a mass of 1200 kg and a volume of 0.952 m^3. Find the glycerin's weight, density, specific weight, and specific gravity.
2. A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N/m^2 to maintain this speed. Determine the fluid viscosity between the plates.
3. If a bubble is equivalent to an air-water interface with σ =0.005 N/m, what is the pressure difference between the inside and outside of a bubble of diameter 0.003 cm?
4. The sliding plate viscometer shown below is used to measure the viscosity of a fluid. The top plate is moving to the right with a constant velocity of 10 m/ s in response to a force of 3 N. The bottom plate is stationary. What is the viscosity of the fluid?



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Power Mechanics Technical Engineering Department

Second Stage

Fluid Mechanics - Static

<u>Chapter Two: Fluid Static</u>

M.Sc Zahraa A. Nadeem

1. Pressure

Pressure is defined as *a normal force exerted by a fluid per unit area*. Since pressure is defined as force per unit area, it has the unit of newtons per square meter (N/m^2), which is called a **pascal** (Pa). The pressure unit pascal is too small for pressures encountered in practice. Therefore, its multiples *kilopascal* (1 kPa $=10^3$ Pa) and *megapascal* (1MPa $=10^6$ Pa) are commonly used.

$$P = \frac{F}{A} \quad , \quad P = \rho g h$$

Three other pressure units commonly used in practice, especially in Europe, are *bar and standard atmosphere*

$$1 \text{ bar} = 10^5 \text{ Pa} = 0.1 \text{ MPa} = 100 \text{ kPa}$$

$$1 \text{ atm} = 101{,}325 \text{ Pa} = 101.325 \text{ kPa} = 1.01325 \text{ bars}$$

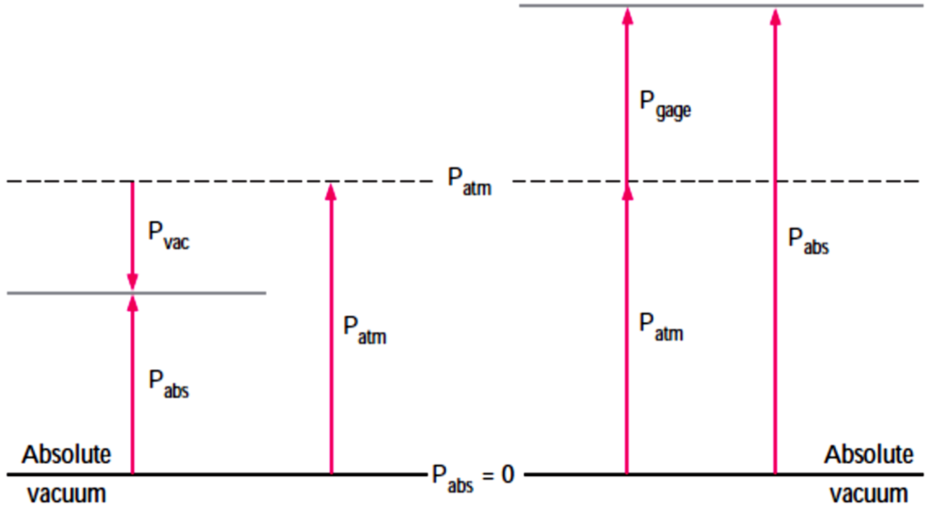
In the English system, the pressure unit is *pound-force per square inch* (lbf/in^2, or psi), and 1atm= 14.696 psi.

The actual pressure at a given position is called the **absolute pressure,** and it is measured relative to absolute vacuum (i.e., absolute zero pressure). Most pressure-measuring devices, however, are calibrated to read zero in the atmosphere (Fig.), and so they indicate the difference between the absolute pressure and the local atmospheric pressure. This difference is called the **gage pressure.**

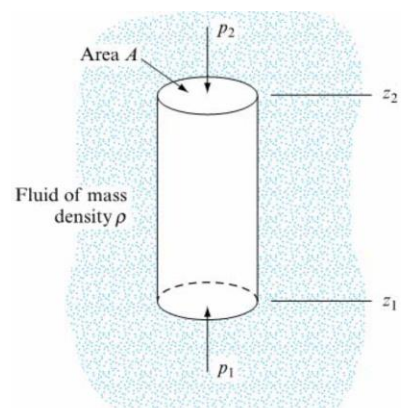
Pressures below atmospheric pressure are called **vacuum pressures** and are measured by vacuum gages that indicate the difference between the atmospheric pressure and the absolute pressure. Absolute, gage, and vacuum pressures are all positive quantities and are related to each other by

$$P_{gage} = P_{abs} - P_{atm}$$

$$P_{vac} = P_{atm} - P_{abs}$$



2. Pressure Variation with Elevation



In the figure shows an element of fluid consisting of a vertical column of constant cross-sectional area A and totally surrounded by the same fluid of mass density ρ. Suppose that the pressure is P_1 on the underside at level z_1 and P_2 on the top at level z_2. Since the fluid is at rest the element must be in equilibrium and the sum of all the vertical forces must be zero. The forces acting are:

Force due to P_1 on area A acting up = P_1 A,

Force due to P_2 on area A acting up = P_2 A

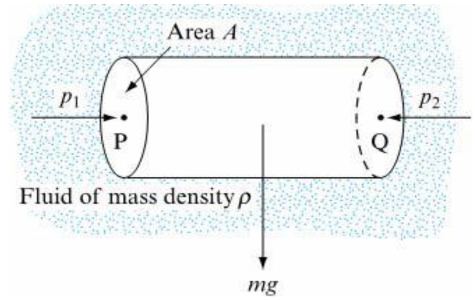
Force due to the weight of the fluid element = mg = Mass density×Volume×g = $\rho gA(z_2-z_1)$

Since the fluid is at rest, there can be no shear forces and, therefore, no vertical forces act on the side of the element due to the surrounding fluid. Taking upward forces as positive and equating the algebraic sum of the forces acting to zero

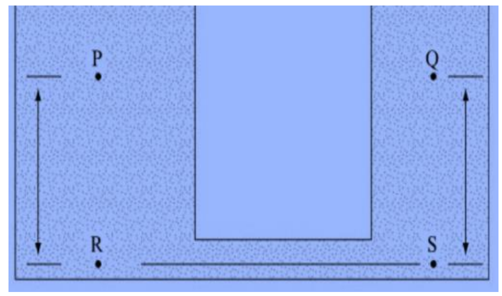
$$P_1A - P_2A - \rho gA(z_2 - z_1) = 0 \qquad \rightarrow \qquad \boldsymbol{P_2 - P_1 = \rho g(z_2 - z_1)}$$

3. Equality of Pressure at the Same Level in a Static Fluid

If **P** and **Q** are **two points at the same level** in a fluid at rest, a horizontal prism of fluid of constant cross-sectional area A will be in equilibrium. The forces acting on this element



horizontally are **(P_1A)** at **P** and **(P_2A)** at **Q**. Since the fluid is at rest, there will be no horizontal shear stresses on the sides of the element. For static equilibrium the sum of the horizontal forces must be zero: **$P_1A= P_2A$** , Thus, ***the pressure at any two points at the same level in a body of fluid at rest will be the same $P_1 = P_2$.***

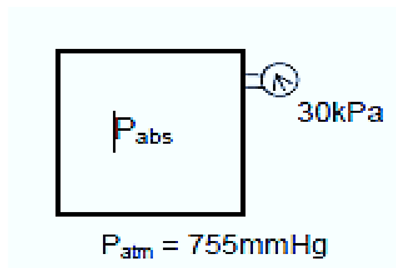


$P_R= P_S$

$P_R = P_P + \rho gh$

$P_S = P_Q + \rho gh$

$P_P + \rho gh = P_Q + \rho gh$



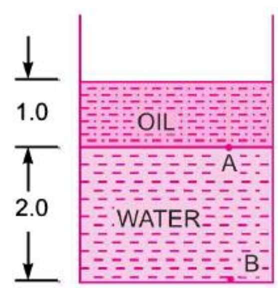
<u>Example(1):</u> *A vacuum gage connected to a tank reads 30 kPa at a location where the barometric reading is 755 mmHg. Determine the absolute pressure in the tank. Take ρ_{HG}=13,590 kg/m^3.*

<u>Solution:</u> h =755 mmHg, $P_{vac.}$= 30 kPa ,$P_{absolute}$=?

$P = \rho gh$ = 13590*9.81*0.755= 100655Pa = 101 Kpa

$P_{absolute} = P_{atm} - P_{vac.}$ = 101-30 =71 Kpa

<u>Example(2):</u> *An open tank contains water up to depth of 2 m and above an oil of (S.G. 0.9) for a depth of 1 m. find the pressure intensity (a) at the interface of two liquids. (b) at the bottom of the tank.*



Solution. Given :

Height of water, $Z_1 = 2$ m

Height of oil, $Z_2 = 1$ m

Sp. gr. of oil, $S_0 = 0.9$

Density of water, $\rho_1 = 1000$ kg/m^3

Density of oil, $\rho_2 =$ Sp. gr. of oil $\times$ Density of water $= 0.9 \times 1000 = 900$ kg/m^3

Pressure intensity at any point is given by

$$p = \rho \times g \times Z.$$

(*i*) At interface, *i.e.*, at *A*

$$p = \rho_2 \times g \times 1.0 = 900 \times 9.81 \times 1.0$$

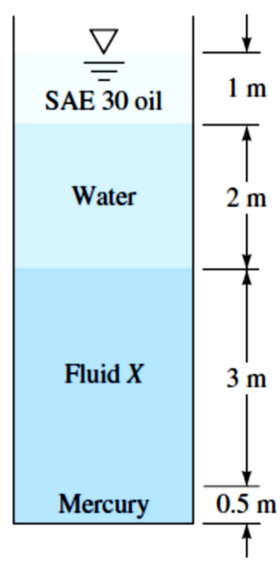
$$= 8829 \frac{\text{N}}{\text{m}^2} = \frac{8829}{10^4} = \mathbf{0.8829\ N/cm^2.\ Ans.}$$

(*ii*) At the bottom, *i.e.*, at *B*

$$p = \rho_2 \times gZ_2 + \rho_1 \times g \times Z_1 = 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$

$$= 8829 + 19620 = 28449 \text{ N/m}^2 = \frac{28449}{10^4} \text{ N/cm}^2 = \mathbf{2.8449\ N/cm^2.\ Ans.}$$

Example(3): *The system in Fig. is at 20°C. If atmospheric pressure is 101.33 kPa and the pressure at the bottom of the tank is 242 kPa, what is the specific gravity of fluid X? take γ in N/m^3 (mercury =133100 , oil = 8720 , water = 9790)*



Solution:

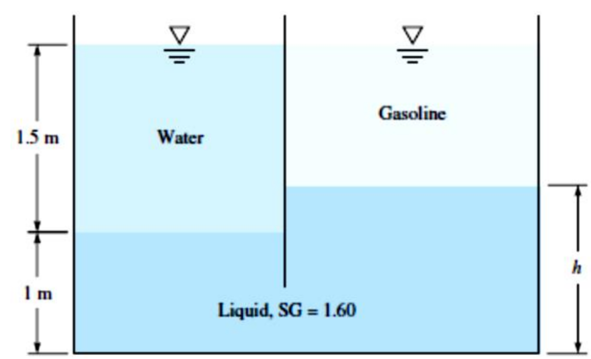
$\mathbf{P_{(mercury)} + P_{(fluid)} + P_{(water)} + P_{(oil)} + P_{atmos.} = P_{bottom}}$

$0.5*133100 + \mathbf{P_{(fluid)}} + 9790*2 + 8720*1 + 101330 = 242000$

$\mathbf{P_{(fluid)}} = 45820\text{Pa} = \rho g h = \rho*9.81*3$

$\rho = 1557$ kg/m^3 , S.G for fluid x = 1.557

Example(4): *In the Figure below the 20°C water and gasoline are open to the atmosphere and are at the same elevation. What is the height h in the third liquid? Take (γ for water = 9790N/m^3, γ for gasoline = 6670N/m^3).*



Solution:

$P_{bottom} = P_{Liquid} + P_{Water} = P_{Liquid} + P_{Gasoline.}$

$\rho gh_{(liquid)} + \gamma h_{(water)} = \rho gh_{(liquid)} + \gamma h_{(gasoline)}$

$1.6*1000*9.81*1 + 1.5*9790 = 1.6*1000*9.81*h + 6670*(2.5-h) \rightarrow \mathbf{h = 1.52m}$

Example(5): *What are the gauge pressure and absolute pressure at a point 3m below the free surface of a liquid having a density of 1.53 *$10^3 kg/m^3$ if the atmospheric pressure is equivalent to 750 mm of mercury ? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m^3.*

Solution. Given :

Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m^3

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$$= \frac{750}{1000} = 0.75 \text{ m of Hg}$$

∴ Atmospheric pressure, $p_{atm} = \rho_0 \times g \times Z_0$

where ρ_0 = Density of Hg = Sp. gr. of mercury × Density of water = 13.6 × 1000 kg/m^3

and Z_0 = Pressure head in terms of mercury.

∴ $p_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75$ N/m^2 $\quad (\because \; Z_0 = 0.75)$

$= 100062$ N/m^2

Pressure at a point, which is at a depth of 3 m from the free surface of the liquid is given by,

$$p = \rho_1 \times g \times Z_1$$
$$= (1.53 \times 1000) \times 9.81 \times 3 = 45028 \text{ N/m}^2$$

∴ Gauge pressure, $p =$ **45028 N/m^2. Ans.**

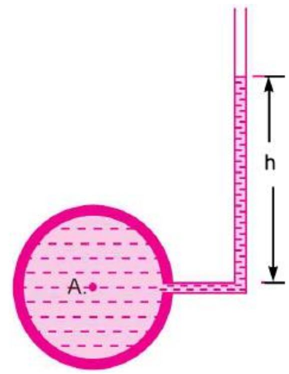
Now absolute pressure = Gauge pressure + Atmospheric pressure

= 45028 + 100062 = **145090 N/m^2. Ans.**

4. Manometer

Manometer are instruments used to measure pressure, their operation is based on a technique involving the rise of the liquid inside a glass tube. The simplest type of this instrument consist of vertical glass tube attached to a container, open at the top. The liquid of the manometer must not mix or react with the other fluid. The relationship between pressure and pressure head forms the basis for the measurement and estimation of pressure by the manometer.

5. Piezometer

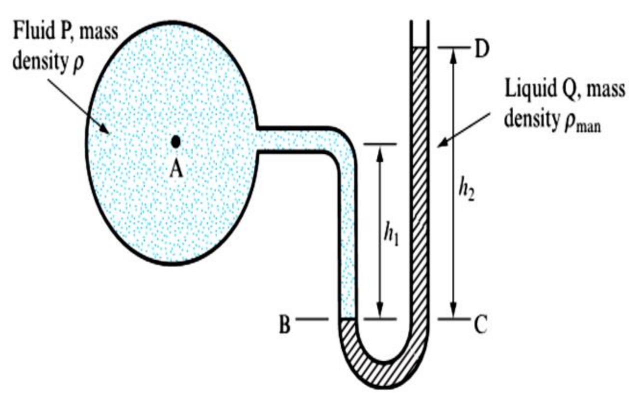


It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. The rise of liquid gives the pressure head at that point. The pressure at **A** is due to the column of liquid **h**, and can be calculated using equation:

$\mathbf{P_A = \rho g h}$

6. U-tube Manometer

It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Figure below. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.



$P_B = P_C$ (at the same level)

$P_B = \rho g h_1 + P_A$

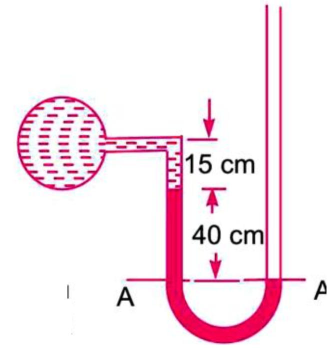
$P_C = \rho g h_2 + P_{atm}$

$P_B = P_C$

$\rho g h_1 + P_A = \rho g h_2 + \cancel{P_{atm}}$

$\mathbf{P_A = \rho g h_2 - \rho g h_1}$

Example (6): *A simple U-Tube manometer containing mercury* ***(S.G = 13.6)*** *is connected to a pipe in which a fluid of* ***(S. G = 0.8)*** *and having vacuum pressure is following. The Other end of the end of manometer is open to the atmosphere Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is* ***40cm*** *and the higher of fluid in the left from the center of pipe is* ***15cm*** *below.*



Solution. Given :

Sp. gr. of fluid, $S_1 = 0.8$

Sp. gr. of mercury, $S_2 = 13.6$

Density of fluid, $\rho_1 = 800$

Density of mercury, $\rho_2 = 13.6 \times 1000$

Difference of mercury level, h_2 = 40 cm = 0.4 m. Height of liquid in left limb, h_1=15 cm = 0.15 m. Let the pressure in pipe = p. Equating pressure above datum line A-A, we get

$$\rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

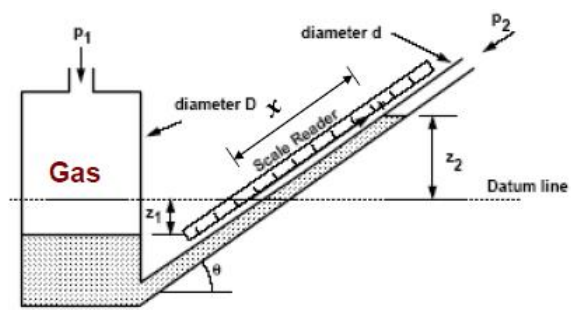
$$p = -[\rho_2 g h_2 + \rho_1 g h_1]$$

$$= -[13.6 \times 1000 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15]$$

$$= -[53366.4 + 1177.2] = -54543.6 \text{ N/m}^2 = \mathbf{-5.454 \text{ N/cm}^2}. \textbf{ Ans.}$$

7. Inclined Manometer

This manometer registers a better reading than a vertical tube and is used to measure small values of pressure difference.

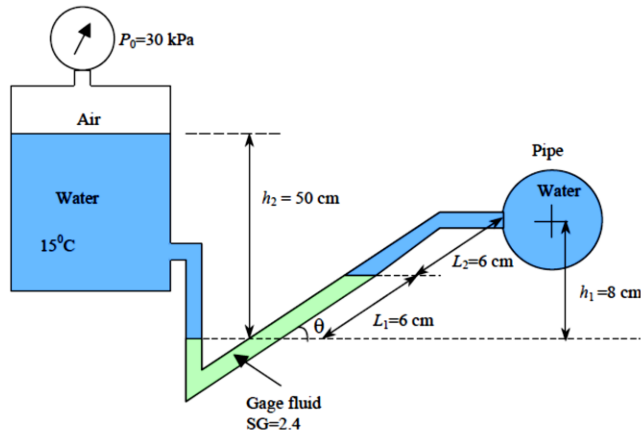


$P_1 = \rho g\, z_2 + P_2$ But, $z_2 = x\, sin\theta$

$$\boldsymbol{P_1 - P_2 = \rho g x\, sin\theta}$$

Example (7): The pressure of water flowing through a pipe is measured by the arrangement shown in Figure. For the values given, calculate the pressure in the pipe.

Solution:



$$P_0 + P_{water2} = P_{gage\ fluid} + P_{water1} + P_{pipe\ water}$$

$$P_{pipe\ water} = P_0 + (\rho_w g h_{w2}) - (\rho_{fluid} g L_1 sin\theta) - (\rho_w g L_2 sin\theta)$$

$$sin\theta = \frac{8}{12} = 0.6667$$

$$P_{pipe\ water} = 30 \times 10^3 + (1000 \times 9.81 \times 0.5) - (2.4 \times 1000 \times 9.81 \times 0.06 \times 0.6667) - (1000 \times 9.81 \times 0.06 \times 0.6667) = \mathbf{33.6\ \mathit{Kp}}$$

8. Difrential Manometer

Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consist of a U-tube containing a heavy liquid whose two ends are connected to the points whose difference of pressure is to be measured. Most commonly types of differential manometers are:

1. U-tube differential manometer and,

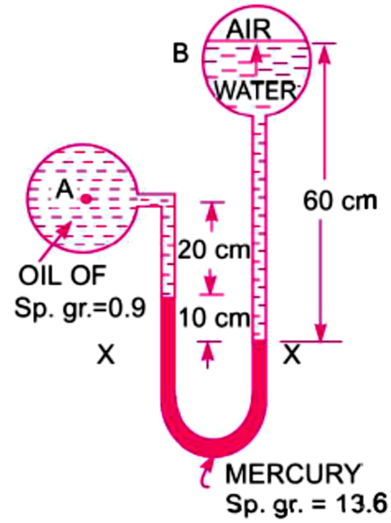
2. inverted U-tube differential manometer.

Example (8): *A differential manometer is connected at the two points A and B as shown in Fig. At B air pressure is 9.81 N/cm² (abs). Find the absolute pressure at A.*

Solution. Given :

Air pressure at $B = 9.81\ N/cm^2$

$p_B = 9.81 \times 10^4\ N/m^2$



Density of oil $= 0.9 \times 1000 = 900 \text{ kg/m}^3$

Density of mercury $= 13.6 \times 1000 \text{ kg/m}^3$

Let the pressure at A is p_A

Taking datum line at X-X

Pressure above X-X in the right limb

$$= 1000 \times 9.81 \times 0.6 + p_B$$
$$= 5886 + 98100 = 103986$$

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + p_A$$
$$= 13341.6 + 1765.8 + p_A$$

Equating the two pressure heads

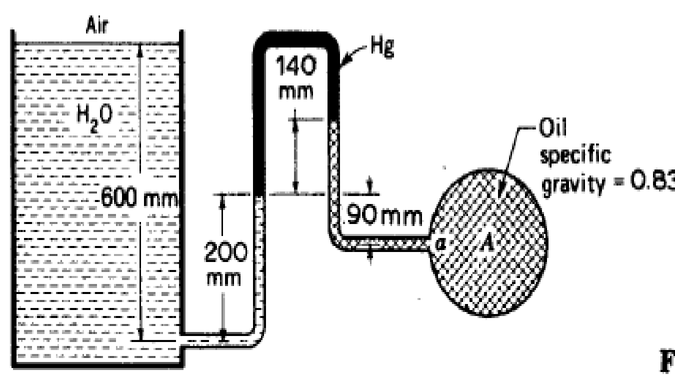
$$103986 = 13341.6 + 1765.8 + p_A$$

$\therefore$ $$p_A = 103986 - 15107.4 = 88876.8$$

$\therefore$ $$p_A = 88876.8 \text{ N/m}^2 = \frac{88876.8\text{N}}{10000 \text{ cm}^2} = 8.887 \frac{\text{N}}{\text{cm}^2}.$$

$\therefore$ Absolute pressure at A = **8.887 N/cm². Ans.**

Example (9): *For the setup shown in Figure, calculate the absolute pressure at A. Assume standard atmospheric pressure, 101.3 Kpa. (sg. $_{Hg}$=13.6)*



Fig

Solution:

$P_B = P_C$

$P_B = -P_{oil} + P_A$

$P_C = -P_{Hg} - P_{water1} + P_{water1} + P_{atm}$

$-P_{oil} + P_A = -P_{Hg} - P_{water1} + P_{water2} + P_{atm}$

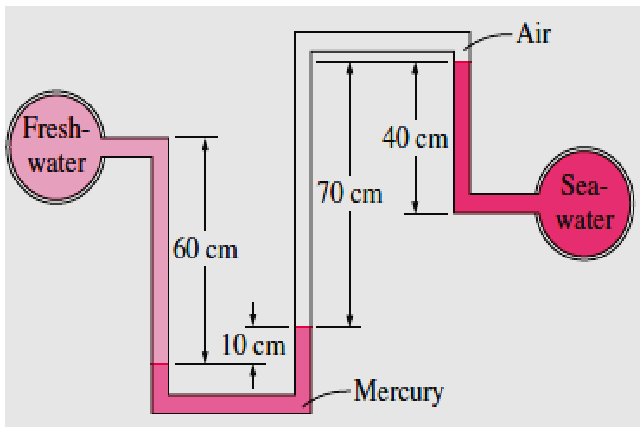
$P_A = -P_{Hg} - P_{water1} + P_{water2} + P_{atm} + P_{oil}$

$P_A = -(13.6*1000*9.81*0.14) - (1000*9.81*0.2) + (1000*9.81*0.6) + (101.3*1000) + (0.83*1000*9.81*(0.09+0.14))$

$\mathbf{P_A = 88.5Kpa}$

9. Multi – Liquid Manometer

Example (10): *Freshwater and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer, as shown in the figure. Determine the pressure difference between the two pipelines. Take the density of seawater at that location to be ρ = 1035 kg/m^3. (Take $\rho_{mercury}$ = 13600kg/m^3, ρ_{air} =1.23 kg/m^3).*



Solution:

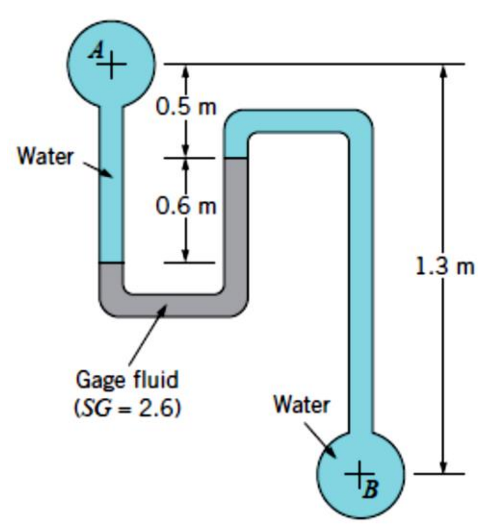
$P_{fresh\ water} + P_1 - P_{mercury} - P_{air} + P_{sea\ water} = P_2$

$(P_1 - P_2) = P_{mercury} + P_{air} – P_{sea\ water} - P_{fresh\ water}$

$(P_1 - P_2) = (13600*9.81*0.1) + (1.23*9.81*0.7) – (1035*9.81*0.4) – (1000*9.81*0.6)$

$(P_1 - P_2)$ = 3396 Pa = 3.396 Kpa

Example (11): Two pipes are connected by a manometer as shown in Fig. Determine the pressure difference, P_A - P_B, between the pipes.



Solution:

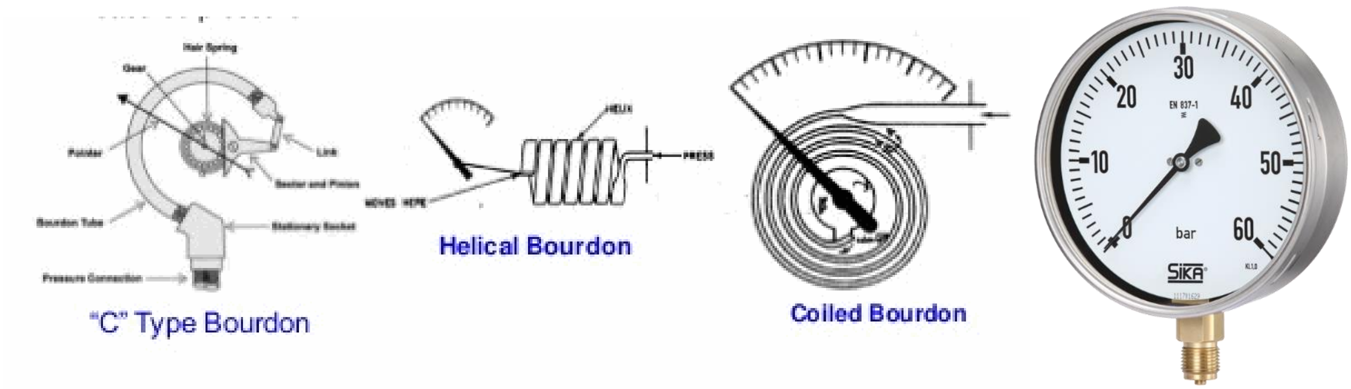
$$P_A + (\rho g h)_{water1} - (\rho g h)_{gf} + (\rho g h)_{water2} = P_B$$

$$P_A - P_B = -(1000 \times 9.81 \times (0.5 + 0.6)) + (2.6 \times 1000 \times 9.81 \times 0.6) - (1000 \times 9.81 \times (1.3 - 0.5))$$

$$\boldsymbol{P_A - P_B = -3.32\ kpa}$$

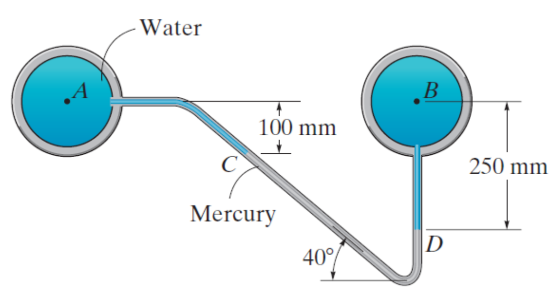
10. Bourdon Gauge

Bourdon tube pressure gauges are the most common type in many areas and are used to measure medium to high pressures. It consist of a hollow elastic curved metal tube, one end is closed and the other is connected to a pressure source such a pipeline or a container. The pressure, due to the fluid inside the tube, causes a movement of a pointer indicating the value of the pressure.

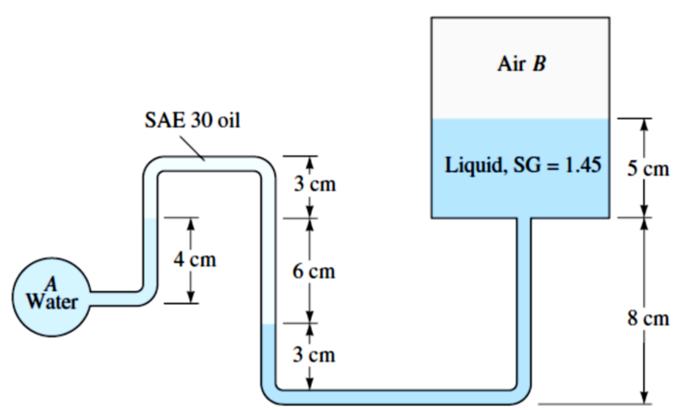


Homeworks

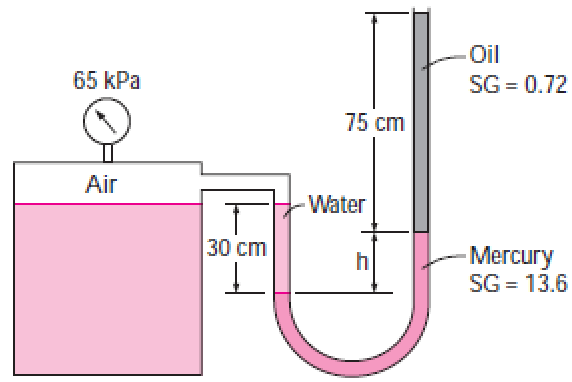
1. For the inclined-tube manometer shown in figure, the pressure in pipe B is 8kPa. What is the pressure in pipe A? ($S.G._{Hg}$=13.55)



2. In Fig. The pressure at point *A* is 172 Kpa. What is the air pressure in the closed chamber *B*? ($s.g._{\text{SAE 30 oil}} = 0.89$).



3. The gage pressure of the air in the tank shown in Fig. is measured to be 65 kPa. Determine the differential height *h* of the mercury column.



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Power Mechanics Technical

Engineering Department

Second Stage

Fluid Mechanics - Static

<u>Chapter Three: Hydrostatic Forces</u>

M.Sc Zahraa A. Nadeem

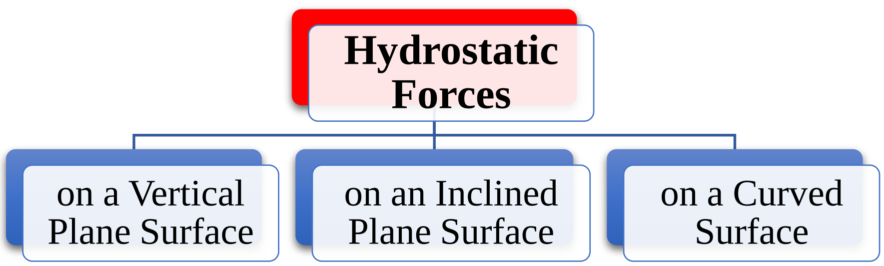
Hydrostatic Forces on Submerged Surfaces

1. Introduction

When a surface is submerged in a fluid, forces develop on the surface due to the fluid. The determination of these forces is important in the design of storage tanks, ships, dams, and other hydraulic structures.

Basic conditions for a Plane surface submerged in a fluid

- Force on the surface: ***Perpendicular*** to the surface (no shearing stresses present).
- Pressure: will vary linearly with the vertical depth.



2. Hydrostatic Forces on a Vertical Plane Surface

For a horizontal surface, such as the bottom of a liquid-filled tank Fig. 1(a), the magnitude of the resultant force is simply $\boldsymbol{F = p\ A}$ where p is the uniform pressure on the bottom and A is the area of the bottom. For the open tank shown, $\boldsymbol{p = \gamma\ h}$.

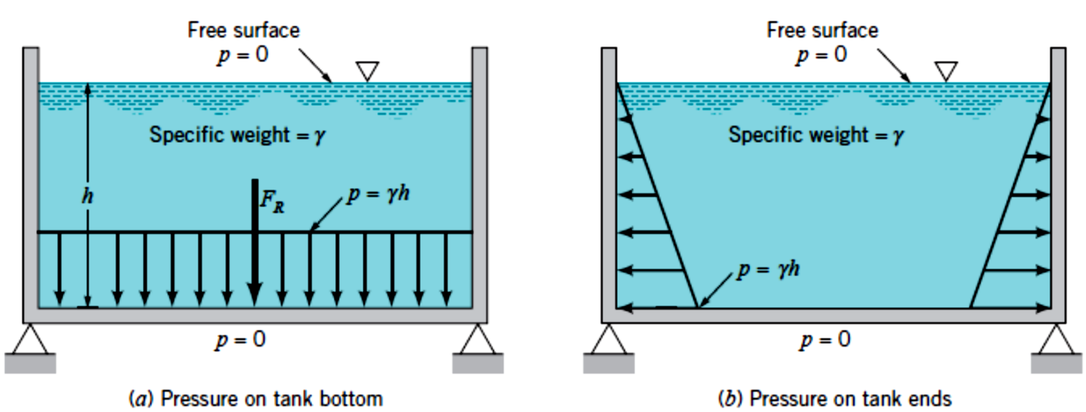


Fig.(1) (a) Pressure distribution and resultant hydrostatic force on the bottom of an open tank.
(b) Pressure distribution on the ends of an open tank.

Since the pressure is constant and uniformly distributed over the bottom, the resultant force acts through the centroid of the area as shown in Fig. 1(*a*). As shown in Fig. 1(b), the pressure on the ends of the tank is not uniformly distributed. The pressure is maximum at the bottom and minimum at the top.

2.1 Pressure Prism

An informative and useful graphical interpretation can be made for the force developed by a fluid acting on a plane area. Consider the pressure distribution along a vertical wall of a tank of width *b*, which contains a liquid having a specific weight ($\gamma=\rho g$).

Since the pressure must vary linearly with depth, we can represent the variation as is shown in Fig. 2, where the pressure is equal to zero at the upper surface and equal to ($\gamma h=\rho g h$) at the bottom.

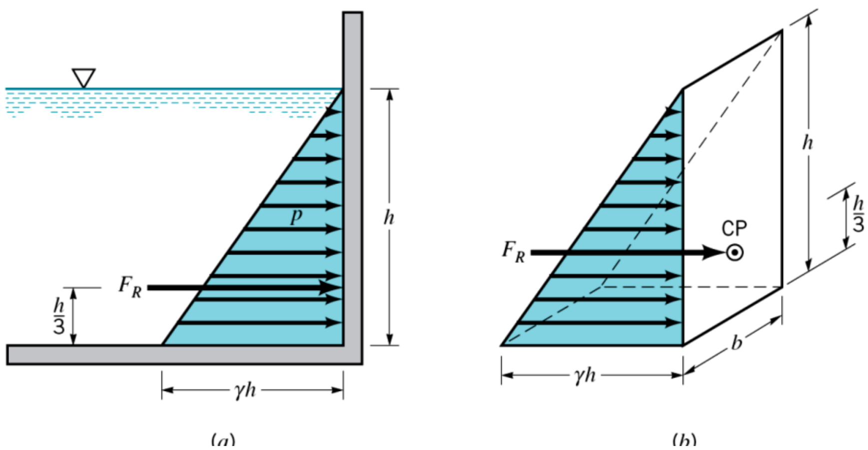


Fig. 2. Pressure Prism

The base of this "*volume*" in pressure-area space is the plane surface of interest, and its altitude at each point is the pressure. This volume is called the ***pressure prism***, and it is clear that the magnitude of the resultant force acting on the surface is equal to the volume of the pressure prism. *"The magnitude of the resultant fluid force is equal to the volume of the pressure prism and passes through its centroid"*

Due to this method,

$$F_R=(\gamma h_c)\cdot(A)=p_{av}\ \text{(at the centroid)}\times\text{area}=\gamma\left(\frac{h}{2}\right)A$$

$$\boxed{F_R=\textbf{Volume of the pressure prism}=\frac{1}{2}(\gamma h)(bh)=\gamma\left(\frac{h}{2}\right)A}$$

The resultant force must pass through the **centroid of the pressure prism** which is located along the vertical axis of symmetry of the surface, and at a distance h/3 above the base (since the centroid of a triangle is located at h/3 above its base).

The **location** of F_R can be determined by summing moments about some convenient axis.

3. Hydrostatic Forces on an Inclined Plane Surface

For the more general case in which a submerged plane surface is inclined, as is illustrated in Fig. 3. The area can have an arbitrary shape as shown. We wish to determine the direction, location, and magnitude of the resultant force acting on one side of this area due to the liquid in contact with the area. Assume that the fluid surface is open to the atmosphere. And angle θ between free surface & the inclined plane.

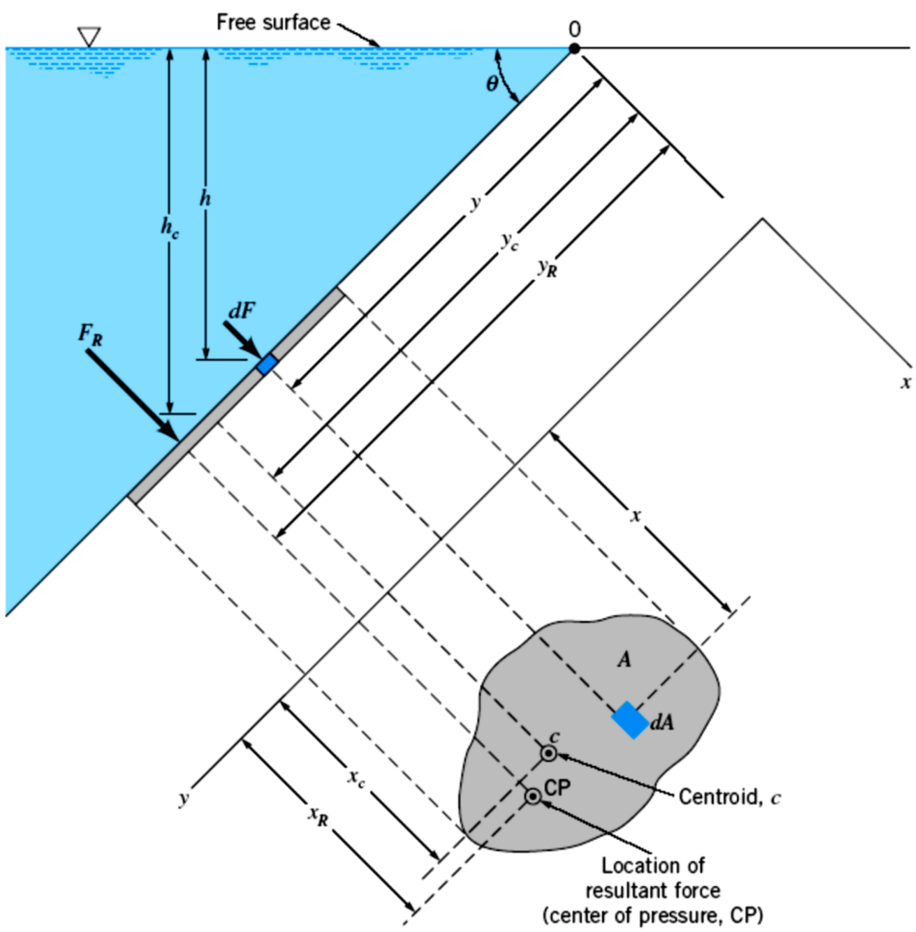


Fig. 3. Notation for hydrostatic force on an inclined plane surface of arbitrary shape.

At any given depth, h, the force acting on dA (the differential area of Fig. 3) and is perpendicular to the surface is:

$$dF = \gamma h \, dA$$

Then, Magnitude of ***total*** resultant force F_R (summing the differential forces over the entire surface)

$$F_R = \int_A \gamma h dA = \int_A \gamma (y \sin\theta) dA \qquad \text{where } h = y\sin\theta$$

$$= \gamma \sin\theta \boxed{\int_A y dA} \longleftarrow \boxed{\begin{array}{l}\text{1st moment of the area} \\ \quad \text{- Related with the center of area}\end{array}}$$

Where y_c is the y coordinate of the centroid of area A. The equation can thus be written as

$$F_R = \gamma A y_c \sin\theta$$

or more simply as

$$\boxed{F_R = \gamma h_c A}$$

Where h_c is the vertical distance from the fluid surface to the centroid of the area.

Although our intuition might suggest that the resultant force should pass through the centroid of the area, this is not actually the case. ***The resultant force should pass through the center of pressure (CP)***. The y coordinate, y_R, of the resultant force can be determined by summation of moments around the x axis. That is, the moment of the resultant force must equal the moment of the distributed pressure force,

$$F_R y_R = \int_A y \, dF = \int_A \gamma \sin\theta \, y^2 \, dA$$

and, therefore, since $F_R = \gamma A y_c \sin\theta$

$$F_R y_R = (\cancel{\gamma} A y_c \cancel{\sin\theta}) y_R = \int_A y dF = \int_A \gamma \sin\theta y^2 dA = \cancel{\gamma} \cancel{\sin\theta} \int_A y^2 dA$$

$$y_R = \frac{\int_A y^2 \, dA}{y_c A}$$

Where $\int_A y^2\, dA$ is the ***second moment of the area (moment of inertia)***, $\boldsymbol{I_x}$, with respect to an x- axis. Thus, we can write

$$y_R = \frac{I_x}{y_c A}$$

The moment of inertia, I_x, with respect to an x- axis. Can be converted to moment of inertia with respect to the centroid **(I_{xc})**. By using parallel axis theorem, where:

$$I_x = I_{xc} + A y_c^2$$

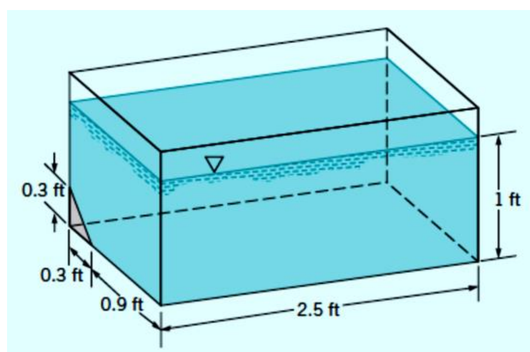
Now, substitute for I_x equation in y_R equation:

$$\boxed{y_R = \frac{I_{xc}}{y_c A} + y_c}$$

Centroidal coordinates and moments of inertia for some common areas are given in **Fig. 4.**

Example (1): *An aquarium contains seawater ($\gamma = 64\ Ib/ft^3$) to a depth of 1 ft as shown in Fig. To repair some damage to one corner of the tank, a triangular section is replaced with a new section as illustrated in Fig. Determine* ***(a)*** *the magnitude of the force of the seawater on this triangular area, and* ***(b)*** *the location of this force.*

Solution:



(a) $y_c = h_c = 0.9$ ft

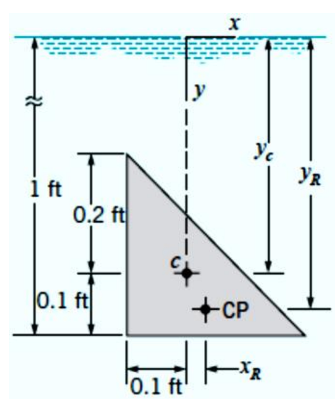
the magnitude of the force is

$$F_R = \gamma h_c A$$
$$= (64.0\ \text{lb/ft}^3)(0.9\ \text{ft})[(0.3\ \text{ft})^2/2] = 2.59\ \text{lb} \quad \textbf{(Ans)}$$

(b) The y coordinate of the center of pressure (CP) is found from

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

and from Fig.4



$$I_{xc} = \frac{(0.3 \text{ ft})(0.3 \text{ ft})^3}{36} = \frac{0.0081}{36} \text{ ft}^4$$

so that

$$y_R = \frac{0.0081/36 \text{ ft}^4}{(0.9 \text{ ft})(0.09/2 \text{ ft}^2)} + 0.9 \text{ ft}$$
$$= 0.00556 \text{ ft} + 0.9 \text{ ft} = 0.906 \text{ ft} \qquad \textbf{(Ans)}$$

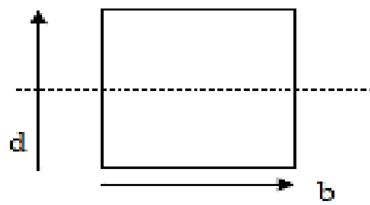
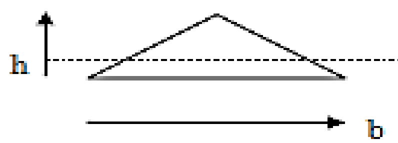
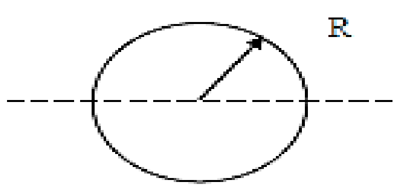
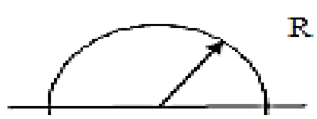
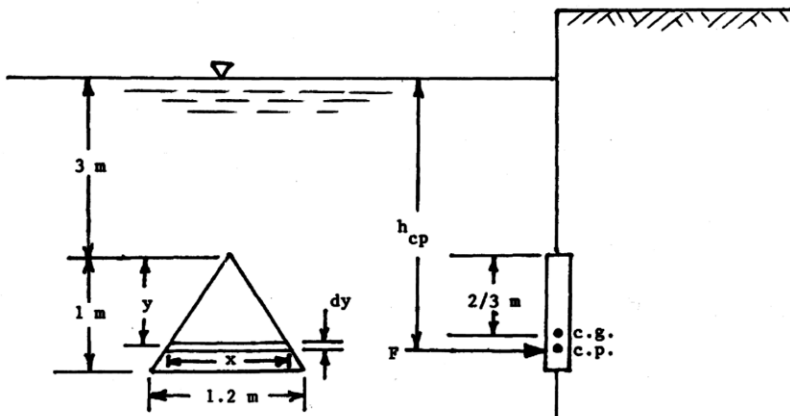
	SHAPE	AREA	Centre of Gravity CG	2^{nd} Moment of area Ic about CG
Rectangle		bd	d/2	$\frac{bd^3}{12}$
Triangle		$\frac{bh}{2}$	h/3	$\frac{bh^3}{36}$
Circle		πR^2	At centre	$\frac{\pi R^4}{4}$
Semicircle		$\frac{\pi R^2}{2}$	$\frac{4R}{3\pi}$	$0.1102R^4$

Fig. 4. Geometric properties of some common shapes.

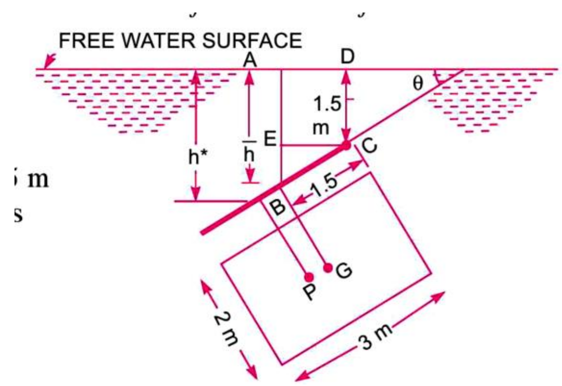
Example (2): *A vertical, triangular gate with water on one side is shown in Fig. Determine the total resultant force acting on the gate and the location of the center of pressure.*

$$F = \gamma h_{cg} A = (9.79)[3 + \tfrac{2}{3}(1)][(1.2)(1)/2] = 21.54 \text{ kN}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg} A} = [3 + (\tfrac{2}{3})(1)] + \frac{(1.2)(1)^3/36}{[3 + \tfrac{2}{3}(1)][(1.2)(1)/2]} = 3.68 \text{ m}$$



Example (3): *A rectangular plane surfaces 2 m wide and 3 m deep lies in water in such a way that its plane makes an angle 30 with the free surface of water. Determine the total pressure and position of center of pressure when the upper edge is 1.5 m below the free water surface.*



Solution:

Width of plane surface, $b = 2$ m
Depth, $d = 3$ m
Angle, $\theta = 30°$
Distance of upper edge from free water surface = 1.5

i. **Total pressure force** is given by

$F = \rho\, g\, A\, h_c$

Where $\rho = 1000\ kg/m^3$

$A = b \times d = 3 \times 2 = 6\, m^2$

$h_c = Depth\ of\ C.G.from\ free\ water\ surface = 1.5 + 1.5\ \sin 30 = 2.25\, m$

$F = 1000 \times 9.81 \times 6 \times 2.25 = \mathbf{132435\ N}$

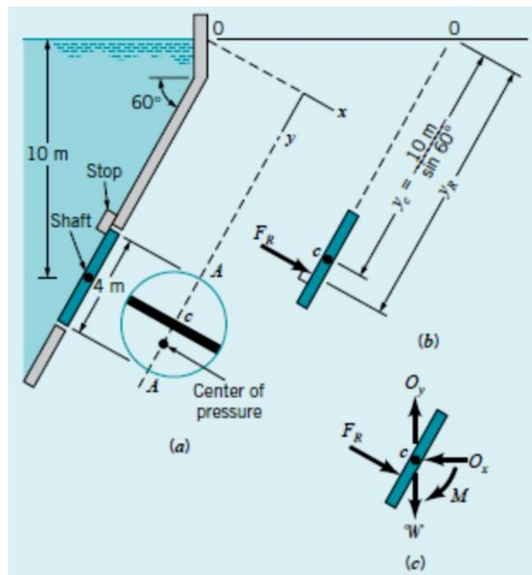
ii. **Center of pressure (y_R)**

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

Where $I_{xc} = \frac{b\,d^3}{12} = \frac{2\times 3^3}{12} = 4.5\ m^4$,

$\sin\theta = \frac{h_c}{y_c}$, $\sin 30 = \frac{2.25}{y_c}$, $y_c = 4.5\ m$

$y_R = \frac{4.5}{6\times 4.5} + 4.5 = \mathbf{4.6667\ m}$, $h_R = y_R \sin 30 = 2.333\ m$



Example (4): *The 4-m-diameter circular gate of Fig. a is located in the inclined wall of a large reservoir containing water ($\gamma = 9.8\ kN/m^3$). The gate is mounted on a shaft along its horizontal diameter, and the water depth is 10 m above the shaft. Determine **(a)** the magnitude and location of the resultant force exerted on the gate by the water and **(b)** the moment that would have to be applied to the shaft to open the gate.*

Solution:

(a) To find the magnitude of the force of the water

$$F_R = \gamma h_c A$$

and since the vertical distance from the fluid surface to the centroid of the area is 10 m, it follows that

$$F_R = (9.80\times 10^3\ \mathrm{N/m^3})(10\ \mathrm{m})(4\pi\ \mathrm{m^2})$$
$$= 1230\times 10^3\ \mathrm{N} = 1.23\ \mathrm{MN} \qquad \textbf{(Ans)}$$

To locate the point 1center of pressure:

$y_R = \frac{I_{xc}}{y_c A} + y_c$ Where $I_{xc} = \frac{\pi R^4}{4}$

and y_c is shown in Fig. b. Thus,

$$y_R = \frac{(\pi/4)(2\text{ m})^4}{(10\text{ m}/\sin 60°)(4\pi\text{ m}^2)} + \frac{10\text{ m}}{\sin 60°}$$
$$= 0.0866\text{ m} + 11.55\text{ m} = 11.6\text{ m}$$

And the distance 1along the gate2 below the shaft to the center of pressure is

$y_R - y_c = 0.0866\text{ m}$ (Ans)

(b) The moment required to open the gate can be obtained with the aid of the free-body diagram of Fig. *c*

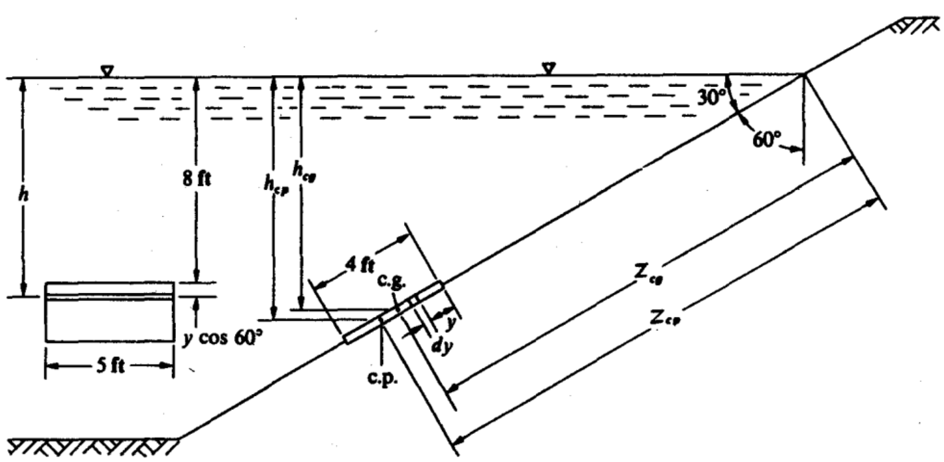
$$\sum M_c = 0$$

$$M = F_R(y_R - y_c)$$
$$= (1230 \times 10^3\text{ N})(0.0866\text{ m})$$
$$= 1.07 \times 10^5\text{ N}\cdot\text{m}$$ (Ans)

Example (5): *An inclined, rectangular gate with water on one side is shown in Fig. Determine the total resultant force acting on the gate and the location of the center of pressure.* $\gamma = 62.4\, lb/ft^3$

$$F = \gamma h_{cg} A = (62.4)[8 + \tfrac{1}{2}(4\cos 60°)][(4)(5)] = 11\,230\text{ lb}$$

$$z_{cp} = z_{cg} + \frac{I_{cg}}{z_{cg}A} = \left(\frac{8}{\cos 60°} + \frac{4}{2}\right) + \frac{(5)(4)^3/12}{(8/\cos 60° + \frac{4}{2})[(4)(5)]} = 18.07\text{ ft}$$



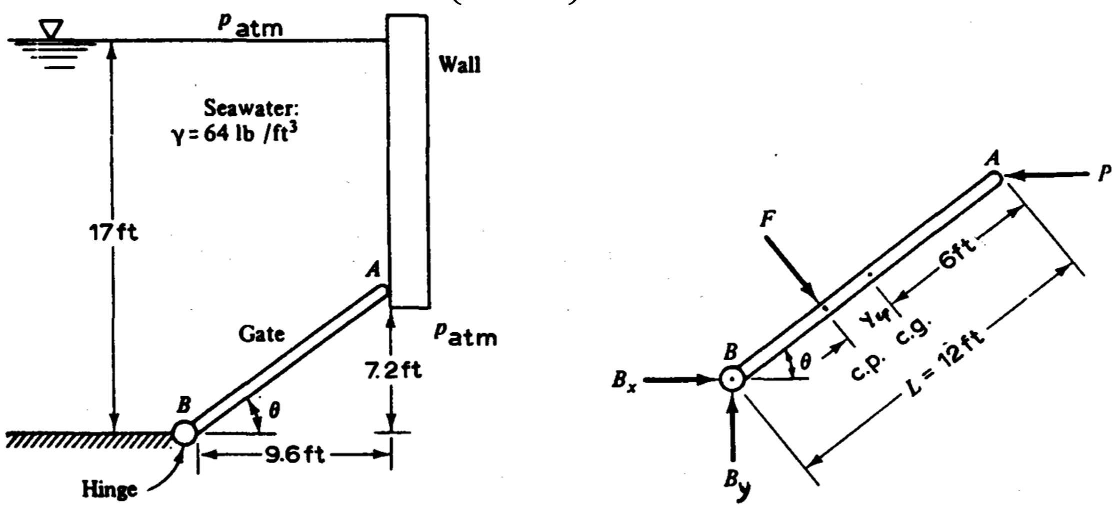
Example (6): *The gate in Fig. is 4 ft wide, is hinged at point B, and rests against a smooth wall at A. Compute (a) the force on the gate due to seawater pressure, (b) the (horizontal) force P exerted by the wall at point A, and (c) the reaction at hinge B.*

Solution:

a) $F = \gamma\, A\, h_c$

$$h_c = Depth\ of\ C.G.from\ free\ water\ surface = 17 - \frac{h}{2} = 17 - \frac{7.2}{2} = 13.4\ ft$$

$$F = 64 \times (4 \times 12) \times 13.4 = \mathbf{30106\ Ib}$$



b) $$y_R = \frac{I_{xc}}{y_c A} + y_c$$

Where $I_{xc} = \frac{b\,d^3}{12} = \frac{4\times12^3}{12} = 576\ ft^4$

$\sin\theta = \frac{h_c}{y_c}$, $\frac{7.2}{12} = \frac{13.4}{y_c}$

$y_c = 22.333\ ft$

$$y_{CP} = y_R = \frac{576}{(4 \times 12) \times 22.333} + 22.333 = 22.87\ ft$$

$L_P = y_R - y_c = 0.537\ ft$

$\sum M_B = 0$, $P \times 7.2 - F\,(12 - 6 - 0.537) = 0$

$\boldsymbol{P = 22843\ Ib}$

c)

$\sum F_x = 0$, $B_x + F\ \sin\theta - P = 0$

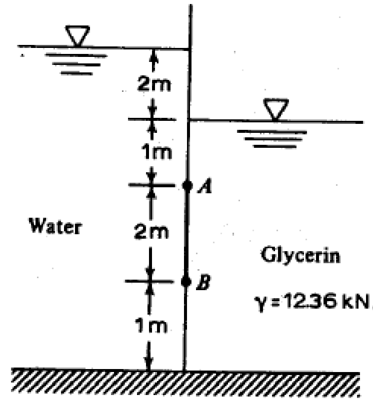
$B_x + 30106 \times \left(\frac{7.2}{12}\right) - 22843 = 0$, $\boldsymbol{B_x = 4779\ Ib}$

$\sum F_y = 0$, $B_y - F\ \cos\theta = 0$

$B_y + 30106 \times \left(\frac{9.6}{12}\right) = 0$, $\boldsymbol{B_y = 24085\ Ib}$

Example (7): *Find the net hydrostatic force per unit width on rectangular panel AB in Fig. and determine its line of action.*

Solution: $F = \gamma\, A\, h_c$



$$F_{H_2O} = (9.79)(2 + 1 + \tfrac{2}{2})[(2)(1)] = 78.32\ \text{kN}$$

$$F_{glyc} = (12.36)(1 + \tfrac{2}{2})[(2)(1)] = 49.44\ \text{kN}$$

$$F_{net} = F_{H_2O} - F_{glyc} = 78.32 - 49.44 = 28.88\ \text{kN}$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

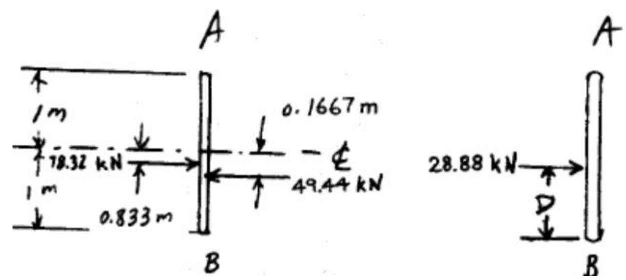
$$I_{xc} = \frac{b\,d^3}{12} = \frac{1 \times 2^3}{12} = 0.6667\ m^4$$

$$y_{Rw} = \frac{0.6667}{(1 \times 2) \times 4} + 4 = 4.0833\ m$$

$$L_{Pw} = y_{Rw} - y_{cw} = 0.0833\ m$$

$$y_{RG} = \frac{0.6667}{(1 \times 2) \times 2} + 2 = 2.1667\ m$$

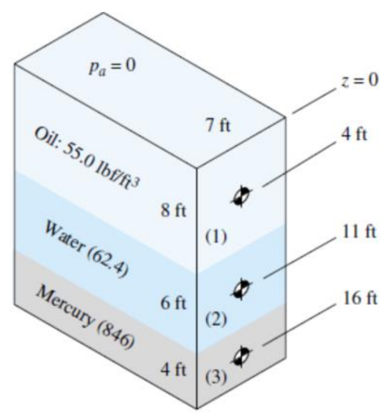
$$L_{PG} = y_{RG} - y_{CG} = 0.1667\ m$$



$$\sum M_B = 0 \qquad (78.32)(1 - 0.0833) - (49.44)(1 - 0.1667) = 28.88D$$

D = 1.059 m (above point *B*, as shown in Fig.)

Example (8): *A tank 20 ft deep and 7 ft wide is layered with 8 ft of oil, 6 ft of water, and 4 ft of mercury. Compute (a) the total hydrostatic force and (b) the resultant center of pressure of the fluid on the right-hand side of the tank.*



Solution: Find the hydrostatic pressure at the centroid of each part,

$$p_{CG_1} = (55.0\ \text{lbf/ft}^3)(4\ \text{ft}) = 220\ \text{lbf/ft}^2$$

$$p_{CG_2} = (55.0)(8) + 62.4(3) = 627\ \text{lbf/ft}^2$$

$$p_{CG_3} = (55.0)(8) + 62.4(6) + 846(2) = 2506\ \text{lbf/ft}^2$$

These pressures are then multiplied by the respective panel areas to find the force on each portion:

$$F_1 = p_{CG_1}A_1 = (220\ \text{lbf/ft}^2)(8\ \text{ft})(7\ \text{ft}) = 12{,}300\ \text{lbf}$$

$$F_2 = p_{CG_2}A_2 = 627(6)(7) = 26{,}300\ \text{lbf}$$

$$F_3 = p_{CG_3}A_3 = 2506(4)(7) = \underline{70{,}200\ \text{lbf}}$$

$$F = \sum F_i = 108{,}800\ \text{lbf} \qquad \textit{Ans. (a)}$$

(b) y_R equation can be used to locate the CP of each force Fi, noting that $\theta = 90°$ and sin 90° = 1 for all parts. The moments of inertia are
$I_{xx_1} = (7\ \text{ft})(8\ \text{ft})^3/12 = 298.7\ \text{ft}^4$, $I_{xx_2} = 7(6)^3/12 = 126.0\ \text{ft}^4$
and $I_{xx_3} = 7(4)^3/12 = 37.3\ \text{ft}^4$
The centers of pressure are thus

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$y_{R1} = \frac{298.7}{(7 \times 8) \times 4} + 4 \ = 5.33\ ft$$

$$y_{R2} = \frac{126}{(7 \times 6) \times 11} + 11 \ = 11.3\ ft$$

$$y_{R3} = \frac{37.3}{(7 \times 4) \times 16} + 16 \ = 16.1\ ft$$

Summing moments about the surface then gives

$$\sum F_i y_{Ri} = F\ y_R$$

$$12300(5.33) + 26300(11.3) + 70200(16.1) = 108800 y_R$$

$$y_R = 13.8\ ft$$

4- Hydrostatic Forces on Curved Surfaces

For example, consider a curved portion of the swimming pool shown in Fig. *5a*. We wish to find the resultant fluid force acting on section *BC* (which has a unit length perpendicular to the plane of the paper). The resultant pressure force on a curved surface is most easily computed by separating it into horizontal and vertical components.
The free-body diagram for this volume is shown in Fig. *5b*. The magnitude and location of forces F_1 and F_2 can be determined from the relationships for planar surfaces. The weight W, is simply the specific weight of the fluid times the enclosed volume and acts through

the center of gravity (CG) of the mass of fluid contained within the volume. The forces F_V and F_H represent the components of the force that the tank *exerts on the fluid*. In order for this force system to be in equilibrium,

$$F_H = F_2$$
$$F_V = F_1 + \mathcal{W}$$

and the magnitude of the resultant is obtained from the equation

$$F_R = \sqrt{(F_H)^2 + (F_V)^2}$$

The resultant F_R passes through the point O, which can be located by summing moments about an appropriate axis. Or to find direction $\tan\theta = \frac{F_H}{F_V}$

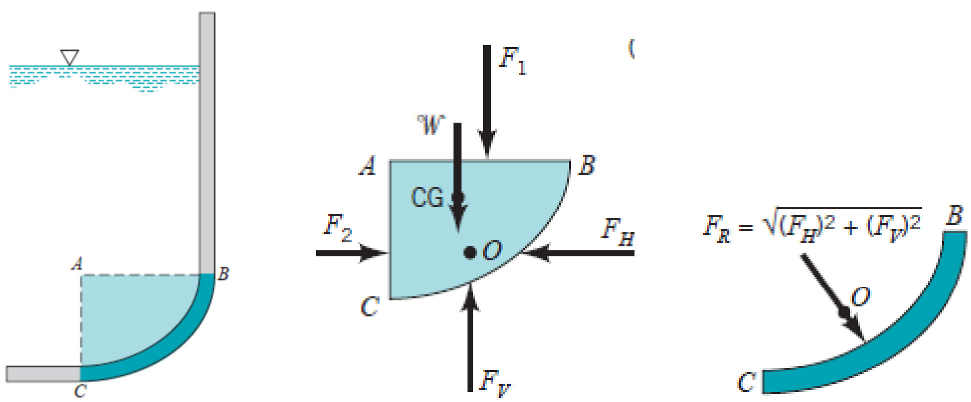
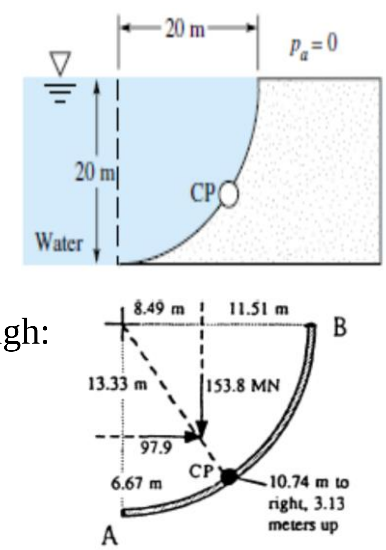


Fig. 5. Hydrostatic force on a curved surface.

Example (9): The dam in Fig. is a quarter circle 50 m wide into the paper. Determine the horizontal and vertical components of the hydrostatic force against the dam and the point CP where the resultant strikes the dam.



Solution:

The horizontal force acts as if the dam were vertical and 20 m high:

$$F_H = \gamma h_{CG} A_{vert}$$
$$= (9790\ \text{N/m}^3)(10\ \text{m})(20\times 50\ \text{m}^2)$$
$$= \mathbf{97.9\ MN} \quad \textit{Ans.}$$

This force acts 2/3 of the way down or 13.33 m from the surface, as in the figure at right.

The vertical force is the weight of the fluid above the dam:

$$F_V = \gamma(\text{Vol})_{dam} = (9790\ \text{N/m}^3)\frac{\pi}{4}(20\ \text{m})^2(50\ \text{m}) = \mathbf{153.8\ MN} \quad \textit{Ans.}$$

This vertical component acts through the centroid of the water above the dam, or $4R/3\pi = 4(20\ \text{m})/3\pi = 8.49$ m to the right of point A, as shown in the figure.

The resultant hydrostatic force is

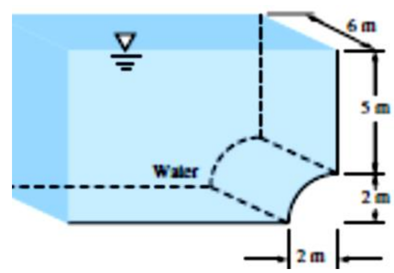
$$= [(97.9\ \text{MN})^2 + (153.8\ \text{MN})^2]^{1/2} = \mathbf{182.3\ MN}$$

acting down at an angle of $\boldsymbol{\theta}$ from the vertical.

$$\theta = \tan^{-1}\left(\frac{97.9}{182.3}\right) = \mathbf{32.5}°$$

The line of action of F strikes the circular-arc dam AB at the center of pressure CP.

Example (10): *Compute the horizontal and vertical components of the hydrostatic force on the quarter- circle panel at the bottom of the water tank in Fig.*



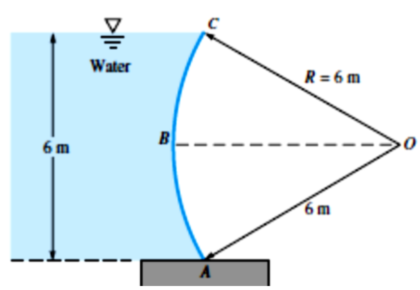
Solution: The horizontal component is

$$F_H = \gamma h_{CG} A_{vert} = (9790)(6)(2\times6)$$
$$= \mathbf{705000\ N} \quad \textit{Ans.}\ (a)$$

The vertical component is the weight of the fluid above the quarter-circle panel:

$$F_V = W(2 \text{ by } 7 \text{ rectangle}) - W(\text{quarter-circle})$$
$$= (9790)(2\times7\times6) - (9790)(\pi/4)(2)^2(6)$$
$$= 822360 - 184537 = \mathbf{638000\ N} \quad \textit{Ans.}\ (b)$$

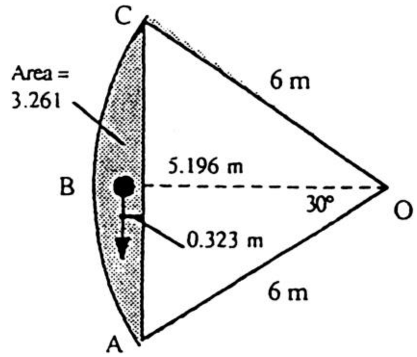
Example (11): *Circular-arc Tainter gate ABC pivots about point O. For the position shown, determine (a) the hydrostatic force on the gate (per meter of width into the paper); and (b) its line of action. Does the force pass through point O?*



Solution: The horizontal hydrostatic force is based on vertical projection:

$$F_H = \gamma h_{CG} A_{vert} = (9790)(3)(6\times1) = 176220\ \text{N}$$

The vertical force is *upward* and equal to the weight of the missing water in the segment ABC shown shaded below.



A (ABC) = (Area (ABCO)) – (Area (ACO))

$$A\ (ABC) = \frac{60}{360} \times \pi \times 6^2 - \left(2 \times \frac{1}{2} \times 3 \times 6\cos 30\right)$$

$= 3.262\ m^2$

The vertical (upward) hydrostatic force on gate ABC is thus

$$F_V = \gamma A_{ABC}(\text{unit width}) = (9790)(3.2611)$$

$\mathbf{F_V = 31926\ N}$

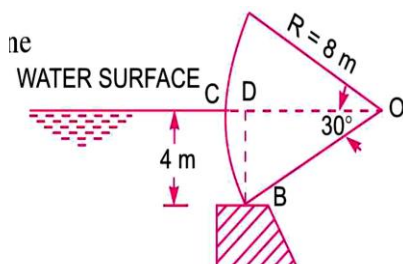
The net force is thus $F = [F_H^2 + F_V^2]^{1/2} = \mathbf{179100\ N}$

Now find direction of action that is angle Ɵ:

tan Ɵ = $\mathbf{R_v}$ / $\mathbf{R_h}$ = 31926/176220 = 0.181→ **Ɵ =10.27°**

<u>Example (12):</u> *Calculate the horizontal and vertical components of water pressure exerted on a tainter gate of radius 8 m as shown in fig. take width of gate unity.*

<u>Solution:</u> The horizontal hydrostatic force is based on vertical projection:



$$F_x = \rho g A\bar{h} = \text{Force on the area projected on vertical plan}$$
$$= \text{Force on the vertical area of } BD$$

where $A = BD \times \text{Width of gate} = 4.0 \times 1 = 4.0$ m

$$\bar{h} = \frac{1}{2} \times 4 = 2 \text{ m}$$

$F_x = 1000 \times 9.81 \times 4.0 \times 2.0 = \mathbf{78480\ N.}$

Vertical component of water pressure is given by

F_y = Weight of water supported or enclosed (imaginary) by curved surface CB

= Weight of water in the portion $CBDC$

= $\rho g \times$ [Area of portion $CBDC$] $\times$ Width of gate

= $\rho g \times$ [Area of sector CBO – Area of the triangle BOD] $\times$ 1

$$= 1000 \times 9.81 \times \left[\frac{30}{360} \times \pi R^2 - \frac{BD \times DO}{2}\right]$$

$$= 9810 \times \left[\frac{1}{12}\pi \times 8^2 - \frac{4.0 \times 8.8 \cos 30^\circ}{2}\right]$$

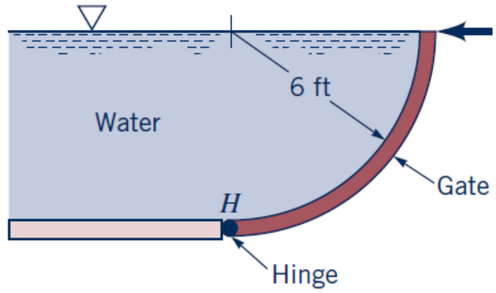
$$\{\because DO = BO \cos 30^\circ = 8 \times \cos 30^\circ\}$$

$= 9810 \times [16.755 - 13.856] =$ **28439 N. Ans.**

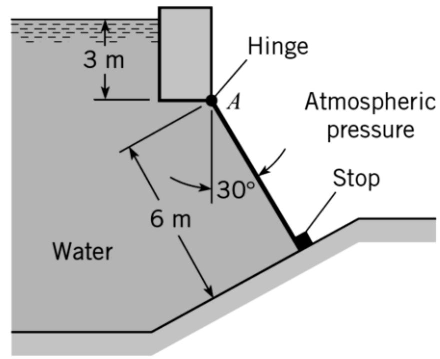
References

1. Bansal, RK. ***A Textbook of Fluid Mechanics and Hydraulic Machines*: **Laxmi publications, 2004.
2. Gerhart, Philip M, Andrew L Gerhart, and John I Hochstein. ***Fundamentals of Fluid Mechanics*: **John Wiley & Sons, 2016.
3. White, Frank M. ***Fluid Mechanics***. Fourth ed.: McGraw-Hill.

Homeworks



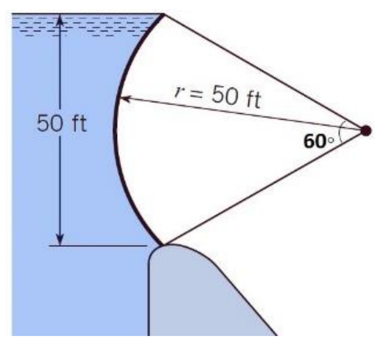
1. 14 ft long gate as shown in Fig.is a quarter circle and is hinged at H. Neglecting atmospheric pressure, compute the hydrostatic resultant force on quarter-circle and its direction.



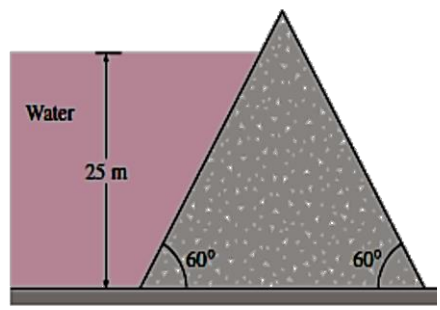
2. The rectangular gat has dimensions 6 m by 2 m. Neglect the weight of the gate. Determine

(a) The magnitude and location of the resultant force exerted on the gate by the water.

(b) and The reaction at point A.



3. The 30 ft wide plate in the form of a quarter-circular arc is used as a sluice gate. (γ_w=62.4 lbf/ft^3) Determine: The magnitude of the resultant force of the water on the gate, and the direction of the resultant force.



4. Water in a 25 m deep reservoir is kept inside by a 150m wide wall whose cross section is an equilateral triangle, as shown in Figure. Determine (a) the total force (hydrostatic - atmospheric) acting on the inner surface of the wall. (P_{atm} = 100kPa) and the location of pressure.

Power Mechanics Technical Engineering Department

Second Stage

Fluid Mechanics - Static

<u>Chapter Four: Buoyancy Forces</u>

M.Sc Zahraa A. Nadeem

1. Introduction

In this lecture the forces due to fluid on floating and submerged bodies is discussed. It is applicable in the design of boats, ships, balloons and other such similar systems. If an object is immersed in or floated on the surface of fluid under static conditions a force acts on it due to the fluid pressure. This force is called **buoyant force**. The calculation of this force is based on Archimedes principle.

Archimedes principle can be stated as

1) **a body immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced and**
2) **a floating body displaces its own weight of the liquid in which it floats.**

The resultant pressure force acting on the surface of a volume partially or completely surrounded by one or more fluids under non flow conditions is defined as ***buoyant force*** **and acts vertically on the volume.** The buoyant force is equal to the weight of the displaced fluid and **acts upwards through the center of gravity of the displaced fluid.** This point is called ***the center of buoyancy*** for the body.

2. Buoyancy Forces

Archimedes principle are easily derived by referring to Fig. 1. In Fig. 1*a*, the body lies between an upper curved surface 1 and a lower curved surface 2. For vertical force, the body experiences a net upward force

$$F_B = F_V(2) - F_V(1)$$
$$= (\text{fluid weight above 2}) - (\text{fluid weight above 1})$$
$$= \text{weight of fluid equivalent to body volume}$$

Alternatively, from Fig. 1b, we can sum the vertical forces on elemental vertical slices through the immersed body:

$$F_B = \int_{\text{body}} (p_2 - p_1)\, dA_H = -\gamma \int (z_2 - z_1)\, dA_H = (\gamma)(\text{body volume})$$

These are identical results and equivalent to Archimedes' law 1 above.

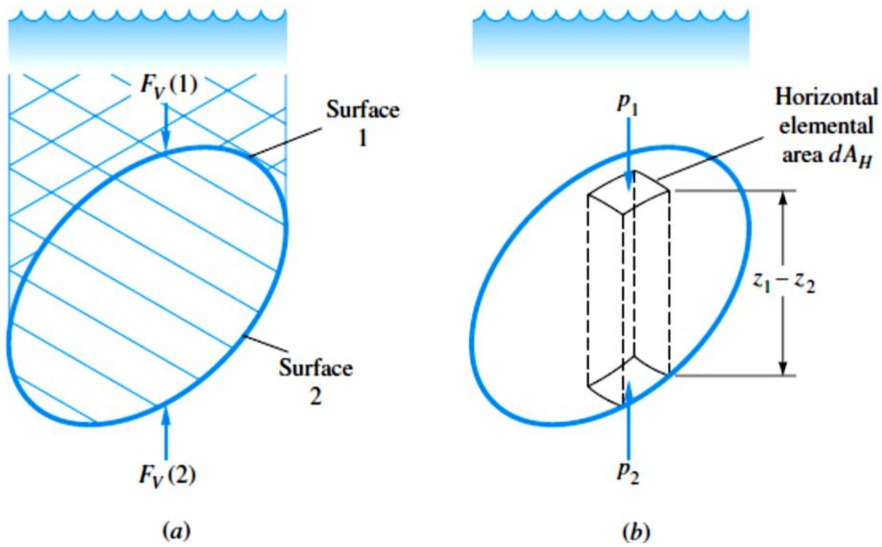


Fig. 1. Two different approaches to the buoyant force on an arbitrary immersed body.

The line of action of the buoyant force passes through the center of volume of the displaced body. This point through which F_B acts is called the *center of buoyancy*, commonly labeled *C* or *CB* on a drawing.

Floating bodies are a special case; only a portion of the body is submerged, with the remainder poking up out of the free surface. This is illustrated in Fig. 2, where the shaded portion is the displaced volume. The equation is modified to apply to this smaller volume is

$$F_B = (\gamma)(\text{displaced volume}) = \text{floating-body weight}$$

This equation is the mathematical equivalent of Archimedes' law 2, previously stated.

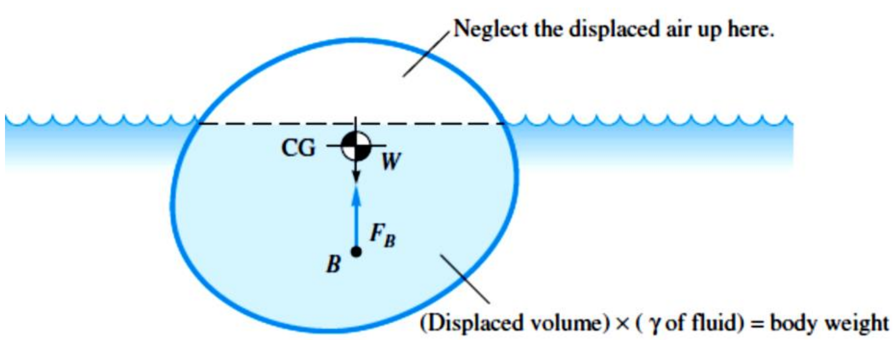


Fig. 2. Static equilibrium of a floating body.

Example (1): *A cylinder of diameter 0.3 m and height 0.6 m stays afloat vertically in water at a depth of 1 m from the free surface to the top surface of the cylinder. Determine the buoyant force on the cylinder.*

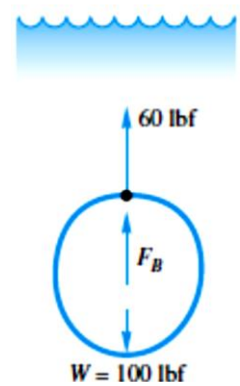
Sol.

Buoyant force = Weight of water displaced = $\rho\ g\ V$

$$= \left(\pi \times \frac{0.3^2}{4}\right) \times 0.6 \times 1000 \times 9.81 = 416.06\ \text{N}$$

Example (2): *A block of concrete weighs 100 lbf in air and "weighs'' only 60 lbf when immersed in fresh water (62.4 lbf/ft^3). What is the average specific weight of the block?*

Solution: A free-body diagram of the submerged block (see Fig.) shows a balance between the apparent weight, the buoyant force, and the actual weight



$$\sum F_z = 0 = 60 + F_B - 100$$

$$F_B = 40\ \text{lbf} = (62.4\ \text{lbf/ft}^3)(\text{block volume, ft}^3)$$

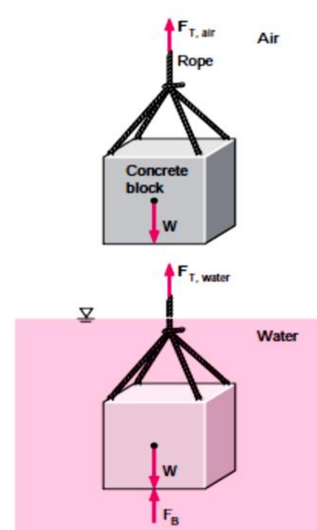
$$V_{block} = \frac{40}{62.4} = 0.641\ ft^3$$

Therefore the specific weight of the block is

$$W_{block} = (\gamma\ V)_{block}$$

$$\gamma_{block} = \frac{100}{0.641} = 156\ lbf/ft^3$$

Example (3): *A crane is used to lower weights into the sea (density = 1025 kg/m^3) for an underwater construction project (Fig. below). Determine the tension in the rope of the crane due to a rectangular 0.4 m × 0.4 m × 3 m concrete block (density = 2300 kg/m^3) when it is (a) suspended in the air and (b) completely immersed in water.*



Solution:

(a) Consider the free-body diagram of the concrete block. Two forces must balance each other, and thus the tension

in the rope must be equal to the weight of the block:

$$V = (0.4\ \text{m})(0.4\ \text{m})(3\ \text{m}) = 0.48\ \text{m}^3$$

$$F_{T,\,air} = W = \rho_{concrete}\,gV$$

$$= (2300\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.48\ \text{m}^3)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 10.8\ \text{kN}$$

(*b*) When the block is immersed in water, there is the additional force of buoyancy acting upward. The force balance in this case gives

$$F_B = \rho_f\,gV = (1025\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.48\ \text{m}^3)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 4.8\ \text{kN}$$

$$F_{T,\,water} = W - F_B = 10.8 - 4.8 = 6.0\ \text{kN}$$

Example (4): *A hollow cube 1.0 m on each side weighs 2.4 kN. The cube is tied to a solid concrete block weighing 10.0 kN. Will these two objects tied together float or sink in water?* $\gamma_{block} = 23496\ N/m^3$.

Solution:

Let W = weight of hollow cube plus solid concrete block, F_{b1} = buoyant force on hollow cube, and F_{b2}= buoyant force on solid concrete block.

$W = 2.4 + 10.0 = 12.4$ kN,

$F_{b1} = (\rho g)_{fluid} V_{body} = 1000 \times 9.79 \times 1^3$= 9.79 kN,

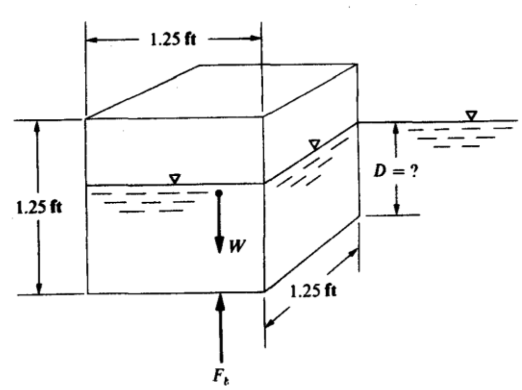
$$V_{block} = \frac{W_{block}}{\gamma_{block}} = \frac{10 \times 1000}{23496} = 0.4256\ m^3$$

$$F_{b2} = (\rho g)_{fluid} V_{block} = 1000 \times 9.79 \times 0.4256 = 4.17\ kN$$

$$F_{b2} + F_{b1} = 9.79 + 4.17 = 13.96\ kN$$

Since $[W = 12.4] < [F_{b2} + F_{b1} = 13.96\ kN]$ the two objects tied together will float in water.

Example (5): *A cube of timber 1.25 ft on each side floats in water as shown in Fig. The specific gravity of the timber is 0.60. Find the submerged depth of the cube.*



Solution:

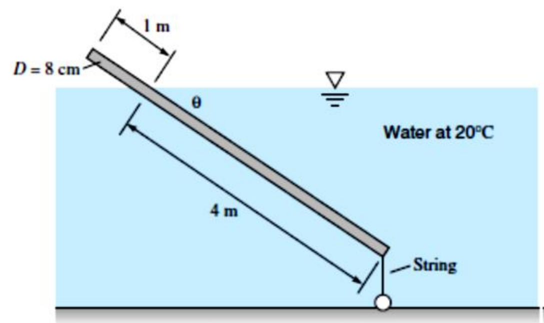
$F_b = W$

$\gamma_{fluid} V_{body} = \gamma_{body} V_{body}$

$62.4 \times (1.25 \times 1.25 \times D) = 0.6 \times 62.4 \times 1.25^3$

$D = 0.75\ ft$

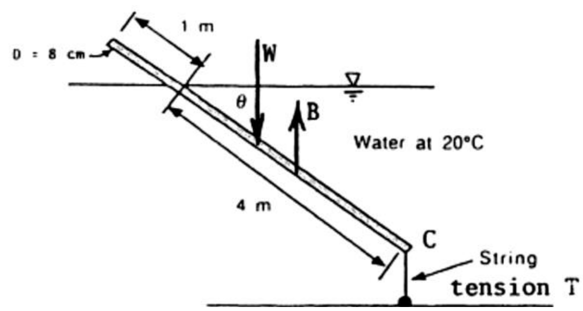
Example (6): *The uniform 5-m-long wooden rod in the figure is tied to the bottom by a string. Determine (a) the string tension; and (b) the specific gravity of the wood. Is it also possible to determine the inclination angle θ?*



Solution:

The rod weight acts at the middle, 2.5 m from point C, while the buoyancy is 2 m from C.

Summing moments about C gives



$$\sum M_C = 0 = W(2.5\sin\theta) - B(2.0\sin\theta), \quad \text{or} \quad W = 0.8B$$

$$B = \gamma_{fluid}\, V_{body} = 9790 \times \frac{\pi 0.08^2}{4} \times 4 = 196.8N$$

$$W = 0.8B = 0.8 \times 196.8 = 157.5\ N$$

$$W = S.G.\gamma_{water} V_{body} = S.G. \times 9790 \times \frac{\pi 0.08^2}{4} \times 5$$

$$S.G. = \mathbf{0.64}$$

Summation of vertical forces yields

String tension T = B−W = 196.8 −157.5 = **39.3 N**

These results are independent of the angle θ, which cancels out of the moment balance.

3. Stability

A body is said to be in a *stable equilibrium* position if, when displaced, it returns to its equilibrium position. Conversely, it is in an *unstable equilibrium* position if, when displaced (even slightly), it moves to a new equilibrium position. Stability considerations are great importance in the design of ships, submarines, bathyscaphes, and so forth.

The stability of submerged or floating bodies is strongly related to the position of the center of gravity with respect to the position of the center of buoyancy. For the *completely submerged* body has a center of gravity below the center of buoyancy. *Floating* bodies are more sensitive to stability then submerged bodies, the reason is that the center of buoyancy may take different positions with respect to the center of gravity.

Stability of completely submerged body:

The location of the center of gravity (CG) in regard to the center of buoyancy (c) is very important. Fig. 3, which has a center of gravity below the center of buoyancy, a rotation from its equilibrium position will create a *restoring* couple formed by the weight, W, and the buoyant force, F_B, which causes the body to rotate back to its original position. Thus, for this configuration the body is stable. It is to be noted that as long as the center of gravity falls ***below*** the center of buoyancy, this will always be true; that is, the body is in a *stable equilibrium* position with respect to small rotations.

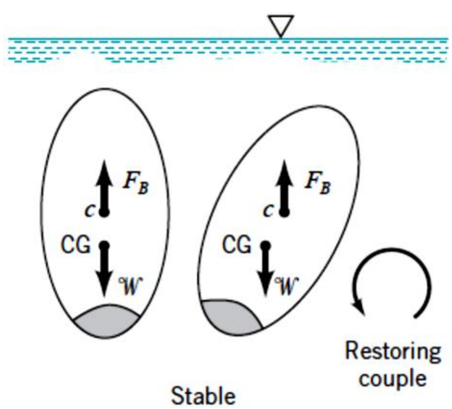


Fig.3. Stability of a completely immersed body—center of gravity below centroid.

In Fig. 4, if the center of gravity of the completely submerged body is above the center of buoyancy. Any minimal force strike, the resulting couple formed by the weight and the buoyant force will cause the body to *overturn* and move to a new equilibrium position. Thus, a completely submerged body with its center of gravity ***above*** its center of buoyancy is in an *unstable equilibrium* position.

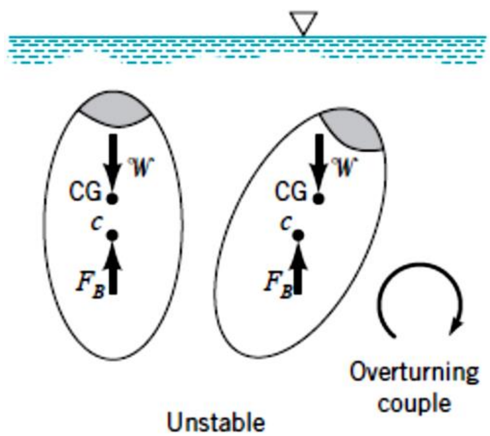


Fig.4. Stability of a completely immersed body—center of gravity above centroid.

A third condition, if the center of buoyancy and center of gravity are coincident. The body **neutrally** stable, that is, has the tendency for neither restoring nor overturning.

Stability of floating bodies:

The stability problem is more complicated and different from submerged bodies. A floating body will overturn at the first opportunity and is said to be statically *unstable*. The least disturbance will cause it to seek another equilibrium position which is stable. Engineers must design to avoid floating instability. Fig.5. illustrates the computation for the usual case of a symmetric floating body. The steps are as follows:

1. The basic floating position is calculated from $F_b = \gamma_{fluid} V$. The body's center of gravity *G* and center of buoyancy *B* are computed.
2. The body is tilted a small angle$\Delta\theta$, and a new waterline is established for the body to float at this angle. The new position $\acute{B}$ of the center of buoyancy is calculated. A vertical line drawn upward from $\acute{B}$ intersects the line of symmetry at a point *M*, called the *metacenter*, which is independent of $\Delta\theta$ for small angles.
3. If point *M* is above *G*, that is, if the *metacentric height* $\overline{MG}$ is positive, a restoring moment is present and the original position is stable. If *M* is below *G* (negative $\overline{MG}$,

the body is unstable and will overturn if disturbed. Stability increases with increasing $\overline{MG}$.

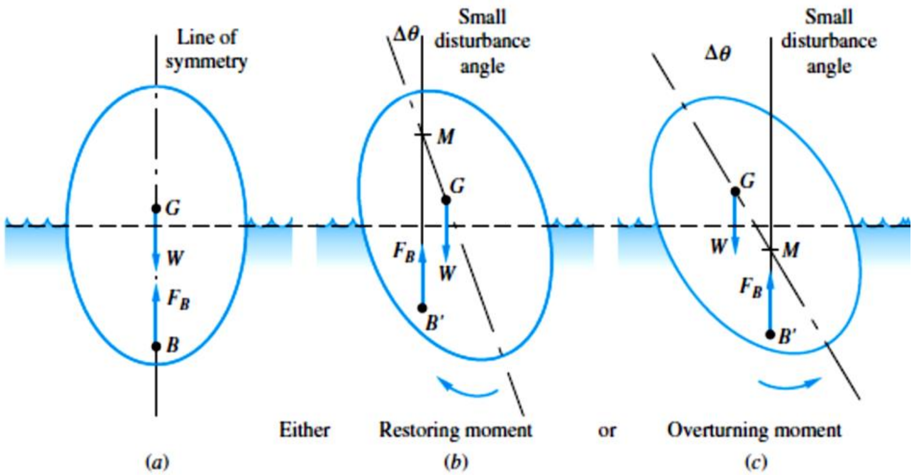


Fig. 5. Calculation of the metacenter M of the floating body.

Thus the metacentric height is a property of the cross section for the given weight, and its value gives an indication of the stability of the body. For a body of varying cross section and draft, such as a ship, the computation of the metacenter can be very involved.

$$\overline{MG} = \frac{I_0}{V} - \overline{GB}$$

I_0= moment of inertia of the area.

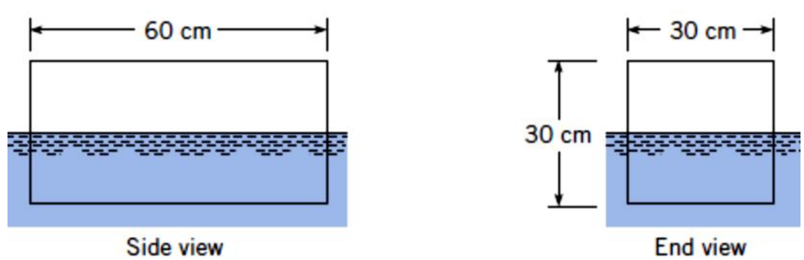
$$I_{0_{reg.}} = \frac{bd^3}{12} \quad , \quad I_{0_{circ.}} = \frac{\pi R^4}{4}$$

V = Volume of displaced fluid.

$\overline{GB}$ = The distance from center of gravity (G) to center of buoyancy B at equilibrium.

Note that if $\overline{GB}$ is negative, that is, B is *above* G, the body is always stable.

<u>Example (7):</u> *A block of wood 30 cm square in cross section and 60 cm long weighs 318 N. Will the block float with sides vertical?*



Solution:

1. Equilibrium (vertical direction)

$$\sum F_y = 0$$

$$-\text{weight} + \text{buoyant force} = 0$$

$$-318\ \text{N} + 9810\ \text{N/m}^3 \times 0.30\ \text{m} \times 0.60\ \text{m} \times d = 0$$

$$d = 0.18\ \text{m} = 18\ \text{cm}$$

2. Stability (longitudinal axis)

$$\overline{MG} = \frac{I_0}{V} - \overline{GB}$$

$$I_0 = \frac{bd^3}{12} = \frac{60 \times 30^3}{12} = 135000\ cm^4$$

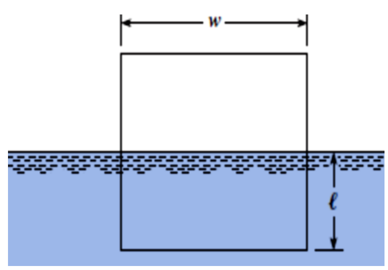
$$V = a \times b \times d = 18 \times 60 \times 30 = 32400\ cm^3$$

$$\overline{GB} = \frac{30}{2} - \frac{18}{2} = 15 - 9 = 6\ cm$$

$$\overline{MG} = \frac{135000}{32400} - 6 = -1.833\ cm$$

Because the metacentric height is negative, the block is not stable about the longitudinal axis.

Example (8): *A floating body has a square cross section with side w as shown in the figure. The center of gravity is at the centroid of the cross section. Find the location of the water line, l/w, where the body would be neutrally stable ($\overline{MG}$=0).*



Solution:

For neutral stability, the distance to the metacenter is zero. In other words

$$\overline{MG} = \frac{I_0}{V} - \overline{GB} = 0$$

where $\overline{GB}$ is the distance from the center of gravity to the center of buoyancy.

Moment of inertia at the waterline

$$I_0 = \frac{Lw^3}{12}$$

where L is the length of the body. The volume of liquid displaced is lwL so

$$\overline{GB} = \frac{Lw^3}{12lwL} = \frac{w^2}{12l}$$

The value for $\overline{GB}$ is the distance from the center of buoyancy to the center of gravity,
Or

$$\overline{GB} = \frac{w}{2} - \frac{l}{2}$$

$$\frac{w^2}{12l} = \frac{w}{2} - \frac{l}{2}$$

$$6(wl - l^2) = w^2 \quad \rightarrow \quad 6\left(\frac{l}{w} - \frac{l^2}{w^2}\right) = 1$$

$$\frac{l^2}{w^2} - \frac{l}{w} + \frac{1}{6} = 0$$

$$\frac{l}{w} = 0.211$$

Example (9): A cylindrical block of wood 1 m in diameter and 1 m long has a specific weight of 7500 N/m^3. Will it float in water with its axis vertical?

Solution:

$$W - F_B = 0 \quad \rightarrow \quad \gamma_b V_b = \gamma_f V_f$$

$$V_f = \frac{7500 \times (\pi \times 0.5^2 \times 1)}{9810} = 0.6\ m^3$$

$$V_f = \pi R^2 d = 0.6\ m^3 \rightarrow d = 0.7645\ m$$

$$B = \frac{0.7645}{2} = 0.3823\ m\ , \quad G = 0.5\ m$$

$$\overline{GB} = 0.5 - 0.3823 = 0.1177\ m$$

$$I_0 = \frac{\pi R^4}{4} = 0.05\ m^4$$

$$\overline{MG} = \frac{I_0}{V} - \overline{GB} = \frac{0.05}{0.6} - 0.1177 = -0.034$$

Thus, block is unstable with axis vertical.

References:

[1] F. M. White, *Fluid Mechanics*, Fourth ed.: McGraw-Hill.
[2] D. F. Elger, J. A. Roberson, B. C. Williams, and C. T. Crowe, *Engineering fluid mechanics* vol. 7: Wiley Hoboken (NJ), 2016.
[3] B. R. Munson, D. F. Young, and T. Okiishi, "Fundamentals of Fluid Mechanics, Six Edition," ed: John Willet & Sons, Inc, 2009.

HOME WORKS

1. A rock weighs 925 N in air and 781 N in water. Find its volume.
2. Is the block in this figure stable floating in the position shown? Show your calculations.